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THE

ASSURANCE MAGAZINE,

AND

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OF THE

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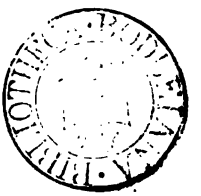

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ERRATA.

Page 220, line 3, for a_z read a_x.

" line 10, for $\frac{9}{9}$ read $\frac{9}{7}$.

Page 234, line 17, for 10,000 read 100,000.

THE

ASSURANCE MAGAZINE,

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INSTITUTE OF ACTUARIES.

New Method for Calculating the Value of an Assurance to the Survivor Nominated, &c. By M. Réboul, *lately of the Paris Observatory, and now Consulting Actuary of the "Impériale" Assurance Company.*

[Read before the Institute the 2nd January, 1860.]

THE following letter from M. Réboul will serve to introduce the several subjects upon which he has addressed the Institute:—

"Gentlemen,—The Institute of Actuaries being the most competent tribunal in matters of assurance, it is to you, as to my natural judges, that I have conceived it proper to submit my first essays in that science.

"The manuscript, which I have the honour to present to you, contains:—

"1st. Five new tables of life annuities.

"2nd. The introduction and plan of a work upon assurance.

"3rd. Note upon a new formula for calculating the assurance of a capital sum to the survivor nominated.

"The first table contains the annuities on a single life, calculated at 5 per cent. from Deparcieux's table of mortality, perfected by the Bureau des Longitudes. For the values of these annuities, are taken the figures common to the results obtained by two completely different methods; this process has permitted the rectifying of some errors, which had slipped in at the end of the table published by Baron Mazères, and which have since been reproduced by every writer quoting that table.

"The four tables which follow contain annuities on two lives, calculated at 4½ per cent., by the same methods and from the same data for mortality, and are designed to complete the repertory of Mazères, and to facilitate interpolations.

"The work, of which I present to you the introduction and plan succinctly, is designed to arouse in France a taste for life assurance, which seems now to slumber; the author is animated by a strong confidence in the usefulness of his undertaking, the success of which appears to him certain, if he have the good fortune to merit your approbation.

"In fine, gentlemen, I particularly call your attention to the method which has conducted me to the formula that I propose for the calculation of a capital sum payable to the survivor nominated, and the exposition of which is more simple than has been the investigation.

"I was connected with the Observatory of Paris during four years; I have applied to the study of astronomy under the auspices of Arago, when I was invited to occupy myself with questions pertaining to assurance. I have found an immense field to explore; and this first memorandum is not, in anywise, other than a specimen of labours which I have undertaken, and which I shall crave permission to submit, should this attempt merit some encouragement at your hands.

"Be pleased to accept, gentlemen, the very respectful expression of my most distinguished esteem.

"E. RÉBOUL.

"*To the Members of the*
"*Institute of Actuaries of London.*"

The annuity tables accompanying M. Réboul's communication are sufficiently described by that gentleman himself. The rate of mortality, and the rates of interest upon which they are based, will, no doubt, be considered by our readers as circumstances affording sufficient reason for not reproducing them here. They appear to have been constructed with great care, and are very neatly executed. The introduction and plan of a work upon assurance consists of a very warm eulogy on the system of assurance in general, and on that of life assurance in particular, and a table of its proposed contents is also appended. A portion of it, which the author has, it seems, already published or sought to publish, and which is, as he says, intended to awaken in France a taste for life assurance, is as follows:—

"DES ASSURANCES SUR LA VIE.

"A qui les assurances sur la vie sont-elles applicables?—A tous.
"A qui sont-elles appliquées?—A personne.

"Les assurances sur la vie, dont les bienfaits ne font pas question pour quiconque a étudié cette branche de l'économie politique, sont passées dans les mœurs en Angleterre, en Allemagne, en Autriche, aux Etats-Unis—c'est-à-dire, dans les trois-quarts du monde civilisé.

"En France elles sont complètement négligées. Le mot seul d'assurance soulève une idée de défiance. Enfin—le dirai-je?—la matière assurée n'est pas la dix-millième partie de la matière assurable. Ce qui ne nous empêche pas de nous croire à la tête de la civilisation.

"Voici une grande vérité: tant pis si elle a l'air d'un paradoxe!

"Il existe, même pour celui qui n'a d'autres ressources que son travail, un moyen sûr, infaillible, à l'abri de toute restriction légale ou fiscale, de

sur la vie, sera dissipée; et cette institution, si utile, sera définitivement naturalisée dans notre pays.

"Il y a dans cette régénération des assurances un travail immense à faire. La matière est inépuisable. Le champ est vaste et inculte. Plus il sera cultivé, plus il produira. Nous ne craignons pas de convier à cette œuvre féconde tous les esprits positifs, qui aiment les affaires sérieuses, morales, satisfaisantes.

"Les bonnes affaires ont en effet ce caractère remarquable d'être bonnes pour tout le monde. Les assurances sont de ce nombre; elles sont bonnes pour les Compagnies et bonnes pour le public.

"Elles sont utiles à l'individu, à qui elles offrent le moyen le plus sûr, en même temps que le plus moral, de faire fructifier ses économies dans un but de prévoyance.

"Elles sont utiles à la société, qu'elles sauvegardent en favorisant l'ordre, qu'elles enrichissent en diminuant le nombre des pauvres. Rappelons-nous, en effet, cette admirable définition de la richesse donnée par G. Varennes, et qu' Achille Guillard appelle avec raison une sanglante critique du pédantisme économique,—'La nation la plus riche est celle qui a le moins de pauvres.'

"Elles sont utiles à l'état, qu'elles aident puissamment dans sa principale, pour ne pas dire dans son unique fonction, qui est, de garantir la sécurité. Car, en arrachant l'homme à l'incertitude du sort, en l'empêchant de devenir la proie du hasard, l'assurance augmente dans une proportion énorme la sécurité, qui est le premier besoin d'une société, puisque sans elle il ne peut y avoir ni épargne, ni travail producteur.

"Ne cherchons pas d'autre criterium. Une institution qui satisfait à cette triple condition—d'être utile à l'individu, à la société elle-même, et à l'état—est éminemment morale; elle est inattaquable.

"'Parmi les établissements fondés sur les probabilités de la vie humaine, les meilleurs sont ceux dans lesquels, au moyen d'un léger sacrifice de son revenu, on assure son existence et celle de sa famille pour un temps où l'on doit craindre de ne plus suffire à ses besoins. Autant le jeu est immoral autant ces établissements sont avantageux aux mœurs, en favorisant les plus doux penchants de la nature. Le Gouvernement doit donc les encourager et les respecter dans les vicissitudes de la fortune publique, car les espérances qu'ils représentent, portant sur un avenir éloigné, ils ne peuvent prospérer qu' à l'abri de toute inquiétude sur leur durée.'

"Qui a dit cela? Laplace, à l'Ecole Normale, in 1795. A voir où nous en sommes, on croirait volontiers que c'est lundi dernier à la séance de l'Institut.

"Pour nous, économiquement parlant, l'humanité se partage en deux classes; ceux qui possédent, ceux qui ne possédent pas. Les uns généralement amis de l'ordre, les autres plutôt partisans de la liberté—qui serait bien la plus belle chose du monde, si l'ordre n'était encore plus beau.

"Or, les assurances sur la vie, en procurant la sécurité, en consolidant l'épargne, en constituant des patrimoines, des dotations, des retraites, &c., créent des valeurs nouvelles, multiplient les capitaux et les propriétaires, suivant le vœu émis par Napoléon I., au Conseil d'Etat. En un mot, elles font passer ceux qui ne possèdent pas dans la classe de ceux qui possèdent. Donc elles enrichissent l'Etat; donc elles accroissent l'ordre, ce qui est toujours le vœu du Gouvernement, qui, en retour, leur doit toute sa sollici-

ment, l'assurance de solvabilité, l'amortissement viager; en un mot—l'organisation du crédit par l'assurance; enfin, nous donnerons une théorie générale des transactions viagères.

"Notre but est suffisamment indiquée; quant à notre route, elle est toute tracée—la vérité avant tout. En moral comme en physique le plus court chemin est la ligne droite. C'est celle que nous suivrons. C'est aussi celle que suit la lumière.

"E. RÉBOUL.

"2 *Décembre*, 1858."

We now come to M. Réboul's "New formula for calculating the assurance of a capital sum to the survivor nominated"; and, that there may be no room for any misunderstanding arising out of an erroneous translation, we give as much of the author's statement as is needful in the original:—

"NOUVELLE MÉTHODE POUR CALCULER LA VALEUR D'UNE ASSURANCE AU SURVIVANT DÉSIGNÉ.

"L'assurance d'une somme s'obtient en retranchant de cette somme sa rente servie jusqu'à l'année à la fin de laquelle doit être payée l'assurance; ainsi, qu'il s'agisse de 1 fr. payable au décès de A, r étant le taux d'intérêt, et A l'annuité viagère sur la tête A, le prix de cette assurance sera—

$$\frac{1-Ar}{(1+r)}.$$

"Les formules d'assurances doivent donc en général se déduire de celles des annuités viagères; c'est ce que nous nous sommes proposé de faire pour l'assurance au *survivant désigné*, qui semblait faire exception.

"Supposons qu'il s'agisse de deux têtes A et B.

"Soit x la valeur de l'assurance de 1 fr. payable au décès de A s'il meurt le premier, et y la valeur de l'assurance de 1 fr. payable au décès de A s'il meurt le dernier.

"En ajoutant ces deux valeurs, x et y, on obtient évidemment le prix de l'assurance de 1 fr. payable sans conditions au décès de A; ainsi on a—

$$[1] \quad x+y=\frac{1-Ar}{(1+r)}.$$

"Tel est le principe bien simple sur lequel nous nous appuyons: x est précisément la valeur de l'assurance de 1 fr. payable à B au décès de A, or on tire de l'équation [1],

$$x=\frac{1-Ar}{(1+r)}-y.$$

"Reste à déterminer la valeur de y. Pour y parvenir, supposons que A et B conviennent d'acheter une *assurance payable au dernier décès*, y sera la part de A dans cet achat; or, sans nous préoccuper de la question pratique qui pourrait conduire à chercher la proportion dans laquelle A et B doivent contribuer au paiement de la dite assurance, remarquons que si ce paiement était fait par primes annuelles, la quotité de ces primes serait—

$$[2] \quad q=\frac{1-(A+B-AB)r}{(1+r)(1+A+B-AB)}.$$

"Telle est la rente qui doit être payée d'avance et jusqu'au dernier décès des têtes A et B pour acquitter l'assurance, et la part de A dans une rente de 1 fr. (servie dans ces mêmes conditions) étant $\frac{1}{2}+A-\frac{1}{2}AB$ (*voir* Baily, tome 1, chap. 4, § 84), sa part y dans une rente de q francs sera $y=q\,(\frac{1}{2}+A-\frac{1}{2}AB)$; d'où en remplaçant q par sa valeur [2], et portant y dans l'équation [1], on tire—

$$[3]\quad x=\frac{1-Ar}{(1+r)}-(\tfrac{1}{2}+A-\tfrac{1}{2}AB)\left[\frac{1-(A+B-AB)r}{(1+r)(1+A+B-AB)}\right].$$

"C'est la formule que nous proposons pour le calcul des *assurances au survivant désigné.*

"Si l'on y remplace A par B, et B par A, on aura la valeur de l'assurance de 1 fr. payable à A ou décès B; désignons là par x'. On trouve

$$x'=\frac{1-Br}{(1+r)}-(\tfrac{1}{2}+B-\tfrac{1}{2}AB)\left[\frac{1-(A+B-AB)r}{(1+r)(1+A+B-AB)}\right],$$

x et x' représentent respectivement les parts de B et de A dans l'assurance au survivant quelconque; comme vérification, leur somme $x+x'$ doit donc se réduire à $\frac{1-ABr}{(1+r)}$, valeur de l'assurance de 1 fr. payable au premier décès.

"Avant d'effecteur, remarquons que la quantité [3] étant symétrique par rapport aux lettres A et B, n'a pas changé par leur permutation, et peut être mise en facteur commun, ce qui donnera—

$$x+x'=\frac{1-Ar+1-Br-(1+A+B-AB)\times\dfrac{1-(A+B-AB)r}{(1+A+B-AB)}}{(1+r)},$$

d'où, en supprimant $(1+A+B-AB)$, facteur commun, et développant, on obtient

$$x+x'=\frac{1-Ar+1-Br-1+Ar+Br-ABr}{(1+r)}=\frac{1-ABr}{(1+r)},$$

expression qui en effet se réduit à

$$x+x'=\frac{1-ABr}{(1+r)}.$$

"Les réductions que comporte le développement de la formule [3] ne permettant pas de la rendre plus élégante, nous nous abstiendrons des transformations, et nous la laisserons sous cette forme, qui nous a paru la plus commode pour le calcul.

"Il est important de remarquer que cette formule [3] ne renferme que quatre quantités, A, B, AB, et r, et que la manière dont elles sont combinées entre elles permet d'arriver trés-simplement au résultat, qui même se déduirait de deux opérations seulement, si l'on puisait les valeurs de q et de $\frac{1-Ar}{(1+r)}$ dans les tarifs d'assurances.

"La formule que l'on trouve dans Baily (chap. 8, prob. 27, § 231) est défectueuse et mal commode, car elle ne se prête pas du tout au calcul logarithmique; la méthode employée pour l'obtenir n'est pas rigoureusement

exacte, puisqu'elle s'appuie sur cette hypothése évidemment fausse à savoir: que dans le cas où les deux têtes s'éteignent dans le cours de la même année, l'une en particulier a autant de chances que l'autre de mourir la première; de sorte que la probabilité de cet évènement est représentée par $\frac{1}{2}$ dans tout le cours du calcul; de plus, l'introduction de têtes auxiliaires amène dans la formule quatre quantités nouvelles qui la compliquent en aggravant la probabilité d'erreur.

"Enfin, les huit quantités contenues dans la formule de Baily doivent être déterminés avec une approximation beaucoup plus grande que celle qu'on veut conserver au résultat, puisqu'il s'obtient en retranchant l'une de l'autre deux nombres qui ne diffèrent pas en général par leurs trois premières figures.

"Aussi les vérifications ne réussissent-elles pas à beaucoup près aussi bien que par l'emploi de la formule [3], comme on peut en juger par les comparaisons qui suivent (les calculs sont faits pour 100 fr.):—

	Méthode nouvelle.	Méthode de Baily.
Assurance à B 25 au décès de A 20 .. .. =	18·527	18·691
Assurance à A 20 au décès de B 25 ..... =	20·923	21·438
Somme ..... =	39·450	40·129
Au lieu de (assurance au survivant quelconque) =	39·452	39·452
Erreur ..... =	−0·002	+ 0·677
Assurance à B 60 au décès de A 20 ..... =	9·979	9·142
Assurance à A 20 au décès de B 60 ..... =	51·737	52·223
Somme .... =	61·716	61·365
Au lieu de (assurance au survivant quelconque) =	61·715	61·715
Erreur ..... =	+ 0·001	−0·350

&c. &c. &c.

"*Exemple du calcul de l'Assurance au Survivant désigné de deux têtes, par les deux Méthodes.*

"£100 payables à B 30 au décès de A 25, à 4°/₀₀ et d'après Deparcieux.

"*Méthode de Réboul:—*

$$\text{Formule: } x=\frac{1-Ar}{1+r}-(\tfrac{1}{2}+A-\tfrac{1}{2}AB)\left(\frac{1-(A+B-AB)r}{(1+r)(1+A+B-AB)}\right).$$

"*Méthode indiquée par Baily:—*

$$\text{Formule: } \frac{s}{2}\left\{\frac{1-ABr}{1+r}-\left(\frac{(1+A'B)a'}{1+r}-A'B.a'\right)\frac{1}{a}\right\}."$$

The writer here works out a particular case by each formula, and proceeds:—

"Ici le calcul ne se compose que d'additions et de soustractions, en tout on a huit logarithmes à chercher, tandis que dans la méthode de Baily, il

faut employer de plus la multiplication et la division; car, outre que la formule ne se prête pas au calcul logarithmique, sept figures ne suffisent souvent pas pour les nombres correspondants.

"Ajoutons que, s'il s'agit de construire un tarif entier, notre formule parait bien préferable, puisqu'en l'employant on peut construire en même temps trois tarifs qui ne donnent pas autant de peine qu'un seul construit par la méthode ordinaire.

"Savoir: 1°. Assurance d'un capital payable au premier décès de deux têtes.

"2°. Assurance d'un capital payable au dernier décès de deux têtes.

"3°. Assurance d'un capital payable au survivor désigné de deux personnes."

Such is M. Réboul's statement as to the construction of his new formula, which, if it were true for all values of A and B, would, no doubt, deserve to be looked at with all the partiality with which he evidently regards it; unfortunately, however, this is not the case, the results given by the formula being nothing more than approximate ones, excepting only when the ages of A and B are the same and the values which they represent equal, under which conditions alone the results obtained by it are quite accurate.

Translating M. Réboul's formulæ into a notation with which most of our readers are more familiar, it will be seen that the expressions [1], [2], and [3], are equivalent to the following, viz.:—

$$\underset{1}{\mathfrak{A}\mathfrak{B}}+\underset{2}{\mathfrak{A}\mathfrak{B}}=\mathfrak{A},\quad q=p,\quad \text{and } \underset{1}{\mathfrak{A}\mathfrak{B}}=\mathfrak{A}-p(\tfrac{1}{2}+\mathrm{A}-\tfrac{1}{2}\mathrm{AB}),$$

(p being the annual premium for assurance of £1 at decease of the survivor of A and B); and, since $\underset{1}{\mathfrak{A}\mathfrak{B}}=\mathfrak{A}-\underset{2}{\mathfrak{A}\mathfrak{B}}$, it follows, according to M. Réboul's reasoning, that

$$\underset{2}{\mathfrak{A}\mathfrak{B}}=p\,(\tfrac{1}{2}+\mathrm{A}-\tfrac{1}{2}\mathrm{AB}).$$

To show that this is not so, let us denote, by $v'd'_{xy}$, $v''d''_{xy}$, $v'''d'''_{xy}$, &c., the value of £1 receivable at the end of the first, second, third, &c., years, in the event of two lives, aged x and y, both dying in them respectively; and by $v''d''_x\,\delta_y$, $v'''d'''_x\,\delta_y$, &c., and $v''d''_y\,\delta_x$, $v'''d'''_y\,\delta_x$, &c., the value of the like sum so receivable in the event of one dying in the second, third, &c., year, the other having died previously. We have, then, for the value of £1 payable at the end of the year in which the survivor of A and B, aged x and y respectively, shall die, the expression

$$\begin{aligned} &v'd'_{xy}+v''d''_{xy}+v'''d'''_{xy}+\text{ \&c.} \quad . \quad . \quad . \quad (\alpha)\\ &\quad +v''d''_x\,\delta_y+v'''d'''_x\,\delta_y+\text{ \&c.} \quad . \quad . \quad . \quad (\beta)\\ &\quad +v''d''_y\,\delta_x+v'''d'''_y\,\delta_x+\text{ \&c.} \quad . \quad . \quad . \quad (\gamma)\end{aligned}$$

the sum of which three series, continued to the extremity of life, may be represented by $\overline{\mathfrak{AB}}$, so that $\overline{\mathfrak{AB}}=\alpha+\beta+\gamma$.

Reasoning in like manner, it will be seen that $\underset{2}{\mathfrak{AB}}=\frac{1}{2}\alpha+\beta$.

Now, $\overline{\mathfrak{AB}}=p\,(1+\mathrm{A}+\mathrm{B}-\mathrm{AB})$, and $\frac{1}{2}\overline{\mathfrak{AB}}=p\,(\frac{1}{2}+\frac{1}{2}\mathrm{A}+\frac{1}{2}\mathrm{B}-\frac{1}{2}\mathrm{AB})=p(\frac{1}{2}+\mathrm{A}-\frac{1}{2}\mathrm{AB})+p(\frac{1}{2}\mathrm{B}-\frac{1}{2}\mathrm{A})=\frac{1}{2}\alpha+\frac{1}{2}\beta+\frac{1}{2}\gamma$.

But $\underset{2}{\mathfrak{AB}}=\frac{1}{2}\alpha+\beta$, and is therefore, obviously, equal to

$$p(\tfrac{1}{2}+\mathrm{A}-\tfrac{1}{2}\mathrm{AB})+\tfrac{1}{2}\beta-\tfrac{1}{2}\gamma+p(\tfrac{1}{2}\mathrm{B}-\tfrac{1}{2}\mathrm{A}),$$

and not to $p(\frac{1}{2}+\mathrm{A}-\frac{1}{2}\mathrm{AB})$ only, as M. Réboul would have us believe. Hence, too, it appears that $\underset{2}{\mathfrak{AB}}=\frac{1}{2}\overline{\mathfrak{AB}}+\frac{1}{2}\beta-\frac{1}{2}\gamma$, and that $\underset{2}{\mathfrak{AB}}=\frac{1}{2}\overline{\mathfrak{AB}}-\frac{1}{2}\beta+\frac{1}{2}\gamma$.

It will be observed that M. Réboul does not give any demonstration of his formula, but he assumes, in effect, that the quantity $p(\frac{1}{2}+\mathrm{A}-\frac{1}{2}\mathrm{AB})$ (which is nothing more than the value of half p payable during the life of A, and of the whole of p during his survivorship) represents truly the single premium for an assurance of £1 at decease of A, provided his life be the second that fails.* He appears, in short, to have argued, that since

$$\overline{\mathfrak{AB}}=p(1+\mathrm{A}+\mathrm{B}-\mathrm{AB}),$$

or, what is the same thing, that since

$$\underset{2}{\mathfrak{AB}}+\underset{2}{\mathfrak{AB}}=p(\tfrac{1}{2}+\mathrm{A}-\tfrac{1}{2}\mathrm{AB})+p(\tfrac{1}{2}+\mathrm{B}-\tfrac{1}{2}\mathrm{AB}) \quad . \quad . \quad (a)$$

that, therefore,

$$\underset{2}{\mathfrak{AB}}=p(\tfrac{1}{2}+\mathrm{A}-\tfrac{1}{2}\mathrm{AB}),$$

$$\text{and } \underset{2}{\mathfrak{AB}}=p(\tfrac{1}{2}+\mathrm{B}-\tfrac{1}{2}\mathrm{AB}),$$

an assumption plainly inadmissible in the absence of a direct demonstration, and, in the present instance, untrue; the fact being, that the quantities on either side of equation (a), are, for the most part, merely complementary to each other; so that, dealing with their sum, the results are truthful, whilst the use of either in the way suggested leads to error. Thus, if we make an addition of the two following equations, viz.:—

* The quantity in question represents, in fact, the value of A's share of the premium paid annually for assurance of £1 at decease of the survivor of A and B, supposing it to be arranged that each is to pay half the premium while both are living, and that the survivor is to pay the whole. How far its value differs in particular cases from what M. Réboul supposes it to be, will be seen in the following instances, which are based on the Three-per-cent. Northampton Table—the first column being derived from M. Réboul's expression, and the second from the $\mathfrak{A}-\underset{1}{\mathfrak{AB}}$ of the tables:—

A *aged* 30, *and* B *aged* 60.		A *aged* 45, *and* B *aged* 65.	
$\underset{2}{\mathfrak{AB}}=2994$	3025	$\underset{2}{\mathfrak{AB}}=3531$	3584
$\underset{1}{\mathfrak{AB}}=1378$	1347	$\underset{1}{\mathfrak{AB}}=1766$	1713
$\overline{\mathfrak{AB}}=4372$	4372	$\overline{\mathfrak{AB}}=5297$	5297

$$\underset{1}{\mathfrak{A}\mathfrak{B}}=\mathfrak{A}-p(\tfrac{1}{2}+\mathrm{A}-\tfrac{1}{2}\mathrm{AB}),$$
$$\underset{1}{\mathfrak{A}\mathfrak{B}}=\mathfrak{B}-p(\tfrac{1}{2}+\mathrm{B}-\tfrac{1}{2}\mathrm{AB}),$$

we find that their sum, viz., $\mathfrak{A}\mathfrak{B}=\mathfrak{A}+\mathfrak{B}-\overline{\mathfrak{A}\mathfrak{B}}$, expresses what is universally true, although each of the equations composing it is defective, except in the particular case of A and B being of equal ages, when it will be seen that the series β and γ become identical, and that those quantities vanish altogether in the equations given above. The addition, accurately stated, would, of course, be

$$\underset{1}{\mathfrak{A}\mathfrak{B}}=\mathfrak{A}-\tfrac{1}{2}\overline{\mathfrak{A}\mathfrak{B}}-\tfrac{1}{2}\beta+\tfrac{1}{2}\gamma$$
$$\underset{1}{\mathfrak{A}\mathfrak{B}}=\mathfrak{B}-\tfrac{1}{2}\overline{\mathfrak{A}\mathfrak{B}}+\tfrac{1}{2}\beta-\tfrac{1}{2}\gamma$$

the sum being . . . $\mathfrak{A}\mathfrak{B}=\mathfrak{A}+\mathfrak{B}-\overline{\mathfrak{A}\mathfrak{B}}$ as before.

M. Réboul shows (*see* p. 8) that the summation of the figures obtained by his method gives a more consistent result than does that of the figures obtained by Baily's formula. That his should be consistent, though erroneous, will not appear surprising after what has been said. That those arising from Baily's formula should be inconsistent, is, no doubt, attributable to the circumstance that the values of the annuities used in the calculation do not contain a sufficient number of fractional parts to bring out the assurance-values more exactly. The authors of *Assurance and Annuity Tables* have shown how great an influence this insufficiency exercises.* As to the objection raised by M. Réboul, that the method in question is not exact, on the score of the adoption of the fraction $\frac{1}{2}$ as the measure of the probability that A will die before B in any particular year, we think those who have read Mr. Peter Gray's unanswerable essay† will attach but little importance to it. It must, perforce, be admitted that Mr. Gray has *proved* that the adoption of such fraction for the purpose is rigorously true, on the hypothesis of a uniform distribution of the deaths of each year,‡ and that he has shown that the errors arising under that hypothesis are, for the most part, very insignificant. Mr. Francis Baily's formula cannot, we submit, be successfully assailed on this score, or, indeed, on any other.

In a note subsequently addressed to the Institute, M. Réboul discusses the several formulæ for the values of an assurance payable earlier than at the end of the year in which the life assured fails. A good deal in reference to these formulæ has already appeared here,

* *See* page 6 of the Introduction to that Work.

† "On the true Measure of the Probabilities of Survivorship between Two Lives (*see* vol. i., p. 137).

‡ *See* also Milne, vol. i., article 110.

and the matter would seem to be nearly exhausted; nevertheless, we shall hope to recur to M. Réboul's observations upon it in a future Number.

From all which has preceded, the readers of this *Journal* will, no doubt, observe that there is nothing in M. Réboul's communication which adds materially to our knowledge of the subject to which it refers. They will, however, we think, not be of opinion that an undue prominence has been given to it in these pages, when all the circumstances under which it has originated are taken into consideration.—ED. *A. M.*

On some Considerations suggested by the Annual Reports of the Registrar-General, being an Inquiry into the Question as to how far the Inordinate Mortality in this Country, exhibited by those Reports, is controllable by Human Agency. (*Part I.*) *By* H. W. PORTER, ESQ., B.A., *Assistant Actuary to the Alliance Assurance Company, Fellow of the Institute of Actuaries and of the Statistical Society.*

[Read before the Institute the 30th January, 1860.]

UPON perusal of the Annual Reports of the Registrar-General, and of the valuable analyses by Dr. Farr that accompany them, we can scarcely fail to be struck with the alarming fatality which appears to prevail in this country from certain classes of diseases; and the inquiry naturally arises in our minds, whether we have any control over this excessive mortality: is, in fact, any proportion thereof preventable?

It is, I believe, an admitted fact, that the maladies most fatal in England are those which are classed under the general head of pulmonary complaints—of which phthisis, or consumption, contributes the greatest number of victims.

Life Assurance Companies have a great enemy to contend with in this class of diseases, as the returns of those Companies which have published their experience in respect of mortality very clearly show. Unfortunately, but few Societies have thought it desirable to make public the information they possess. It is to be regretted that this should be the case, as, by a combination of the statistics of the large number of Assurance Companies which now exist, some light could not fail to be thrown upon the subject, and the means might possibly be afforded of enabling Assurance Companies

to deal with the very numerous cases in which an hereditary taint of consumption is traceable, and which are now necessarily ruthlessly rejected by them, owing to the absence, at present, of any reliable data on the subject.

It is important, however, not to lose sight of the fact, that it is not among the class of life assurers, but among the labouring community, that the intensity of mortality from consumption is to be found; and when the inordinate mortality to which we are now referring is made clearly apparent, and the matter is forced upon the attention of the public, as it is likely in the course of time to be through the medium of the Registrar's Reports, a very serious responsibility will, I apprehend, be thrown upon our legislators if they neglect to consider this question with the view to the prevention of some portion at least of the unnecessary sacrifice of human life, both from disease and accidental causes, which now takes place every year.

With reference to phthisis. This disease, Dr. Farr tells us, "is the greatest, the most constant, and the most dreadful of the diseases that afflict mankind." "It is the cause," he says, "of nearly half the deaths that happen between the ages of 15 and 35 years."

It will not, perhaps, be considered that I am asserting too much when I say that the mortality under this head is, in a great measure, produced by causes which are more or less under human control; and the classification of the occupations followed by those who fall a prey to consumption, which it will come within the province of the Registrar to make, will serve, in course of time, to point out in what direction public attention must be turned, with the view to ascertain what may be the influence of the unhealthy nature of certain occupations and of peculiar local influences upon the duration of life, and to show our legislators how it is within their power to ameliorate the unhappy condition, in respect to health and longevity, which may, by such means, be made apparent, and possibly, moreover, to place in the hands of the medical profession the means of grappling with a disease which is the great terror of this country, and which has hitherto defied all human skill, and entirely baffled medical science.

It is only by bringing these subjects before the public that we can hope to find any remedy for evils which, though they are now beginning to be known to exist around us, it seems to be no one's particular business to help to remove.

The first step towards providing a remedy for an evil, is to promulgate a knowledge of it.

Unfortunately, the Reports of the Registrar, which supply data, the reliability of which, from the elaborate system now in operation for recording facts, is undoubted, are not of a character likely to come into the hands of the general public.

It may, I imagine, be considered to fall peculiarly within the province of the Institute of Actuaries to endeavour to draw attention to matters of this nature, and to place before the public information in a shape in which alone it can be made available for general information.

The Institute, by making itself a medium for the diffusion of facts of the class under consideration, will confer a lasting benefit on the public, who will have good reason to be thankful for the establishment of a Society which, I may be allowed to say, has already, to no inconsiderable extent, assisted in the promulgation, by means of its published transactions, of much useful knowledge; and an acquaintance with facts is clearly essential before any remedy can be suggested with the view to check existing evils.

This Institute may have the proud feeling of having always promulgated its information for the use and advantage of the whole assurance body, an example that might, with advantage, be more generally followed.

Now, however valuable the voluminous returns of the Registrar's office may be, they are, in fact, practically, almost useless, unless the conclusions that may be drawn from them are made clearly apparent.

The public have to thank the indefatigable Dr. Farr for conducting them through the almost Egyptian darkness of those masses of figures, and for acting as an interpreter of the hieroglyphics of the Registrar. But, when we have deduced our inferences from the data presented to us, any deduction we may draw would prove, like the diagnosis of the physician, highly injurious unless it were strictly correct, and, like that diagnosis, however logically it might be made and however correct it might be, would be entirely useless unless we can contrive to combine with our skill in diagnosis the additional resources of the therapeutic art. Supposing, then, that Dr. Farr has most skilfully interpreted the facts recorded in the Reports, and that he has scientifically diagnosed the disease from which the country is suffering, we next have to look to the means of cure. This rests with the Legislature to effect, but, as is the case with most reforms, it is to public opinion that we must really look for taking the initiative in the matter—and how are we to influence public opinion?

It is, probably, by the diffusion of sanitary knowledge only that we are likely to gain our end; and by that means alone that we shall be enabled to introduce remedies—to force them upon our rulers, in fact—for the many preventable evils which are found to exist, and which, if even suspected before the establishment of the registration department, were certainly not positively known to be so bad as they turn out to be. I allude principally to the mortality caused by pulmonary and other cognate diseases, and by fevers and small-pox—to the excessive mortality among infants—and to deaths from accidental causes.

I will beg to point out, however, that my paper is intended to be more in the nature of an essay than of a statistical analysis of the returns of the Registrar-General, a work which, if I felt inclined to undertake, would be rendered supererogatory by the elaborate inquiry into the subject by Dr. Greenhow, in his capacity of Lecturer on Public Health at St. Thomas's Hospital.

In endeavouring to bring about any sanitary reform, we have a great amount of ignorance to contend with, and it is painful to observe how great a tendency ignorance unfortunately has to reproduce itself. This fact we see very clearly established by the statistics of marriage. The results observable from some of the returns give rise to the consideration, the Registrar tells us, "whether the ignorant have a tendency to marry the ignorant in a greater or less proportion than the learned, up to the writing point, marry the ignorant, or than those so far learned marry the learned." The Registrar, after investigating this question, comes to the conclusion that "the ignorant evidently intermarry, by choice and the force of circumstances, to a much greater extent than would be inferred from their numbers; and this," he goes on to say, "is important, as the result is, that of 24 in every 100 of the families that are now constituted every year by marriage, the children are without the advantage of having either the father or the mother able to write." Hence the reproduction of ignorance.

In any steps that may be taken by the philanthropist for the amelioration of the sanitary condition of the population, prejudices, the result of ignorance, are a great stumbling-block in the way; and, probably, until we are enabled to improve the education of the people, we shall not make great progress in improving their position in respect to health and longevity.

Our legislators are, perhaps, not free from blame in allowing the present state of things to continue; and I fear it is a slur upon us as a nation, that no general plan of national education has, up to

this period, been determined upon; but most sad it is to contemplate, that, while the evils which result in so many different forms from the existing ignorance of the masses are so patent, the whole question remains at a standstill—not because we are unable to organize a scheme of national education, but because we allow the conflicting strife of different religious sects to interfere with the progress of education among the people. These sects, like the rival physicians of Le Sage, allow the patient to die while they argue about systems.

The urgency of the case, which is continually becoming more apparent, seems almost to warrant, if not to demand, that, to some degree at least, the compulsory education of the young should be made the subject of legislative enactment—notwithstanding the objections to what has been termed "meddling legislation."

There is, probably, no doubt that the failure in the satisfactory settlement of this question has arisen from the difficulty of dealing with the religious element involved; but surely it is time that the contentions of the religionist and the secularist should cease—looking to the urgent necessity for increased education, so very fully made apparent by the annual marriage returns. From the last published Annual Report, the 20th—viz., that for the year 1857—it appears that the number of persons signing the marriage register with marks was no less than 105,778—the proportions per cent. being of the men 72, and of the women 61 only, who were able to write their names, notwithstanding the progressive educational improvement which we are told has been going on since the passing of the Reform Bill in 1832.

The Dean of Carlisle, Mr. Close, no mean authority on the subject of education, attributes the failure of the several attempts to settle this question to the circumstance that too much has been attempted in all Bills that have hitherto been introduced into the House. He objects to one general plan of education, and is opposed to the institution of local rates for the purpose of supplying the necessary funds, whether such rates be voluntary or compulsory. In a pamphlet on the subject, he enters fully into the question; but it will be sufficient here to state, that a voluntary rate would clearly be most likely to be objected to by a majority of ratepayers in the very localities in which education was most required—in small towns and villages—where the ratepayers were but slightly in advance, in point of education, of those intended to be educated; and that the great argument against a compulsory rate is, that the ratepayers in that case would have a fair ground

to insist that the management of the schools which they supported should remain in their own hands, and with this they could not be trusted at present.

Mr. Close considers that the course which would be most easily carried out in practice would be the extension of the present system of Government aid now actually in operation, leaving it to a trustworthy executive to determine in what manner, and to what extent, the ample public funds with which they might be entrusted should be distributed among the schools, in such manner as the very many different circumstances in which schools in large towns and small villages, in rich or poor neighbourhoods, and affected by manifold local influences, might seem to demand.

In some cases, the total expense of the schools might advantageously be borne by the Government; in others, such a course might be productive of evil, and have the effect of destroying the school; in such cases, either no assistance should be given, or some partial aid only afforded.

With due respect to Mr. Close, however, I must be allowed to express an opinion that his idea of "leaving it to a trustworthy executive to determine in what manner and to what extent the ample public funds with which they might be entrusted should be distributed," is just begging the whole question, and is, in fact, supposing the matter he has to provide for already decided. How is this "trustworthy executive" to be appointed, and of what religious party is this executive to consist, which is to have the disposal of large national funds? It would appear, from the statement of this gentleman—and it must be remembered that the name of the Dean of Carlisle has been intimately associated with the educational question for many years past—that, in practice at least, the religious element is not found to create the difficulty that might be imagined; and his plain common-sense view of the case would seem to show that the difficulties in the way of the satisfactory settlement of the matter are, to say the least, not of an insuperable character.

In all existing schools (of course, those assisted by Government funds are referred to), the Bible, we are told, is used, with one or two exceptions only; the Church Catechism, too, is taught in many schools in which the children of Dissenters are among the pupils; more or less tolerance upon points of religious teaching is displayed by the managers, and it is stated that, except in a few instances, those engaged in the actual instruction of the children of the working classes cannot complain of having met with any serious obstacles in preserving the religious portion of education.

In the hands of this executive—which would be composed, I apprehend, of all religious sects—Mr. Close, a clergyman, remember, of the Established Church (and one who may therefore be fairly considered to have his own bias in favour of keeping all education which is the result of a Government grant in the hands of the clergy), would be content to leave the power of determining, according to the local circumstances of each case, whether any particular school should be Church of England, Dissenting or mixed, or in the principles of the British and Foreign; and how, by whom, and in what proportion, the Bible should be read and the religious instruction given for the good of the whole community.

But, above all, he considers that the education of the children of the working classes should be made compulsory. "Slowly and reluctantly, and after struggling against this necessity for nearly 40 years," this practically-experienced authority says, "I am an absolute convert to this necessity;" and, looking to the figures just quoted from the marriage records, will anyone present say that this necessity is not apparent? The same influence that rendered legislation an absolute requirement of the age with the view to preventing the too early employment of children in factories—the covetousness, Mr. Close calls it, but I should prefer to say the necessities, of the parents—will ever be at work to restrain them from voluntarily giving their children the advantage of education, and probably, too, this effect will be increased in proportion to the improved education that the schools may provide; for it seems that it has been proved, and the opinion of one of Her Majesty's Inspectors of Schools is quoted in confirmation of the fact, that, "in many of our best schools, where there is every possible attraction of superior cultivation, where the best attention is devoted to religious training, and where every encouragement to success is held out, there the children are taken away at an earlier age than ever"—and why? because, by reason of the improved standard of education, boys of 9 years of age are more advanced than those of 12 used to be, and they are accordingly removed by their parents from school, to be placed at work—a strong reason, I submit, for the compulsory system of education.

It is satisfactory to find, by the last Report of the Inspectors of Factories, "that some instances have recently occurred of mill-owners giving a preference to the employment of young persons who are able to read."

Lord Brougham—who has done more for education than, perhaps, any man living—stated, in the House of Lords, towards the

end of last Session, after alluding to the increase in the number of Day and Sunday Schools, and of the number of pupils who attended them, since the year 1818, that the difficulties of the question were increased by religious differences, and expressed his own opinion that he should prefer education even under the Pope of Rome to no education at all. As a means of improving the defective education of the next higher class in the social scale, he proposed that the Privy Council should have the power of inspection of the middle-class schools throughout the country, in addition to the inspection they now have the power to make of all schools assisted by a Government grant. This proposition, however, was objected to on the part of the Government, on the ground of the additional labour that it would impose if carried out. The suggestion was a most important one, for the recent middle-class examinations that have been established have shown that the middle-class schools of this country are far from imparting generally as sound an *elementary education* as even some of our national schools which are attended by the children of the working classes—the failures in the first of these examinations which was held having shown that a very much less number of candidates than could possibly have been expected gave evidence of being even decently grounded in elementary knowledge of the humblest character.

The National Education question at present stands thus:—

It having been, up to this time, found to be impossible, for the reasons adverted to in the foregoing remarks, to organize any permanent system, the education of the children of the working classes of this country has been carried on by means of Government grants, subject to certain conditions, which have from time to time been increased, in aid of the funds collected by private efforts in different parts of the kingdom.

The amount of Parliamentary grants voted for education in the year 1857 was £541,233.

A commission is now sitting, with the view to inquire into the whole question of Government grants for educational purposes; and the result of the report of the commissioners will possibly lead, at last, to some legislative measure for settling this long-debated question.

This may appear a lengthened digression from the exact subject of this paper, but I am anxious to show that the extension of education is what we must very much regard if we desire to amend the sanitary condition and so to improve the health of the labouring community, of which the bulk of the population is composed, and

to diminish the unnecessary mortality that is shown at present to exist in this country, seeing that our efforts are so much checked by the ignorance and prejudices that we have to contend against.

The efforts of the Industrial Pathology Committee, in connection with the Society of Arts, are stated, in a paper by a member of that Society, to have almost entirely failed in their object, which was, to endeavour to mitigate some of the evils at present inherent in certain unhealthy employments, owing, I apprehend, to the apathy resulting for the most part from ignorance of the power of science to influence the matter.

In immediate connection with the too early employment of children in factories, just referred to, as affecting the question of education, the consideration arises as to the effect upon their health and longevity, which we shall consider hereafter.

Now, not only does the existing want of education tend to promote and engender disease, because parents, owing to their ignorance of the evils that are likely to result from such a course, persist in placing their children far too early at laborious employments; but the higher standard of education now happily beginning to be afforded to the present generation, tends, as has been shown, to increase the evil as affects the parents. It is not, therefore, probable that, until the last generation has passed away altogether, we shall advance as we ought; but, then, the present generation, having had the means of obtaining a higher class of information in their youth, will hardly fall into the errors committed by their fathers—of placing in turn their children so early at work as to render their proper education impossible; and so the tendency will be to increase the standard of education, and to diminish the evil consequences which result, in so many different ways, from a want of knowledge among the working classes. This is so far as the ignorance of the parents affects the question—how far the greed of gain, or the necessities of the heads of families, may still tend to bring about the same results, is another question; and hence the crying necessity that education should be compulsory, at least for many years to come.

Whether such a state of things should be allowed to exist, in which the labour of the young children of a family should be an actual necessity to enable the whole family to live, as now really is the case in some manufacturing districts, is a problem in political economy that cannot be discussed here.

To return to phthisis: the total number of deaths registered in the year 1857 under this head was 50,106.

If to this number we add the deaths which are classed under other heads, but which are likewise of a tubercular character—viz., hydrocephalus, tabes mesenterica, and scrofula—we obtain a total of deaths from tubercular diseases of 65,762; adding to this total the deaths from other diseases of the respiratory organs—viz., bronchitis, pneumonia, asthma, disease of the lungs, laryngitis and pleurisy, all of which are more or less of a character cognate to phthisis—we increase the total to 124,082; and if we further add one-half of the number of deaths ascribed to influenza, and one-half of those set down to hæmorrhage, we obtain a final total of 125,385. My object in including a proportion of the deaths under the two last heads is, as regards influenza, because this complaint is scarcely likely to prove fatal except in cases where there is a tendency to pulmonary disease, which influenza tends to develop; and as regards hæmorrhage, because a great proportion of the deaths recorded under this head no doubt resulted from hæmorrhage of the lungs following disease of those organs. It appears, therefore, that more than half a quarter of a million of the population—that 1 in every $5\frac{1}{3}$, or nearly 19 per cent. of the whole number born in a year, fall a prey annually to that class of diseases which is so peculiarly fatal in this country. In other words, 1 in every $3\frac{1}{3}$, or nearly 30 per cent. of the whole number of deaths occurring in this country in the course of a year, arise from one class of diseases alone.

Looking to such facts as these, it can hardly, I submit, be considered otherwise than an undoubted duty incumbent upon every one to render any assistance that may lie in his power, however slight his opportunities may be, in the investigation of the causes that may tend to produce such fatal results as the Reports of the Registrar-General annually record.

Phthisis, the indigenous disease of this country—a disease that has hitherto baffled our most eminent physicians—we do, I fear, our utmost to foster. Hear Dr. Guy—no mean authority on statistical and medical subjects.

In connection with a paper read before this Institute on the subject of "The analogy existing between the aggregate effects of the operations of the human will and the results commonly attributed to chance," Dr. Guy gave us some interesting verbal information on the subject of the mortality among the printers in the metropolis—a class that helps to swell largely the returns of deaths from phthisis. From some notes I took at the time, I am enabled to state the result of his investigation into the subject.

He had visited, he told us, the principal printing offices in London, which are, most of them, situated in the parish of St Clement's, in the Strand, or in that vicinity.

The offices, he found, were all close and dark, as most of the business premises in that locality are, and a great deal of gas is necessarily consumed, particularly in the winter months, a good light being required for printing operations, more especially in the composing-rooms, and every compositor requires a gas-burner to himself. The rooms in which the men work are, for the most part, small and low, and are heated with hot water. All the fire-places in the composing-rooms are found to be blocked up, for the twofold reason of saving space and of avoiding the risk of fire, so that all ventilation from the chimneys and open fire-places, which form so ready and satisfactory a kind of ventilation, is rendered nugatory. The men employed in the process of printing suffer very much from colds and are very sensitive to draughts of air, and dread even the slight current produced by a ventilator, if the proprietor of the establishment has been induced to fit one up, to such a degree, that they very generally take steps to check its action altogether. Above all, many printers are much addicted to drinking.

Looking to such facts as these, we cannot be surprised that phthisis is very prevalent among this class of operatives—the peculiar position in which they are placed, or in which they place themselves, being exactly one which, as medical men perfectly well know, fosters this disease, and thus printing offices become actual hot-beds for its propagation; and we find, that while 1 in 5 or 6 of the whole population die of phthisis or other cognate diseases —and this is a fearful proportion—among compositors no less than 2 in 7 fall victims to these maladies, the disease being in some cases brought on, and in others accelerated in its progress, by the bad local influences surrounding its victims.

With the view to obtain additional information on this subject, and to test the facts above stated from my own observation, at the present time—Dr. Guy's information having been obtained some years ago—I recently visited a large printing establishment employing about 300 hands, that of Messrs. Petter and Galpin.

The premises of this firm have lately been rebuilt upon an improved plan, and form a favourable contrast to those to which I have referred—and, no doubt, as the old printing offices come in course of time to be rebuilt, the evils now existing will gradually disappear.

From the information I obtained from the intelligent foreman of these works—Mr. Wilson—I am quite satisfied that printers' compositors generally, in London, are in a very unfortunate position as regards health and longevity.

Phthisis is their great enemy, brought on to a positive certainty if the faintest taint exists in the system, and in an incredibly short space of time; and generated, as it were, spontaneously in numerous cases where no such taint is discernible, but in which, possibly, there might be a latent hereditary disposition to the disease.

The principle causes operating to produce this result appear to be—

1. The closeness of the work rooms, the imperfect ventilation, and the impurity of the atmosphere of the small, low, and overcrowded apartments in which, for the most part, printers work—the impurity of the air being increased (in fact, rendered, I may say, deadly) by the quantity of gas—often very impure in quality—consumed, and the fact that little attention is paid to the necessity of providing an outlet for the products of its combustion.

2. The quantity of drink imbibed by the printers in some departments of the business. Many drink, it seems, not only beer, but gin, rum, and brandy, and no surer means than these could be suggested for increasing the bad results likely to ensue from the evils under the first head.

3. The inhalation of dust from the types, which are composed of antimony and lead. This dust is being constantly disturbed, owing to the necessity of shaking up the masses of distributed type in order that the compositor may take quickly in his fingers the letters he requires to use, and that he may be enabled to place them in the necessary position for performing the operation of composing easily and quickly—a great desideratum in printing.

With reference to No. 1, it is clear that the evils complained of under this head are quite capable of remedy; and the same may be said of No. 2. Some inconvenience under No. 3 must, I suppose, continue to exist.

Thackrah, in his treatise *On the effects of Arts, Trades, and Professions, on health and longevity*—a work published as long ago as 1832, and which might, therefore, be considered not to be applicable to the state of things at the present time—refers to the injurious effect produced by the heated types upon the health of the workmen. Although many of the processes of manufacture to which he alludes are certainly now obsolete, and some of his

remarks therefore do not now apply, it is remarkable how applicable some of his observations on many points still continue to be, showing how little has been done towards remedying those evils which affect the health and longevity of the operatives, and which were pointed out nearly 30 years ago. Thackrah mentions that printers suffer injury from the types, which, being composed of lead and antimony, emit, when heated, a noxious fume injurious to respiration, and which produces partial palsy of the hands; and adds, that all danger may be avoided by abstaining from using the types until they are cold. This evil, I believe, no longer exists—in London, at all events—though it has taken many years to abolish. It is well known to intelligent workmen in the trade that this consequence would ensue from handling the types when in a heated state, owing to the antimony in their composition; but there is no necessity in the world for their doing so.

The types are cleaned with lye, an alkaline mixture composed of potash, pearlash, and soda—in which composition there is nothing injurious—and the lye is washed away with cold water, so that the types are not necessarily heated at all, and it is only when heated that antimonious fumes are generated.

It used to be the practice of some printers to dry the types when wet (and it is necessary that they should be wetted, for in that state they can be more easily distributed) upon a stove, and there is no reason, I believe, to doubt that if this were done to any extent, and the types used while still heated, paralysis of the hands and arms would be the result; but this fact is now so well practically known to the trade, from cases that have occurred, that, I am credibly informed, the practice has long since been discontinued.

Compositors frequently fall victims to phthisis at the early age of 30, and probably 40 would be too high a mean age to assign to them as a body.*

* Since writing the foregoing, I have been favoured by Mr. David Arnot, the Secretary of the Metropolitan Typographical Widows' Fund, with the Annual Reports of that Society extending over 10 years, from which I find that the average age at death of the printers who were members of the Fund was 48 years.

The number of deaths caused by phthisis and other diseases of that class among the members, in the ten years ending 31st December, 1859, was 101 out of a total number of 173, being 58¾ per cent. of the whole.

We find, accordingly, from these returns, that 4 in 7 die of these diseases.

Dr. Guy, it will be remembered, ascertained that in the cases he investigated among printers, 2 in 7 fell victims to phthisis; probably, however, he did not, as I have done, include other diseases of the respiratory organs, which would account for some difference in our results.

It must be remembered, however, that men who become members of provident societies are naturally of a superior class. They are, besides, in the enjoyment of good health at the time of entry; and, accordingly, more or less select lives.

Fortunately, they have their sick clubs and houses of call, which are not now, as formerly, public-houses, but private establishments supplied with reading rooms and libraries, where the men, when out of work, are to be heard of if wanted at a moment's notice, as is often the case in the printing business.

I say they have such establishments to which they can go in their leisure hours, but, unfortunately, many still prefer to frequent the public-houses.

This sort of evil is, no doubt, much encouraged by the arbitrary regulations enforced by the combinations that exist among the men engaged in printing, in common with many other trades, as the recent strike among the builders has shown, and by which the hands are compelled to obey certain rules and conditions in taking engagements, many of which encourage and necessitate the expenditure of money in drink—the object of the combination being to keep up wages, which free and independent competition, it is thought, might tend to lower. The sight of compositors is frequently very much weakened; ostensibly, by the constant close application to minute type, but, possibly, in a measure, as well by the quantity of snuff that they take, which cannot fail to be injurious in its effect. The practice is commenced with the view of stimulating the faculties, and particularly the sight; this effect, however, which it does seem to have at first, soon ceases, but the habit then has become confirmed.

Besides the injury from the use of tobacco as snuff, the adulteration, particularly of the commoner sorts, with sundry compounds and, amongst others, with ground glass, tends to increase the injurious effect, and renders the use of snuff most pernicious. The ground glass is mixed with the snuff to make up for the want of pungency, which is diminished by other adulterations. Carbonate or sub-carbonate of lead, too, is found in snuffs in considerable proportions, and is formed by a chemical combination of carbonic acid, which is stated to be gradually evolved from the snuff, with the metallic lead in which the article is very generally packed. The carbonate thus produced penetrates into the mass, and has been found in the interior of a packet of snuff to the extent of ·951 per cent., and on the outside layer of a sample which had been in actual contact with the metallic lead, to the extent of 2·743 per cent., according to the experiments of M. Wicke. Taking, then, the mean of these two percentages, we find nearly 2 per cent. of this pernicious salt in any given portion of the mass. This carbonate or sub-carbonate of lead, which is also known by

the name of cerusse or white lead, is probably the only deleterious salt of that metal, and is of such a nature that not only is it seldom used internally in medicine, but when used externally, in the form of a powder or ointment, great caution is necessary in its application; and yet the habitual snuff-taker, daily, without fear of the consequences, takes this deleterious poison into his system. A moderate snuff-taker will consume about two ounces of snuff a week, so that, allowing for waste, he will take say 100 ounces in the year; of this, accordingly, nearly two ounces will consist of subcarbonate of lead, a highly-poisonous salt of that metal. This is a moderate estimate; some snuff-takers consume considerably more. The Reports of the Analytical Sanitary Commission, recorded in the *Lancet*, showed that of forty-three samples of snuff, chromate of lead was found to be present in nine, and oxide of lead in three cases. The presence of the lead was attributed by the commission to intentional adulteration. Saturnine disease has been clearly shown to be caused by the use of snuff thus adulterated, no less than by other well-known means by which the system becomes impregnated with lead. Printers begin the habit of snuff-taking by using Grimstone's eye-snuff, which is the mildest form in which snuff is manufactured, and so get on until they come to take Gillespie's, the strongest preparation made.

It has been considered that the digestive functions may suffer injury by reason of the long-continued standing posture which habit has necessitated in the printing trade. I have not, however, been able to ascertain that printers suffer in this respect, or from rupture or varicose veins, more than other classes of operatives that are obliged to stand long at a time—all such being liable to injury in these respects, but to a much less serious extent, I apprehend, than those whose labour necessitates the stooping posture or any pressure against the chest, as in the Sheffield grinders, in shoemaking and wig-making, for example. There is, however, no reason for the maintenance of the standing posture by compositors; contrivances have been invented to support the body and relieve the pressure on the legs entirely, but from the habit of a sort of circular motion round the case, during the process of composition, being acquired by the men, they seem always to prefer to work standing; and at times they work very long hours—thirty-six hours, perhaps, without cessation, except for meals. This practice, however, is only, I believe, owing to habit, and it is extraordinary to observe how the working classes cling to old habits and customs, at the expense often of their health, and all owing to a want of

knowledge of the evil results that are certain to be engendered by a perseverance in them; and this affords another instance of how much an extension of education is called for among the working classes.

So with the shoe-makers and wig-makers: these workmen are very likely to be injured by the pressure of the last and block against the chest, but they cannot be made to see it.

An arrangement to prevent the necessity for stooping over their work, and to obviate the pressure of the last, has long since been invented for shoe-makers; and I have ascertained that a similar contrivance, arising perhaps out of the former, has been arranged for wig-makers, consisting of an apparatus for fixing the wig-block, to remedy the injurious effects that had been observed to exist; but neither of these inventions are in general use.

Numerous instances of this class might be adduced, to show the indifference exhibited by artizans in availing themselves of means that may be suggested by the ingenious and philanthrophic for remedying the pernicious results that arise from some species of occupations. This "sure it was always so" and "never heed it" spirit will be found to prevail, with greater or less intensity, in proportion to the indigence and ignorance of the unhappy sufferers.

Some injury to health is likely to be caused by attendance in the drying-room of printing offices, where the newly struck-off sheets are hung up to dry, and which is heated by hot water or steam to a temperature of about 70° Fahrenheit. The men who are much exposed to the high temperature and moist air of this department, if constitutionally disposed to suffer from heat and damp, are not unlikely to be injured by such exposure.

The laborious movement of the heavy levers by which the hand presses in printing offices are worked, would strike an observer as being likely to be injurious, from the tendency to contract the chest, which the action certainly has; but this labour is alternated with that of inking the types by passing the roller over them, a far from heavy work; and since at each press two men are always employed, the heavy mechanical labour is divided between them, so that no injurious result is likely to happen to strong men from this cause.

Of late years, owing to the street improvements in the populous neighbourhood in which printing offices abound, the small houses where printers, as well as other men engaged in works in the vicinity, used to live, have, in many instances, been, and are being, pulled down; and the consequence has been, that the operatives

have been driven over the bridges, to the evident improvement of their health and social position; for the walk to and from the scene of their labours cannot fail to conduce to health, both on account of the exercise they obtain in the open air, and by the change of scene which acts on the mind. The fact of their living in a purer air too, and the tendency to resort less to the public-house, which is brought about by their having a more airy and more pleasant residence a little way over the water, and a garden, perhaps, with which to amuse themselves, has tended to improve, to some extent, both the health and habits of this class of men.

I have mentioned that, in course of time, as the old printing offices come to be rebuilt, more attention will, no doubt, be paid to promote the health and comfort of the workmen. This is evidenced in the establishment of Messrs. Petter and Galpin, who carry on a general printing business in which no such appearance as that which Dr. Guy describes is visible. In the first place, the men are more favourably situated in an ordinary printing office than in the office of a newspaper, where the principal work is at night. The men engaged on newspapers are always, so to speak, working at high pressure, since the papers must be published within a very few hours after the events they record have occurred; and night work is always more trying than work by day. In the establishment just referred to, the composing-room is 8 feet in height, and its dimensions 86 feet by 50; and in it every possible attention seems to have been paid to ventilation, by means of siphon ventilators in the roof, the effect of which is to ensure a proper amount of ventilation without at the same time creating draughts of cold air, which printers seem so much to dislike, owing, of course, to their having been so much accustomed to be shut up all day, and perhaps half the night as well, in low-pitched and heated apartments. The ventilation is also increased by a large opening in each floor for the purpose of working lifts from the basement to the top of the building. A bad effect of this, however, is, that the foul and heated air is carried up to the composing-room, which is in the upper story; and it is, of course, the most vitiated air that finds its way up there. These openings, however, cannot be done without, as it is absolutely necessary, in a large establishment, to have certain powerful steam or hydraulic lifts, for the purpose of raising the heavy forms of type which require to be conveyed from the composing-room to the machine-room. Under these circumstances, then, it would appear to be advantageous to increase the altitude of the composing room, making it at least as high as the

machine-room, which is 14 feet in height—6 feet higher than the former. The position in the building of these two rooms it would scarcely be possible to reverse, as the great weight of the machinery employed renders the desirableness of placing it on the ground floor, if possible, clearly apparent. A slight alteration in the method of constructing the roofs of printing offices and factories generally, and one that could be carried out very inexpensively, would tend materially to obviate an inconvenience of another kind, viz., that arising from the heat during the summer months, caused by the sun's rays shining directly on the heads of the workmen through the skylights with which the upper stories of such buildings are usually lighted. It is only when the sun's rays are so strong as to prove injurious in this way, that the gas, which has so evil an effect, and which it is necessary to use at all other times, is capable of being dispensed with, and another source is thus opened up whereby suffering is unnecessarily entailed upon the operatives. This inconvenience may be remedied by means of dormar windows in the roof, in place of the obnoxious skylights. I have seen this plan carried out at the printing office of Messrs. Wertheimer, to the great comfort of the men employed. Of course, when the roof is sloping, as is often the case, these dormars are more than ever desirable, as increased height is gained by their adoption. Care should be taken not to glaze those sides of the dormars which, from their position, are most exposed to the sun's rays; or, at least, they should be fitted with shutters. I have been induced to dwell somewhat on this subject, because the same remarks apply to factories of all descriptions. The suggestions I have thrown out, it will be borne in mind, are not those of an architect, but of an unprofessional man, who, from personal observation, has seen and regretted the unnecessary injury to health and longevity to which large classes of our labouring community are often most cruelly subjected.

If the metropolitan inspectors of buildings were vested with some discretionary powers to see to the appropriateness of the buildings under their *surveillance*, as respects their answering satisfactorily, in a sanitary point of view, the objects for which they were intended, it is possible that some diminution of the evils I have pointed out might be the result.

It is very difficult to ascertain satisfactorily the causes that tend to produce phthisis, and to reconcile the apparently anomalous results of the observations of different writers on the subject.

Even with respect to the effect produced by the inhalation of

dust, which has generally been considered to be the most fertile cause of this complaint, great difference of opinion seems to prevail, and the statistics of the most celebrated investigators of the question are far from agreeing in their results.

Dr. Knight, of Sheffield, has shown that the grinding and polishing of steel causes phthisis in an unusually short period; and that out of 250 workmen engaged in the occupation of polishing steel, 154 suffered from affections of the chest; and that there was no case of a person engaged in polishing forks reaching his 36th year—magnets, wire masks, currents of air, and moisture, having been successively tried for the purpose of arresting the passage of the metallic particles to the lungs, but without diminishing the mortality: while Dr. Cowan, in his notes to the translation of the celebrated work on phthisis, by M. Louis—the greatest authority, probably, in Europe, on the subject of phthisis—considers that the evidence brought forward on this point tends to an opposite conclusion. Dr. Cowan seems to consider that the grinders, who, he says, were not an unhealthy set of men previous to the commencement of the last century, have become so from the fact of their now pursuing their trade in close rooms, whereas they used to carry on their business in the country in large open rooms; from their working 16 hours a day at grinding only, instead of working 4 or 5 hours a day, and combining with the operation of grinding other less injurious departments of the trade, such as hafting and forging. The division of labour is now carried to great excess in this trade. He attributes much of the injury caused by grinding to the posture in which the grinders labour. They work with the shoulders elevated, the elbows resting on the knees, and the body inclined forwards. Thackrah, again, considers that the angular or pointed form of the particles detached is the chief cause of the injury to workers in iron; for the dust from old iron, which is thrown off, he tells us, so copiously as to deposit a thick brown layer on the dress of the dealers in the article, produces no inconvenience. M. Lombard, from inquiries made at Geneva, found that one of the principal causes tending to produce phthisis was the inhalation of air in which mineral and other dusts were floating; and showed that, among polishers, sculptors, stonecutters, plasterers, watch-hand makers, and others of that class, the proportion of consumptive complaints was 177 in 1,000—the general average number of such cases, in all the professions in Geneva combined, being 114. Again, with reference to mineral, vegetable, and animal particles, the results of the inquiries of

MM. Benoiston and Lombard appear to be exactly the reverse of each other—the former making animal particles the most injurious and mineral the least, and the latter entirely reversing the order. The inquiries of Drs. Young, Leblanc, and Alison, are generally considered to determine the question of the liability of stonemasons to consumption. Dr. Cowan, however, does not entirely assent to their dicta. Certainly, from the observations of M. Benoiston, it would appear that the proportion of cases of phthisis among quarrymen, stonecutters, and marble-workers, was less than the general average; and it is stated, that in a department of France (Meusnes) celebrated for the fabrication of millstones, in a period of 17 years no increase of mortality from phthisis, as compared with other departments where stone-cutting was not general, was to be detected.

M. Lombard, however, found sculptors, stonecutters, and plasterers, afflicted with the disease in a high degree; and Thackrah found this class of persons to be phthisical and short-lived, dying generally before attaining the age of 40.

Upon the whole, Dr. Cowan considers, that, "were the existence of tubercular disease ascertained, the influence of dust in its production is still undecided; and that the hard nature of the material, the partial exercise of the body, exposure to varieties of temperature, their habits of intemperance, &c., probably exert a far more powerful influence."

M. Lombard considers, on the whole, "that the influences which modify the *system in general*, such as a sedentary life, &c., &c., are more active in the production of phthisis than those acting *locally* upon the lungs, as dust, vapours, &c.; and if we reflect that the *latter* agents are never present independently of the former, their *real* activity, as causes of pulmonary tubercles, is, at best, but problematical."

Dr. Clarke, who has written on this subject in the *Cyclopædia of Practical Medicine*, observes, that "in almost every instance the sufferers are exposed to causes, independently of dust, fully adequate to the production of the tuberculous cachexia."

It will be seen, from these considerations, how difficult it is to disunite and sever the manifold predisposing causes that may lead to the production of phthisis; the effect of this is, that we find among the conditions tending to create this disease, many which, to say the least, we are accustomed to consider as most favourable to ensure immunity from the complaint.

We are likely more especially to observe this in the returns of

M. Lombard, since his investigations comprehend so large a number of cases, and, in proportion as the number of facts recorded increases, so do we more nearly approximate to an investigation of the mortality of the whole population from this disease; for, of course, if we were to inquire into the number of deaths from phthisis in every calling, we should in fact be investigating the mortality from phthisis of the entire community, and it is absurd to suppose that there is necessarily a predisposing cause to the disease in question in *every* possible condition of life; hence it follows that, in any very extensive inquiries into the relative effect of different occupations in the production of phthisis, a certain proportion of deaths from that disease is attributed to local influences, which may possibly have no effect whatever on their production; and this the anomalies apparent between the statistics of different inquirers into the subject seem clearly to show. In many conditions of life, therefore, we ought perhaps to endeavour to discover other causes for the disease than those set down by the statist, who merely records the fact that, out of a certain number of deaths from phthisis, so many occurred to persons who were exposed to peculiar influences, such as, for example, "muscular exercise, and active life." We should hardly say this could be the cause of the large percentage of deaths recorded under that head in the returns of M. Lombard. In point of fact, all writers on the question appear to unite in recommending muscular exercise and active life as amongst the best safeguards against the ravages of phthisis. We may therefore look for some other cause, and may probably fairly consider, from what we know as to the hereditary nature of this complaint—and this fact is not made an element in the classification of deaths, under the head of occupations —that of 100 persons dying of phthisis, in a certain percentage of the number no particular predisposing cause could be traced; but it was observed, that this percentage of persons was placed in a position to conduce to the use of muscular exercise, and to the leading of an active life, which, so far from being the *cause* of their death from phthisis, had, in all human probability, the effect of retarding the fatal issue of a disease which had, perhaps, been hereditarily transmitted to them, and in which the taint was too strong to be overcome by the favourable circumstances in which the victims were placed—such an hereditary taint being, unhappily, too well known to exist in families, and to be capable of transmission from generation to generation, as we shall presently see.

With reference to the effect on the constitution of the most laborious muscular exercise, I may mention, that from an inquiry

into the subject, instituted by Mr. John Whitterom, into the health of the forgemen in the low moor district, it was shown, in a paper read at the recent meeting at Bradford of the Society for promoting Social Science, that the mortality of that class of men whose muscular labours were of the most laborious nature, was but slightly in excess of the general average of the country; and further, that notwithstanding the slight excess apparent in respect of mortality, they were not exposed to the same amount of disease as the ordinary out-door labourers employed in the same district.

Similarly of the "exercise of the voice": when not carried to a most unusual extent, provided the lungs are not seriously advanced in disease, the due exercise of the voice has always been considered as likely to improve their strength, and to increase, not only the capacity of the chest, but the muscular development of the whole of the upper part of the body. This we see frequently evidenced in singers; but even in persons with weak lungs, where actual disease does not exist, judicious exercise of those organs is one of the best methods of increasing their strength.

It is related of Cuvier, the naturalist, that he ascribed his own exemption from phthisis, with which disease he was threatened at the period of his appointment as Professor at the College of Surgeons, to the practice of public lecturing, which this position necessitated. This result was due, no doubt, to his lungs being in such a state as to be improved by the exercise of speaking aloud. Had the disease been further advanced, its progress would doubtless have been accelerated.

We may, perhaps, be justified in considering that the exercise of the voice, which appears as a cause of disease in the records of M. Lombard, was excessive, or that the hereditary taint was implanted in the lives under observation, or, perhaps, that both reasons aided in bringing about the result recorded, if, after what has been said, we are to take any note of this condition as a cause of the complaint at all. "Living in the open air" has always been considered to be most conducive to health, and persons whose avocations necessitate this mode of life are usually healthy and long-lived; and this is the case, I apprehend, even with persons exposed to such vicissitudes of the weather as might be deemed to be excessive —exposed, in fact, as much as they can be. We can only, therefore, imagine the existence of the hidden hereditary taint in this class too, supposing, in this as well as in the previous cases, that no other more patent reason for the creation of the disease were to be discovered. In confirmation of the opinion as to the advantage

to be derived from open-air employments, I may refer to the Reports of the Registrar-General, in one of which it appears that of twelve classes in the tables—consisting of tailors, shoemakers, farmers, carpenters, butchers, persons engaged in wool, cotton, and silk manufactures, bakers, inn and beershop keepers, grocers, miners of all descriptions, labourers, and blacksmiths — among which a comparison was instituted, the farmers were found to be the longest lived.

The Registrar, in alluding to this class, and in speaking of the advantage to Assurance Offices and to Friendly Societies of a knowledge of the facts contained in these tables, states that "it is evident that the lives of farmers, for example, may be safely insured at much lower rates than the lives of licensed victuallers."

Now, with all due deference to the Registrar, this is not quite the way to put the case; and probably the actuaries present will agree with me, that the experience of Life Offices shows, that if the lives of farmers may be safely assured at the ordinary rates, the lives of licensed victuallers, if they are taken at all, should be assured only at an extra premium—the rate of mortality among the class of innkeepers being in excess of that among all the other eleven classes just referred to, at all ages, except the class of butchers in one particular decade of ages, viz., those between 55 and 65, the butchers dying at the rate of 41 in 1,000, while the innkeepers died at the rate of 39 in 1,000; so that even in this decade the innkeepers were not much longer lived than the butchers, and at all other ages the mortality among the innkeepers was the highest of the twelve classes under observation.

The rate of mortality among the whole population at the ages just quoted, 55 to 65, was only 30 in 1,000.*

Of late years, Assurance Companies have been on their guard against this class of persons, and some Companies decline to entertain proposals for assurance on the lives of innkeepers altogether.

As a class, they are exposed, the Registrar-General tells us, "by their business, to unusual temptation, live intemperately, and enjoy less quiet than the rest of the community. They are exposed also to zymotic diseases, by intercourse with large numbers of people."

The result of this investigation points very clearly to the fact, that intemperance may have no inconsiderable effect in unduly swelling the records of the Registrar-General.

With reference to the deaths set down to the cause of "animal emanations," much difference of opinion, I find, prevails as to whether

* *Vide* the 14th and 15th Reports of the Registrar-General, p. xv.

such a cause as this may be considered generally to affect health injuriously.

The deaths from phthisis, among the class of persons, according to the statement of M. Lombard, exposed to animal exhalations, though not so numerous as those that are stated to have occurred from the other causes quoted, are nevertheless considerable in number. The Registrar seems to consider that the great mortality among butchers may be owing to what he states to be the most probable cause, viz., the element of decaying matter by which they are surrounded in the slaughter-house and its vicinity.

This is, however, nothing but matter of opinion, though no one perhaps is more competent to form a correct one on such a subject than the Registrar-General. It certainly is not found, however, that those persons who are exposed to animal exhalations of this nature are generally *unhealthy*. We may, therefore, fairly look for some other cause as tending to induce the great mortality of this class.

The high death-rate among butchers could perhaps be traced to other causes than those just referred to; indeed, the Registrar only throws out for consideration the suggestion, whether what he mentions may not be the most powerful reason for the high mortality among them. The other probable causes that he mentions are, the excess of animal food in their diet, the fact that they often drink to excess, and that they are much exposed to heat and cold.

It is not impossible that the actual inoculation of the systems of butchers with decaying animal matter may be the cause of not a few deaths; and the constant exposure in their open shops to draughts of air—the extent of which, in windy weather, is very apparent from the effect on the gas-burners—may not improbably have an injurious effect; while the quantity of foul gas consumed in the unprotected burners will hardly tend to purify the atmosphere. The Registrar observes, that the red injected face of the butcher is an indication of disease—to the ordinary observer this might be an indication of robust health. Similarly with respect to brewers' draymen: their appearance would indicate that they were blessed with strong constitutions; this is not, however, the case. It is found, in the hospitals, that these men are a very difficult class to treat when attacked with inflammatory symptoms; and they are very prone to such attacks, and succumb readily to them. Their horses, which are good matches often for their masters in appearance, have very much the same attributes as the men; and as the draymen probably acquire their peculiarity of

appearance and of constitution from the quantity of beer they imbibe, and owing to the insufficient amount of active walking exercise they take in proportion to their potations, so, perhaps, are the horses affected by much the same causes, by being fed to some extent on the brewers' grains, which act, no doubt, on their livers as the beer does on those of their masters. These horses are far from strong, though they look so fat and comfortable as to have given rise to the expression, "I would I were a brewer's horse;" and they can endure about the same amount of bodily labour, proportionally, as the draymen, and that is not much.

In connection with the subject of the inhalation of foreign particles, I may state that the workmen employed in flour mills are grievous sufferers from a spasmodic affection of the lungs, caused by the inhalation of minute particles of dust with which the atmosphere of most flour mills, as usually constructed, is impregnated; and I have been informed by the proprietor of some extensive flour mills in Birmingham, that if the workmen who suffer in this way continue too long in the business, a fatal result would be certain to ensue. I have great pleasure, however, in adding, that this gentleman—Mr. Evans—has introduced into his mills an apparatus for remedying the evil consequence just mentioned. He was kind enough to take me over his works, and personally to explain the action of the improvements. In the portion of the mill into which the ground grain is delivered into bins from the upper part of the mill, where the grinding process goes on, the atmosphere in which the men have to work had, prior to the introduction of his improvements, been completely impregnated with fine particles of flour dust; and these, from being the portion that was affected by the air, you will understand, consisted of the very finest particles of the flour—being, in fact, an almost impalpable powder—and, of course, by consequence, the more penetrating and the more likely to be inhaled and to be carried into the minute air cells of the lungs in exact proportion to its impalpability.

The effect of such inhalation is the production of a spasmodic action, which is excited by the physical contact of these minute particles with the delicate surfaces with which they are thus brought in contact. The lower parts of Mr. Evans' mills are now free from dust, and the effect is produced as follows:—

The ground grain descends from the upper portion of the mill through tubes of wood, and is delivered into bins placed on the ground floor. As the flour runs into these bins, in mills of ordinary construction, a cloud of fine dust is emitted, of the before-described

injurious nature. In these mills, however, by means of a fan, worked by the steam-power used for driving the ordinary mill machinery, a current of warm moist air is made to take such a direction as to cause the light obnoxious particles of flour-dust to ascend up a wooden tube to an upper floor, where the dust accumulates, and is swept up twice a day.

There is, accordingly, no reason whatever why flour mills should continue to remain the death-dealing receptacles many still are. Surely, looking to the disastrous results entailed under the ordinary system, it should be made compulsory on every proprietor of such an establishment to introduce improvements of this nature, when it is shown by actual trial how fully they answer in practice, and by how inconsiderable an amount of expense the decimation of a portion of our labouring community may be prevented!

It is required, by legislative enactment, that the machinery used in driving mills should be so fenced off as to prevent accidents to the lives and limbs of the operatives—why should not the preservation of their lungs, and thus of their health and lives, be equally attended to? We can exist without our legs and arms, but the free action of our respiratory organs is an essential requirement of our being.

Mr. Evans has set a noble example, in spontaneously taking the initiative in a sanitary mill-reform of so much importance.

Injurious, however, as the effects of the inhalation of soft vegetable dusts may be, they are probably less serious than those resulting from that of metallic and stony particles.

With reference to stone-cutters, to which class previous reference has been made, the late Hugh Miller, in the history of his life, thus speaks as to the effect of the inhalation of the dust of stone.

It will be remembered that this eminent geologist was originally a mason. He found, he tells us, that during his apprenticeship the dust of the stone had an injurious effect upon his lungs; and that after he had been engaged in the occupation for two years, he began to suffer so severely that he was obliged, though a poor man, to discontinue his occupation for a time.

This observant writer states, as the result of his own personal knowledge of the subject, that, at the stage of the malady at which he had arrived, poor workmen who may be unable to give up their employment sink, in six or eight months, into the grave. So general, indeed, is the affection, that few of the Edinburgh stone-cutters pass their 40th year unscathed, and not one out of every 50 ever reaches the 45th year.

In his case, after being engaged in hewing stone only two seasons (I should mention that the masons in the north only work during the six summer months), the dust of the stone inhaled at every breath had exerted the usual weakening effects on the lungs; and, after working day after day with wet feet in a water-logged ditch, he began to be sensibly informed, by a dull depressing pain in the chest and by a blood-stained mucoidal substance expectorated with difficulty, that he had already taken harm from his employment.

Hugh Miller, however, seems to have been blessed with a good constitution, and being enabled to give up his occupation for some months, he threw off the disease with which he was threatened.

The disease which causes death in these cases seems to be created by the mechanical irritation of the fine and hard particles of stone, which fret the delicate air passages of the respiratory organs, which accordingly become charged with matter and stone dust, and the eventual result appears to be death by phthisis or some allied complaint.

I may mention a very curious circumstance bearing on this question, and one which it is difficult to understand, but yet which is found to be a fact, viz., that the most sober workmen engaged in this occupation are the first to suffer from the results just detailed.

Hugh Miller was a most temperate man, and it was confidently predicted by his comrades that he would suffer for his sobriety, by being the sooner affected by the stone-cutters' malady. "A good bouse," they said, gave a wholesome fillip to the constitution, and "cleared the sulphur off the lungs;" and that his would suffer for want of the medicine which kept theirs clean. Whether there was any virtue in their remedy or not, it is just possible—Hugh Miller considered—that the shock given to the constitution by an over-dose of strong drink might, in certain cases, be medicinal in its effects.

Judging, however, from the information possessed by Life Assurance Companies, and from what the Registrar-General tells us about innkeepers, very few in this room would be willing, I imagine, to try the prescription of the stone-cutters of the north.

In this case, however, they were right in their predictions as to their fellow-workman being the first of his party to be affected by the malady; but his recovery, after a few months' cessation from the work, and his ultimate power of contending with and conquering the disease, arose, no doubt, as well from his extreme temperance as from the original strength of his constitution.

Having read in an article by Mr. William Jones, treating of the effect of different occupations on health, published some time ago in the *New Monthly Magazine,* that a preparation of the lungs of a mason who had died from phthisis was deposited in the anatomical museum of the Edinburgh College, showing the air cells literally blocked up with fine particles of dust, I have taken some trouble to ascertain whether this is really the fact, as a strong opinion appears to prevail that, in cases of this nature, the stony character of the deposit in the lungs is due to disease, and not to inhalation. I have positively ascertained that there is no foundation whatever for this statement.

In the Museum of the Royal College of Surgeons of Edinburgh, there are two preparations, consisting of sections of the lungs of stonemasons, put up in spirit.

In both there is a deposit of tubercle, and cavities have been formed, lined by false membrane.

These have just been examined by Dr. Sanders, the Conservator of the Museum, at the request of Dr. Douglas Maclagan, the President of the College, with the special object of determining the point now under discussion.

I have to record my obligations to both these gentlemen, for their courtesy, and to the latter, in particular, who has been so good as to enter very fully into the subject with me.

Portions of the lung tissue, of the tubercular matter, and of the false membrane of the cavities, were examined by Dr. Sanders, under the microscope, and with the addition of nitric acid, &c., but in none of these was there any appearance of particles of stone-dust. The sediment, moreover, that had fallen to the bottom of the jars, was likewise examined, and was found to consist of organic matter only, chiefly of cholesterine, but no particles of silica or carbonate of lime were found to be present.

It did not, therefore, appear to Dr. Sanders that there was any stone-dust in the portions of the lungs he examined.

There is, also, a preparation of the lung of a stonemason in the University Museum, which is, probably, the one referred to by Mr. Jones.

Dr. Maclagan informs me, that this specimen does not present any appearance different from that of an ordinary tubercular lung. This gentleman has taken the trouble to investigate the origin of the story, which he considers to be a pathological romance.

It seems that the deceased was a mason who worked in granite; he died phthisical; and his lungs were found to contain some grey

tubercles, mixed with some of the usual carbonaceous deposit seen in most adult lungs. The Museum porter was an acquaintance of the deceased, and remarked that his friend's lung *looked like* a bit of the granite with which he worked.

It will be seen, from this investigation, how careful it is necessary for inquirers into such subjects as these to be, to ascertain the truth of the statements that are met with in unscientific writings.

Dr. Maclagan informs me, that he has had an opportunity of examining the lung of a Sheffield fork-grinder, in the museum of Dr. W. Gardner, who exhibited to him a sample of the dust separated by the grindstones. The latter was composed entirely of black particles, some of an irregular shape, and others in round globules, which were entirely attracted, Dr. Maclagan says, by the magnet—being either metallic iron or the metallic oxide. There were also a few particles of silicious matter, from the grindstone, mixed with the metallic particles.

The section of this lung was examined microscopically by Dr. Cleland, Demonstrator of Anatomy in the University, who could see, it appears, none of the iron particles.

Dr. Maclagan further examined this lung, by drying, pulverising, and then bringing the magnet over it, but not a trace of iron was separated from it.

Any attempt, therefore, on the part of these gentlemen to find inhaled particles was fruitless.

Dr. Maclagan, however, is of opinion, that this is no proof that such particles may not have been inhaled, and thus have been an exciting cause of disease; for he does not see, he informs me, how we could expect particles inhaled into the pulmonary cells to remain there in a disease in which there is so much expectoration, and in which the victims must be off work for some time before death.

This opinion, it will be seen, agrees exactly with what I have stated elsewhere, as the result of other inquiries into the subject.

It may also be open to conjecture whether a deposit of such a nature may not be the result of some chemical action taking place in the system, it being well known that carbonic acid will hold a large quantity of lime in solution, forming carbonate of lime, which would be deposited upon the acid being set free.

Patissier seems to consider that the stone-dust is deposited bodily in the lungs. He says, in his *Traité des Maladies des Artisans,* "La poussière qui se détache des pierres pénètre par la bouche dans les poumons, s'arrête dans les voies respiratoires, se

mêle aux mucosités dont ces organes sont lubrifiés, *et forme parfois de vrais calculs,* qui donnent naissance à la toux aux hémoptysies, et peuvent même occasionner des pleurésies des péripneumonies dangereuses."

Deslandes, on the other hand, seems to hold an opinion exactly the reverse of that of Patissier. He says, in his *Dictionnaire de Médecine et de Chirurgie Pratiques,* "Longtemps les matières crétacées que l'on rencontre dans le voisinage des bronches, et même les tubercules des poumons, ont été attribués à des agglomérations, et même à l'absorption de molécules pulvérulentes. Cette opinion ne mérite plus aujourd'hui qu'on s'arrête pour la réfuter. Quant à séjour des poussières dans les voies aériennes, il y a lieu de penser, avec Laënnec, que le plus souvent, il doit être fort court."

Dr. Hodgkin, in his *Lectures on the Morbid Anatomy of the Serous and Mucous Membranes,* mentions that "An idea has been advanced, that earthy collections in the lungs are sometimes produced by the inhalation of impalpably fine dust, to which some persons are, by their occupations, exposed; and the workmen of the quarries of Montmartre have been cited as instances of this fact. Although there can be no doubt that the inhalation of the atmosphere highly charged with mineral particles, however minute, must be a highly deleterious source of irritation, and, consequently, be very liable to become a fruitful cause of those affections which I have mentioned as producing calculi in the lungs; yet it appears extremely unlikely that, under ordinary circumstances, the earthy particles should either penetrate profoundly into the substance of the lung, or be so partially collected as to form concretions. It is more probable that they would be arrested by the mucus of the air-passages, and voided by the efforts at expectoration excited by the irritation which they induce."

Probably, on the whole, the opinion of the medical profession generally is in favour of the assumption that any calcareous *deposits* that may be found in the lungs after death are not the result of inhalation, but are either due to change of structure, produced by disease, or to chemical deposit, as previously suggested.

It is not, however, only particles of a hard nature, or of a high specific gravity, that cause injury to the delicate tissues of which the respiratory organs are composed; soft and light woolly matter, which floats in the air, is found to have a highly-injurious effect, when inhaled into the bronchial tubes and lungs.

(*To be continued.*)

The Life Assurance Companies of Germany; their Business and Position in the Year 1858. *By* G. HOPF, *Manager of the Gotha Life Insurance Bank, Corresponding Member of the Institute of Actuaries in London, of the Commission Centrale de Statistique at Brussels, and of the Useful Sciences at Erfurt.*

[Read before the Institute, 26th March, 1860.]

WIDOWS' Funds and Burial Societies have existed in Germany already for more than 200 years, and in very great numbers. There was no town of any importance that did not possess one or more establishments of that kind. The guilds (corporations) of artisans usually maintained such institutions for their members; seeking in them, at the same time, a tie of stronger combination, and uniting their corporate interests. The love of these institutions, and the endeavour to provide, by means of them, for those left behind, spread from the guilds of artisans to the other classes of the people, in consequence of which, many hundreds, even thousands, of those little Burial Societies and Funeral Funds are to be found in Germany, from which, at the death of a member, a certain sum of money is paid to the survivors, to defray the expenses of interment, and to provide for the family immediately after the death of the subscriber. An extensive use was also made at all times of these establishments by such classes of people as were not so wealthy, because even the poorest are ambitious to take measures that they may have after their death a decent burial furnished at a fixed expense. This motive induces many, and particularly women, to subscribe to a Funeral Fund in Germany.

The number of Widows' Funds, from which, after the death of the husband, a yearly pension is paid to the widow, is much smaller in Germany than the number of Burial Societies. Such institutions were, principally, founded by Governments and Princes for their public functionaries and officers, and there is scarcely one country in Germany that has not one or several Widows' Funds for the officers of state, and all the persons in the service of the sovereign, generally also for clergymen, schoolmasters, and military officers. Duke Ernest the Pious, of Gotha, the ancestor of all the princes of the Ducal Saxon houses, who has deserved very well of the churches and schools of his country, laid the foundation of a Widows' Fund for the clergy of the churches of his country, as far back as the year 1645, and a similar one for the schoolmasters, in 1662 —establishments which subsisted until recent times, with many changes in their previous regulations, until they were united, about

40 years ago, with the Widows' Fund, founded in 1775, for all public functionaries and officers of the Duchy.

All those establishments, which had their origin in ancient times, were very imperfectly regulated, and many of them, which did not require the forced accession of the members of certain corporations, have been ruined for this reason. The scientific elements were totally wanting at the time of their foundation. Partly the laws of mortality were not yet sufficiently investigated, partly the mathematicians had not yet paid the necessary attention to the parts of their science relating to life contingencies. In both respects, little was done till the eighteenth century. There were, indeed, facts already collected relating to mortality in Germany; and it is known that Halley calculated his table of mortality from observations collected in the town of Breslau during the years 1687–91. But even though the defective method followed by him in this calculation had not then been perceived, yet it was evident that the law of mortality could not be found out from such a limited experience as could be gathered in one town only, during the above-named short period, and that, therefore, no confidence was to be bestowed upon the table of Halley. It has, therefore, never been made use of in Germany for calculations relating to Burial or Widows' Funds, or similar establishments.

It was not till the middle of the last century that the inquiry into the law of human mortality made great progress. The man who distinguished himself, not only in this branch of statistics, but who examined into all the theory of population, and raised it to a science, was Süssmilch. The work published by him, *Die göttliche Ordnung in den Veraenderungen des menschlichen Geschlechts*, appeared first in 1741, and was followed by several much enlarged editions. It forms the basis of all the later inquiries in this wide subject. Not long after a table of mortality had been constructed by Süssmilch, the mathematical rules for the calculations as to life insurance and annuities, founded on such observations, were also made public by several learned men of Germany. The most prominent, at that time, were Euler,[1] Florencourt,[2] Karstens,[3] and Tetens.[4] The latter treated the whole method of these compu-

[1] *Histoire de l'Academie Prusse, Année* 1760; Berlin, 1761; p. 163. *Neues Hamburger Magazin;* Leipzig, 1770; No. 43.

[2] Florencourt: *Abhandlungen aus der Juristischen und Politischen Rechenkunst*; Altenburg, 1781.

[3] Karstens: *Theorie der Wittwencassen;* Halle, 1784.

[4] Tetens: *Einleitung zur Berechnung der Leibrenten und Anwartschaften, die von Leben und Tode einer oder mehrerer Personen abhaengen;* 2 vols.; Leipzig, 1785–6. Mr. Hendriks has drawn attention to this work in a clever paper "On the early History of Auxiliary Tables for the Computation of Life Contingencies," *Ass. Mag.*, vol. i., p. 1.

tations with such sagacity and judgment, and gave, at an early period (1785), such practical rules for performing the calculations, that his directions are not, even to this day, to be considered as antiquated, and they form the foundation of all the later labours in this field. But it was long before the principles laid down by those learned men were employed by the institutions depending on human life, and a great many Widows' and Burial Funds were consequently established with the old imperfect regulations, even after the appearance of their works.

The Burial Funds are Life Insurance Societies upon a small scale. Every member pays a premium, either once only or periodically, to the fund, and after his death a certain sum is paid out of it to his heirs. Though the idea of establishing Insurance Societies for higher sums, and for the admission of members from more extended localities, was made very intelligible by the great number of Burial Funds in Germany; yet it required the observations made in similar Societies in England to stimulate the Germans to found such institutions. This was done at first at Gotha, where, after the establishment of the Mutual Fire Insurance Bank for Germany, which is still flourishing there, the idea was seized and the resolution taken to create a Life Insurance Company, based on similar principles. After various preparations and calculations, in which the work of Charles Babbage (*A Comparative View of the various Institutions for the Assurance of Lives;* London, 1826; translated into German: Weimar, 1827) did useful service, the first plan was published in 1827, and from that date members were enlisted for it. A considerable number of members was soon found, notwithstanding that their policies were not delivered till the 1st January, 1829; consequently, a period of 30 years, closing with the year 1858, has passed since the opening of this branch of Assurance on German soil. Since the establishment of the Gotha Company, scarcely any year has passed without the formation of one or more similar Companies.

Table I. shows what results all these Companies have had, up to the end of 1858; and Table II., what premiums are payable to them for an assurance for life. Both the tables are arranged like those which we published in the former volumes of this *Journal.* If we compare Table I. with the same table for the year 1857 (p. 163, vol viii.), we see that since then two new Life Insurance Companies were established in Germany — the Life Assurance Branch of Nuova Società at Trieste, and the Anker at Vienna. They are both Share-Companies. The latter is founded with a

share capital of 2,000,000 Austrian florins, and has been called into existence principally by Belgian resources and influence; the former is founded with a capital stock of 4,000,000 florins.

By comparing the two tables, it appears also that one of the older Companies, the Hammonia of Hamburgh, has ceased to exist. It was reduced to such a sadly ruinous condition by the unsoundness of its operations, especially by too high expenses of administration, that it could no longer meet its obligations, and was obliged to declare itself insolvent. The state of this Society was very unsatisfactory. The share-capital was very small, and did not exceed the sum of £35,714, of which not more than £4,587 were paid up. Many shareholders now refuse to pay up the balance of their shares, for which reason many actions must be brought against them, the result of which is very doubtful. It is doubted whether the creditors of the Hammonia will receive more than 30 per cent. of their claims, at the settlement of the bankruptcy.

The number of German Life Insurance Companies, which were in active operation during the year 1858, amounted to 26. The number of the English, French, and Belgian Companies, which have extended their business to Germany, is as great, if not greater. Of these, the English Companies especially do a considerable business in the large commercial and seaport towns. In the greater part of the States of Germany, the permission of Government is necessary for establishing an agency, and this licence is often refused, or connected with troublesome conditions. Legislation is, in this respect, very different in the different German States. Whilst some Governments, as those in the Hanseatic towns, in the kingdom of Saxony, and in the Thuringia countries, proceed liberally, and leave free the establishment of agencies for Life Insurance, or require only the observation of formalities easily to be performed, it is very difficult in others to obtain the requisite permission. And these prohibitive measures extend as much to German Companies as to foreign ones. The severest restrictions are in Austria, which has not allowed admission into its States to any foreign Assurance Company, whereas Austrian Companies have been allowed to carry on operations in many German countries. But the protection afforded, by this method of proceeding, to the few Life Insurance Companies in Austria against foreign competition, has by no means brought them to a flourishing condition. On the contrary, we maintain that it has turned rather to their prejudice than to their advantage; and to prove this, we refer to Table I. Of the five older Life Insurance Companies in Austria,

only two have hitherto published reports of their results; and of these, only one, the Assicurazioni Generali of Trieste, has made remarkable, but by no means brilliant, progress—the operations of the other having remained till now of very limited extent; and we think we are not mistaken in affirming, that the affairs of the three Companies which have not yet made a report are still less satisfactory. Likewise, it cannot be denied, that the legislation which excludes entirely all foreign Assurance Companies from Austria, turns also to the prejudice of the Austrian people. From want of competition, they have not been duly made acquainted with the nature, use, and the varied applicability of Life Insurance; and those who are acquainted with it, have often felt the want of an institution which suited their necessities and wishes; and their people have been obliged to disregard an important means of increasing their prosperity.

Casting a glance on Table I., we see that, in 1858, 14,645 persons have newly entered the 20 German Life Assurance Companies which, out of the existing 26, have made public reports; and that, the subsequent assurances (Nachversicherungen) included, they have insured, upon the whole, a sum of £2,340,298 in case of death—for our table comprehends only the insurances effected for the case of *death*, the life assurances in the proper sense of the word. The state of assurance of those 20 Companies was—

At the beginning of the year,	£12,875,128	on the lives of	80,523	persons.
At the end of the year .	14,383,010	„ „	90,128	„
Showing an increase of	£1,507,882	„ „	9,605	„

According to this statement, the total amount of assurances increased, in the course of 1858, by 11·71 per cent. of its amount at the beginning of the year, and the number of assured persons by 11·93 per cent. The average sum assured on one life is about £160.

The receipts of premiums and interest increased in a greater degree than the assurances. According to Table I. on p. 163, vol. viii., these receipts were—

In the year 1857 . . .	£509,007
„ 1858 . . .	618,382
Increase equal to .	£109,375, or 21 per cent.

Whilst £251,545 were paid for claims on 1,510 persons who died in the year 1857, a payment of £303,438 was made for 1,900 cases of death in 1858.

The total of assurance funds was—

At the end of 1857	. .	£2,258,088
„ 1858	. .	2,437,844
Increase	. .	£179,756, or 8 per cent.

As every new enterprise in Germany always requires time for its development, and people learn but by slow degrees to accustom themselves to new institutions and to make use of them, such also was the case with life assurance. It requires considerable time to give the German a clear idea of the nature and manifold utility of life assurance, and to inspire him with confidence in it; but when this is once done, the interest in this means of advancing the welfare of families increases from year to year. During the last few years, life assurance has developed itself in Germany in the following rapid manner:—

Year.	Number of the reporting Offices.	New Assurances in the course of the Year.		Assurances at the end of the Year.	
		Persons.	£	Persons.	£
1852	12	5,236	841,844	46,980	8,224,130
1853	13	5,558	939,854	50,019	8,750,239
1854	14	5,224	841,459	52,876	9,150,885
1855	18	9,366	1,361,711	61,832	10,411,549
1856	18	12,778	1,633,272	71,169	11,487,487
1857	19	13,601	1,930,649	81,348	12,893,086
1858	20	14,645	2,340,298	90,128	14,383,010

It results from these figures that the use people make of life assurance has increased, especially in the latter years, since the termination of the oriental war, during which a stagnation took place on account of the doubtful security of our position, and the predilection for insuring has made considerable progress. It is to be expected that, with security, this rate of increase will continue a long time before it reaches its culminating point.

The Gotha Company has reached the greatest development of all the Companies. Table I. shows that their insurances comprehend more than one-third of the sums assured in all German Life Insurance Companies. As it is founded on mutuality, all surplus, annually ascertained, is returned as dividend to the assured, and deducted from the premiums, or paid in cash if the assurance is extinguished—a proceeding adopted also by the other German Companies which give dividends to the assured. On each premium paid a dividend is granted—generally five years after payment of the premium—in proportion to the surplus of the corresponding year for which the premium is paid, no matter whether the assurance be still in existence or not at the time of distribution. The

practice of adding the bonus to the sum assured is not found in any German Life Office. The dividends in the Gotha Office amounted, upon an average, to 28 per cent. of the premium, and, consequently, the cost of assuring in this Company is the lowest of all German Offices, as Table II. shows. It is also the Company which is managed at the most reasonable rate. All the expenses of administration, the commission for the agents included, do not now amount to quite 4 per cent. of the annual receipts of premiums and interest.

The Gotha Company passed through the third decennium at the end of 1858. Table III. shows the working of the business in each one of these periods. The new assurances amounted in the

1st decennium (1829–38), including the preliminary period of 1827 and 1828, to .	10,648	persons, with	18,387,200	thalers.
2nd decennium (1839–48), to	10,404	„	16,692,000	„
3rd decennium (1849–58), to	12,380	„	20,959,000	„
Total . .	33,432	„	56,038,200	„

The new members were, therefore, considerably more numerous in the last period than in either of the two former; and as they exceed the withdrawals by more than double, we may surely hope that the assurances in the Gotha Office will reach a still greater extent than they have already done.

The calculations of the Gotha Office are founded on the table computed by Charles Babbage according to the experience of the Equitable Society, with the modification that the mortality of the higher ages above 55 years, which that table seemed to represent as too small, was augmented. It will not be without interest to see how true and safe that table has proved in the 30 years now past. This is shown by the Table IV.; according to it—

Should die	6787·68	persons, with	11,215,898	thalers	assured sum.
Died .	6628·00	„	10,988,974	„	„
Therefore, *less* died	159·68	„	226,924	„	„

There have been, then, disbursed in the 30 years past, 226,924 thalers, or about 2 per cent. *less* for deaths than might have been spent according to the basis of calculation adopted; and the number of the claims was about 2½ per cent. below the expectation. The adopted table of mortality has therefore proved, upon the whole, convenient and safe. If, however, we follow mortality into the single classes of age, there appear, indeed, greater deviations; for mor-

tality at the ages under 55 years was regularly less, and at the ages above 55 years greater, than the table in use led us to expect, as this was formerly (vol. iv., p. 54; vol. v., p. 58; vol. vi., p. 295; vol. vii., p. 177) proved by the results of single years. A nearer examination has shown that the greater mortality at the ages above 55 years is not caused by such of the assured as are admitted at these ages, but by those who, having been assured at the younger periods of life, attain to the higher ages. The health of the latter, examined at earlier periods, is naturally, on an average, not so good at their entrance into the higher ages, as the health of those who did not undergo the examination till later in life. The examination of health, however, just at the period from the 40th to the 55th year is of great consequence, because at that period of life essential change takes place in the constitution of man—not only of females, but also of males—which have a decided influence on their health. Of those who pass an examination as healthy lives at the age of 40, a greater number is doubtful and bad after 10 years than of those who have been selected as healthy at ages 30 and 50, and are reconsidered after an equal period.

During the 30 years past (1829–58), 6,779 cases of death occurred amongst the persons assured in the Gotha Office, which are distributed in the different classes of age, as shown in Table V. This table has been constructed in the same manner as the table published vol. v., p. 329, for the first 25 years (1829–53), during which period 4,521 cases of death had happened. The experience has, therefore, been enriched by 2,258 cases in the last five years. However, the results formerly obtained have not been essentially modified thereby, as is seen by a comparison of cols. 9 and 10 in Table V. This table is founded on the experience gathered from one calendar year to another. Beside the registers destined for them, on the basis of which the annual balances are made and the reserves are computed, there are arranged registers of the persons assured and the cases of death according to the years of assurance (vol. vi., p. 1), which do not require a correction for admission and withdrawal, as was necessary in Table V., cols. 6 and 7, and they furnish, therefore, more exact data. A table of mortality deduced from these data will be computed as soon as the experience in the younger and higher ages has become somewhat fuller, and, in consequence, more certain in its results.

Table VI. exhibits a comparative view of mortality according to different tables. It results from this table that the mortality in the Gotha Company in the ages under 55 years agrees almost

entirely with the mortality in the English Life Insurance Companies, as it is expressed in the Actuaries' Table, founded on the experience of 17 Companies. Beyond the 55th year of age, however, the mortality is essentially greater among the assured in the Gotha Office than in the English Companies. This difference has a general reason, because, in our opinion, mortality is in general less at the higher ages in England than on the continent. It may be that people enter there the higher ages with less weakened vitality, or that they are preserved from adverse influences at this period of life more in England than on the continent, or that both causes co-operate. The fact itself seems no longer doubtful, and the same result will be noticed on a comparison of the tables of mortality founded on the experience of the mixed population, though these tables are less accurate and certain than those which are based on the experience of the Assurance Companies. We refer to the data furnished by Nos. 8–12 of Table VI. According to them, the proportions for the higher ages seem to be more unfavourable, particularly in Germany (Hanover and Saxony), because mortality after the 50th year is greater there, even considerably greater, than in England. The mortality at the higher ages does not appear so favourable either in Belgium or in France, as in England, which country is, therefore, a particularly fortunate locality for aged persons.

It is rather remarkable that mortality amongst the members of the Friendly Societies in England is essentially less than amongst the members of Life Insurance Companies, though they are not obliged to undergo, at their entrance, so severe an examination respecting their health as the latter. The correctness of this fact is not liable to any doubt whatever, in consequence of the carefulness of the investigations made by the two highly-respectable authorities, Finlaison and Neison, the results of which are shown by Nos. 4 and 5 in Table VI. This observation, however, is of weight only for England—it is just the reverse in Germany.

In conclusion, we must not omit to draw attention to the mortality amongst the members of the Mutual Life Assurance Society at New York, which is *less* at most ages than amongst the members of the English and German Life Insurance Companies, though part of them live in the southern regions of North America, which are injurious to health. We owe the statement of this interesting fact to the elaborate researches of the able actuary of that Society, Mr. Homans, in his valuable report on the experience of the Society during the first 15 years of its existence.

TABLE I.—*New Business and Position of the Life Assurance Companies of Germany in the Year* 1858.

Established.	Name of Company.	Assurances existing at the beginning of the Year.		New Assurances during the Year.		Assurances existing at the end of the Year.		Income (Premiums and Interest).	Claims paid.		Expenses of Management.			Assurance Fund.				Average Dividend during the last ten years in per Cent. of Premium.	Share Capital.	
														In General.		Reserve and advanced Premiums.	Clear Surplus.			
		Persons.	Sums.	Persons.	Sums.	Persons.	Sums.		Persons.	Sums.	Absolute.	Per Cent. of Income.	Per £1000 of Assured Sums at end of Year.	Amount.	Per Cent. of Assured Sums at end of Year.				Nominal.	Paid up.
			£		£		£	£		£	£			£		£	£		£	£
1827	Gotha	20,841	4,792,614	1,316	331,329	21,470	4,944,028	231,597	517	125,871	9,208	3·98	1·87	1,324,212	26·78	1,080,968	216,177	28·	Mutual	Mutual
1828	Lubeck ..	6,936	984,571	2,396	272,512	8,777	1,171,793	48,062	133	24,872	6,217	12·94	5 31	122,656	10·47	?	?	0·251 On Sum Assured	72,857	7,286
1830	Leipsig ..	4,881	786,786	357	51,886	5,041	807,171	38,924	133	21,086	2,404	6·18	2·98	233,135	28·88	198,838	27,126	16½	Mutual	Mutual
1830	Hanover ..	*2,889	251,685	*105	10,100	*2,878	251,671	9,186	59	4,979	?	?	?	44,943	17·86	?	?	None	Mutual	Mutual
1834	Trieste.... (Generali)	†4,400	971,429	†2,000	471,428	†5,600	1,285,714	57,143	†110	†25,714	?	?	?	?	?	?	?	None	420,000	?
1836	Berlin	8,390	1,469,486	480	105,843	8,510	1,508,043	72,532	256	42,671	5,286	7·29	3·51	836,486	22·31	284,872	51,614	14⅔	142,857	28,571
1836	Munich ..	2,424	262,278	318	40,424	2,570	286,016	6,286	47	4,718	?	?	?	46,441	16·24	?	?	None	Capital of the Loan Bank.	
1839	Vienna ..	10,419	426,963	1,322	56,819	11,142	455,916	18,013	184	8,337	2,143	11·90	4·70	48,181	10·57	36,815	11,366	13⅓	Mutual	Mutual
1842	Brunswick	1,020	79,314	70	6,086	1,063	83,243	3,286	19	1,329	?	?	?	14,285	17·16	?	?	c. 6	Mutual	Mutual
1844	Frankfort .	1,336	256,637	322	53,009	†1,600	285,714	13,428	15	2,343	1,857	13·83	6·50	17,149	6·00	?	?	6⅔	244,898	24,489
1847	Hamburg . (Janus)	*6,235	716,880	*1,270	131,196	*7,032	792,878	28,550	84	10,284	4,955	17·36	6·25	61,185	7·72	?	?	7⅔	71,428	7,143
1852	Leipsig .. (Teutonia)	*1,259	95,447	*327	32,463	*1,440	117,181	6,375	53	2,661	1,875	29·42	16·00	12,286	..	21,222 Debit	None	None	85,714	12,857
1853	Cologne .. (Concordia)	†3,400	973,262	†1,000	221,812	†4,200	1,112,865	32,857	†50	16,548	?	?	?	109,433	9·84	79,351	23,341	None	869,714	173,943
1853	Schwerin..	*292	51,900	*73	14,600	*349	63,886	1,888	5	1,114	?	?	?	3,586	5·61	2,059	1,527	..	14,286	1,428
1854	Halle .... (Iduna)	*1,868	182,956	575	61,736	*2,232	217,349	10,455	62	4,932	3,046	29·14	14·02	7,392	3·40	12,870 Debit	None	None	28,571	?
1854	Stuttgart..	1,539	247,460	516	87,640	2,002	323,795	12,102	19	1,796	734	6·06	2·27	27,728	8·56	16,594	10,354	..	Mutual	Mutual
1855	Darmstadt	861	49,720	283	14,825	1,073	62,744	2,149	42	820	?	?	?	4,435	..	?	None	None	Capital of the Annuity Society.	
1856	Magdeburg	*933	154,311	*573	111,233	†1,353	252,608	13,015	94	1,526	4,689	36·03	18·56	8,864	..	8,837	3,056	None	282,000	56,400
1857	Stettin ...	†600	121,429	†1,150	228,571	1,606	323,832	11,251	16	1,614	?	?	?	9,798	..	14,362	6,184	None	428,571	85,714
1857	Zurich....	..	..	192	36,786	190	36,563	1,283	2	223	513	..	..	5,649	..	?	202	None	Capital of the Credit Bk.	
	Total ..	80,523	12,875,128	14,645	2,340,298	90,128	14,383,010	618,382	1,900	303,438	..	..	..	2,437,844	..	..	..	..	..	..

* Policies. † Approximated.

TABLE II.—*Showing the Gross Premium per Cent., the Average Dividends,*

Established.	Name of Company.	Average Dividend per Cent.	Age 25.			Age 30.			Age 35.		
			Gross Premium.	Dividends.	Net Premium.	Gross Premium.	Dividends.	Net Premium.	Gross Premium.	Dividends.	Net Premium.
1827	Gotha	28	2·356	0·660	1·696	2·633	0·737	1·896	2·969	0·831	2·138
1828	Lubeck	*Old Tariff*: 0·251 on sum assured	2·403	0·251	2·152	2·669	0·251	2·418	2·992	0·251	2·741
		New Tariff: non profit, 1857	..	..	1·828	..	..	2·106	..	..	2·461
1830	Leipsig	16½	2·356	0·389	1·967	2·633	0·434	2·199	2·969	0·490	2·479
1834	Trieste (Assicurazioni Generali)	Non profit	..	..	2·14	..	..	2·42	..	..	2·76
1836	Berlin	*Old Tariff*: 14⅔	2·406	0·346	2·060	2·672	0·385	2·287	2·992	0·431	2·561
		New Tariff, 1854: Not yet divided	2·133	?	?	2·433	?	?	2·817	?	?
		Non profit	..	..	2·000	..	..	2·283	..	..	2·658
1836	Munich	Non profit	..	..	2·167	..	..	2·433	..	..	2·750
1839	Vienna (Mutual)..	c. 13	2·150	0·279	1·871	2·483	0·323	2·160	2·883	0·375	2·508
1842	Brunswick	c. 6	2·125	0·127	1·998	2·382	0·143	2·239	2·722	0·163	2·559
1844	Frankfort	Non profit	..	..	1·992	..	..	2·242	..	..	2·550
1847	Hamburg	Non profit	..	..	1·950	..	..	2·225	..	..	2·542
1851	Trieste (Asienda Assicuratrice)..	Non profit	..	..	2·12	..	..	2·40	..	..	2·76
1852	Vienna (First Austrian Assurance Company)	Non profit	..	..	2·100	..	..	2·383	..	..	2·717
1852	Leipsig (Teutonia)	Not yet divided	2·181	?	?	2·500	?	?	2·861	?	?
1853	Trieste (Riunione Adriatica di Sicurtà)	Non profit	..	..	1·96	..	..	2·25	..	..	2·57
1853	Cologne	Non profit	..	..	1·783	..	..	2·042	..	..	2·383
1853	Schwerin	Only two dividends	2·203	?	?	2·521	?	?	2·906	?	?
1854	Halle (Iduna)	Not yet divided	2·192	?	?	2·500	?	?	2·861	?	?
1854	Stuttgart	Only one dividend	2·230	?	?	2·457	?	?	2·760	?	?
1855	Darmstadt	Non profit	..	..	1·750	..	..	2·017	..	..	2·350
1856	Magdeburg	Non profit	..	..	1·867	..	..	2·083	..	..	2·358
1856	Erfurt (Thuringia)	Non profit	..	..	1·758	..	..	2·008	..	..	2·333
1857	Stettin (Germania)	Non profit	..	..	1·817	..	..	2·075	..	..	2·408
1857	Frankfort ..... (Providentia)	Non profit	..	..	1·883	..	..	2·125	..	..	2·425
1857	Zurich	Non profit	..	..	2·174	..	..	2·439	..	..	2·778
1857	Trieste (Nuova Società Commerciale)	Non profit	..	..	2·050	..	..	2·330	..	..	2·660
1858	Vienna (Anker) ..	Non profit	..	..	1·920	..	..	2·240	..	..	2·550
	Average Premium	..	2·249	..	1·958	2·535	..	2·221	2·885	..	2·542

An average dividend is only computed for those

and the Net Premium after Reduction, in the German Life Assurance Companies.

Age 40.			Age 45.			Age 50.			Age 55.			Age 60.		
Gross Premium.	Dividends.	Net Premium.	Gross Premium.	Dividends.	Net Premium.	Gross Premium.	Dividends.	Net Premium.	Gross Premium.	Dividends.	Net Premium.	Gross Premium.	Dividends.	Net Premium.
3·386	0·948	2·438	3·961	1·109	2·852	4·733	1·325	3·408	5·742	1·608	4·134	7·161	2·005	5·156
3·400	0·251	3·149	3·894	0·251	3·643	4·531	0·251	4·280	5·317	0·251	5·066	6·367	0·251	6·116
..	..	2·928	..	..	3·564	..	..	4·417	..	..	5·561	..	..	7·133
3·886	0·641	3·245	3·961	0·654	3·307	4·733	0·781	3·952	5·742	0·947	4·795	7·161	1·182	5·979
..	..	3·21	..	..	3·81	..	..	4·66	..	..	5·78	..	..	7·25
3·400	0·490	2·910	3·898	0·561	3·337	4·533	0·653	3·880	5·317	0·766	4·551	6·367	0·917	5·450
3·333	?	?	3·950	?	?	4·750	?	?	5·858	?	?	7·417	?	?
..	..	3·150	..	..	3·758	..	..	4·558	..	..	5·650	..	..	7·142
..	..	3·183	..	..	3·750	..	..	4·517	..	..	5·533	..	..	6·933
3·333	0·433	2·900	3·950	0·513	3·437	4·733	0·615	4·118	5·667	0·737	4·930	6·967	0·906	6·061
3·187	0·191	2·996	3·805	0·228	3·577	4·660	0·280	4·380	5·854	0·351	5·503	7·479	0·449	7·030
..	..	2·950	..	..	3·483	..	..	4·192	..	..	5·142	..	..	6·422
..	..	2·950	..	..	3·425	..	..	4·100	..	..	5·125	..	..	6·492
..	..	3·24	..	..	3·84	..	..	4·64	..	..	5·70	..	..	7·23
..	..	3·167	..	..	3·767	..	..	4·550	..	..	5·633	..	..	7·183
3·300	?	?	3·900	?	?	4·667	?	?	5·624	?	?	6·833	?	?
..	..	2·99	..	..	3·62	..	..	4·35	..	..	5·55	..	..	7·07
..	..	2·825	..	..	3·417	..	..	4·208	..	..	5·267	..	..	6·692
3·391	?	?	4·005	?	?	4·802	?	?	5·906	?	?	7·661	?	?
3·317	?	?	3·850	?	?	4·600	?	?	5·750	?	?	7·200	?	?
3·182	?	?	3·757	?	?	4·572	?	?	5·688	?	?	7·077	?	?
..	..	2·800	..	..	3·433	..	..	4·300	..	..	5·550	..	..	7·317
..	..	2·758	..	..	3·333	..	..	4·108	..	..	5·033	..	..	6·300
..	..	2·750	..	..	3·317	..	..	4·083	..	..	5·267	..	..	6·717
..	..	2·842	..	..	3·417	..	..	4·192	..	..	5·208	..	..	6·592
..	..	2·821	..	..	3·396	..	..	4·167	..	..	5·212	..	..	6·671
..	..	3·226	..	..	3·846	..	..	4·545	..	..	5·556	..	..	7·143
..	..	3·100	..	..	3·710	..	..	4·500	..	..	5·660	..	..	7·160
..	..	2·990	..	..	3·600	..	..	4·390	..	..	5·430	..	..	6·830
3·329	..	2·980	3·903	..	3·526	4·665	..	4·271	5·679	..	5·285	7·063	..	6·670

Companies who have divided at least five dividends.

TABLE III.—*Assurances in the Gotha Life*

Year.	Assurances proposed.		New Assurances effected.		Assured in the course of the Year.		Decreased. By discontinued, expired, and lapsed Policies.	
	Pers.	Thlrs.	Pers.	Thlrs.	Pers.	Thlrs.	Pers.	Thlrs.
1829	1,581	2,746,600	1,285	2,379,200	1,285	2,379,200	..	.
1830	607	1,121,700	504	1,044,700	1,777	3,408,800	14	26,200
1831	1,491	2,501,400	1,244	2,348,400	2,991	5,696,700	46	126,000
1832	1,574	2,513,500	1,165	2,168,100	4,083	7,637,500	212	490,100
1833	1,351	1,975,300	1,041	1,738,100	4,857	8,832,800	120	358,700
1834	1,218	1,749,800	902	1,424,500	5,593	9,825,700	154	335,500
1835	1,318	1,877,900	989	1,577,600	6,361	10,958,000	155	298,300
1836	1,635	2,390,400	1,213	1,932,500	7,333	12,422,600	170	383,800
1837	1,581	2,435,600	1,151	1,886,200	8,204	13,743,300	170	290,200
1838	1,555	2,224,900	1,154	1,887,900	9,061	15,114,200	156	338,200
Total 1829–38	13,911	21,537,100	10,648	18,387,200	51,545	90,069,000	1,197	2,647,000
1839	1,419	2,143,000	1,024	1,743,200	9,803	16,304,700	195	378,900
1840	1,485	1,942,800	1,089	1,568,400	10,570	17,283,500	193	354,600
1841	1,414	2,002,100	983	1,604,800	11,217	18,255,700	178	292,400
1842	1,352	1,939,800	1,013	1,594,700	11,688	19,264,100	167	348,600
1843	1,503	2,082,600	1,049	1,653,400	12,572	20,259,200	193	398,600
1844	1,493	2,101,700	1,073	1,747,500	13,249	21,281,800	183	291,800
1845	1,412	2,046,000	1,018	1,646,800	13,884	22,281,000	178	327,100
1846	1,484	1,961,300	1,096	1,643,500	14,564	23,218,700	208	388,800
1847	1,706	2,425,200	1,235	1,948,200	15,361	24,412,400	240	396,500
1848	1,151	1,790,200	824	1,531,500	15,652	25,070,700	277	543,200
Total 1839–48	14,419	20,434,700	10,404	16,692,000	128,760	207,631,800	2,012	3,720,500
1849	1,377	1,946,500	1,011	1,629,800	16,047	25,641,000	239	449,500
1850	1,537	2,138,000	1,138	1,860,500	16,609	26,479,600	196	402,300
1851	1,667	2,369,200	1,262	2,018,800	17,342	27,522,000	190	359,000
1852	1,868	2,671,600	1,423	2,326,600	18,278	29,028,900	217	421,200
1853	1,804	2,576,900	1,324	2,153,900	19,039	30,182,300	213	418,800
1854	1,534	2,240,700	1,073	1,855,900	19,500	30,971,100	252	443,400
1855	1,699	2,466,300	1,156	2,032,600	20,014	31,927,200	180	304,300
1856	1,910	2,768,800	1,314	2,183,100	20,677	33,071,800	169	309,800
1857	1,913	3,090,400	1,363	2,578,500	21,461	34,637,900	167	332,300
1858	1,901	2,938,200	1,316	2,319,300	22,157	35,867,600	151	341,700
Total 1849–58	17,210	25,206,600	12,880	20,959,000	191,124	305,329,400	1,974	3,782,300
Total	45,540	67,178,400	33,432	56,038,200	371,429	603,030,200	5,183	10,149,800
Average for one year..	1,518	2,239,280	1,114	1,867,940	12,381	20,101,007	173	338,327

Assurance Bank during the Years 1829–58.

Decreased (*continued*).				Net Increase.		Existing at the end of the Year.				
By Death.		Total Decrease.						Average Sum on one Life.	Average Age of Assured.	
Pers.	Thlrs.	Pers.	Thlrs.	Pers.	Thlrs.	Pers.	Thlrs.	Thlrs.	Yrs.	Ms.
12	15,100	12	15,100	1,273	2,364,100	1,273	2,364,100	1,857	42	7
16	34,300	30	60,500	474	984,200	1,747	3,348,300	1,917	42	9
27	51,300	73	177,300	1,171	2,171,100	2,918	5,519,400	1,892	41	10
55	102,700	267	592,800	898	1,575,300	3,816	7,094,700	1,859	42	4½
46	72,900	166	431,600	875	1,306,500	4,691	8,401,200	1,791	42	9
67	109,800	221	445,300	681	979,200	5,372	9,380,400	1,746	43	1
86	169,400	241	467,700	748	1,109,900	6,120	10,490,300	1,714	43	3½
110	181,900	280	565,700	933	1,366,800	7,053	11,857,100	1,681	43	5
127	226,800	297	517,000	854	1,369,200	7,907	13,226,300	1,673	43	8
126	214,500	282	552,700	872	1,335,200	8,779	14,561,500	1,659	44	0
672	1,178,700	1,869	3,825,700	8,779	14,561,500	.	..	17,789	..	
127	210,700	322	589,600	702	1,153,600	9,481	15,715,100	1,658	44	4½
143	278,000	336	632,600	753	935,800	10,234	16,650,900	1,627	44	8¾
164	293,900	342	586,300	641	1,018,500	10,875	17,669,400	1,625	45	1
198	314,700	365	663,300	648	931,400	11,523	18,600,800	1,614	45	5
203	326,300	396	724,900	653	933,500	12,176	19,534,300	1,604	45	10
200	355,800	393	647,600	690	1,099,900	12,866	20,634,200	1,604	46	0
238	383,700	416	710,800	602	936,000	13,468	21,570,200	1,602	46	4
230	365,700	438	754,500	658	894,000	14,126	22,464,200	1,590	46	7
293	476,700	533	873,200	702	1,075,000	14,828	23,539,200	1,588	46	10
339	516,300	616	1,059,500	208	472,000	15,036	24,011,200	1,597	47	4
2,135	3,521,800	4,147	7,242,300	6,257	9,449,700	..	..	16,109	..	
337	572,400	576	1,021,900	435	607,900	15,471	24,619,100	1,591	47	7
333	574,100	529	976,400	609	884,100	16,080	25,503,200	1,586	47	10
297	460,700	487	819,700	775	1,199,100	16,555	26,702,300	1,584	48	0
346	579,300	563	1,000,500	860	1,326,100	17,715	28,028,400	1,582	48	2
399	648,300	612	1,067,100	712	1,086,800	18,427	29,115,200	1,580	48	3
390	633,100	642	1,076,500	431	779,400	18,858	29,894,600	1,585	48	6
471	734,200	651	1,038,500	505	994,100	19,363	30,888,700	1,595	48	9
410	702,600	579	1,012,400	735	1,170,700	20,098	32,059,400	1,595	48	11
453	757,300	620	1,089,600	743	1,488,900	20,841	33,548,300	1,610	49	0
536	917,700	687	1,259,400	629	1,059,900	21,470	34,608,200	1,612	49	2
3,972	6,579,700	5,946	10,362,000	6,434	10,597,000	..	..	15,920	..	
6,779	11,280,200	11,962	21,430,000	21,470	34,608,200	..	..	49,818	..	
226	376,007	399	714,333	716	1,153,607	..	..	1,661	..	

TABLE IV.—*Claims payable during* 1829–58.

Year.	Amount.					Expected according to the Table in use.			Difference.			
			Average for one Life.	Per cent. of the Assurances.				Average for one Life.	There died Plus.		Minus.	
	Pers.	Thlrs.	Thlrs.	Pers.	Thlrs.	Pers.	Thlrs.	Thlrs.	Pers.	Thlrs.	Pers.	Thlrs.
1829	12	15,100	1,258	0·93	0·63	15·19	27,979	1,842	..	..	3·19	12,879
1830	16	34,300	2,144	0·90	1·01	22·84	43,688	1,913	..	..	6·84	9,388
1831	27	50,300	1,863	0·90	0·88	32·38	63,254	1,954	..	..	5·38	12,954
1832	53	102,300	1,930	1·30	1·33	50·25	94,681	1,884	2·75	7,619	..	..
1833	46	71,032	1,544	0·95	0·80	64·81	117,700	1,816	..	..	18·81	46,668
1834	65	108,300	1,666	1·16	1·10	77·19	135,909	1,761	..	..	12·19	27,609
1835	85	165,400	1,946	1·34	1·51	89·34	156,092	1,747	..	9,308	4·34	..
1836	108	179,200	1,659	1·47	1·44	103·42	178,234	1,723	4·58	966	..	..
1837	123	217,417	1,768	1·50	1·58	117·99	202,162	1,713	5·01	15,255	..	..
1838	124	211,900	1,709	1·37	1·40	133·88	228,054	1,703	..	..	9·88	16,154
Total 1829–38	659	1,155,249	17,487	11·82	11·68	707·29	1,247,753	18,056	12·34	33,148	60·63	125,652
1839	123	206,400	1,678	1·25	1·26	149·74	254,226	1,699	..	..	26·74	47,826
1840	136	256,200	1,884	1·29	1·48	164·25	276,936	1,686	..	..	28·25	20,736
1841	158	278,525	1,763	1·41	1·53	179·02	299,402	1,672	..	..	21·02	20,877
1842	191	300,800	1,575	1·61	1·56	193·93	322,746	1,664	..	..	2·93	21,946
1843	201	323,800	1,611	1·60	1·60	208·55	345,219	1,655	..	..	7·55	21,419
1844	197	340,400	1,728	1·49	1·60	224·81	370,748	1,649	..	..	27·81	30,348
1845	235	381,200	1,622	1·69	1·71	241·13	397,277	1,648	..	..	6·13	16,077
1846	224	353,800	1,579	1·54	1·52	257·03	421,986	1,642	..	..	33·03	68,186
1847	289	470,000	1,629	1·88	1·93	275·53	450,174	1,634	13·47	19,826	..	..
1848	326	496,500	1,523	2·08	1·98	290·27	474,505	1,636	35·73	21,995	..	..
Total 1839–48	2,080	3,407,625	16,592	15·84	16·17	2184·26	3,613,219	16,585	49·20	41,821	153·46	247,415
1849	335	566,500	1,691	2·09	2·21	302·28	494,798	1,638	32·72	71,702	..	..
1850	326	556,600	1,707	1·96	2·10	318·57	520,490	1,634	7·43	36,110	..	..
1851	293	459,000	1,567	1·69	1·67	337·52	550,166	1,630	..	..	44·52	91,166
1852	335	554,900	1,656	1·83	1·91	359·54	584,449	1,626	..	..	24·54	29,549
1853	392	635,800	1,625	2·06	2·11	382·07	618,850	1,620	9·93	16,950	..	..
1854	375	608,600	1,623	1·99	2·04	400·52	649,828	1,622	..	..	25·52	41,228
1855	466	721,450	1,548	2·41	2·34	417·83	681,135	1,630	48·17	40,315	..	..
1856	406	692,450	1,706	2·02	2·16	437·73	715,080	1,633	..	..	31·73	22,630
1857	444	749,700	1,688	2·06	2·16	459·28	752,017	1,637	..	..	15·28	2,317
1858	517	881,100	1,704	2·33	2·46	480·79	788,113	1,639	36·21	92,987	..	..
Total 1849–58	3,889	6,426,100	16,515	20·44	21·16	3896·13	6,354,926	16,309	134·46	258,064	141·59	186,890
Total....	6,628	10,988,974	50,594	48·10	49·01	6787·68	11,215,898	50,950	196·00	333,033	355·68	559,957
Average for one Year	221	366,299	1,687	1·60	1·63	226·26	373,863	1,698	6·53	11,101	11·86	18,665

Table V.—*Persons Assured in the Gotha Life Bank, and Deaths during* 1829–58.

Ages.	Number of Persons assured.	Number of Persons entered.	Number of Persons gone in lifetime.	Total of 3 and 4.	Half of the Numbers in Col. 5.	Number exposed to the Risk of Mortality for a whole Year.	Deaths.	Rate of Mortality per cent. 1829–58.	Rate of Mortality per cent. 1829–53.
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
15–25	2,833	1,241	165	1,406	703	2,130	12	0·56	0·46
26–30	14,643	4,625	557	5,182	2,591	12,052	100	0·83	0·87
31–35	37,584	7,304	896	8,200	4,100	33,484	290	0·87	0·92
36–40	56,680	7,180	873	8,053	4,027	52,653	508	0·97	1·00
41–45	64,213	5,201	850	6,051	3,025	61,188	628	1·03	1·04
46–50	61,802	3,715	799	4,514	2,257	59,545	868	1·46	1·45
51–55	52,195	2,464	523	2,987	1,493	50,702	963	1·90	1·82
56–60	38,599	1,446	323	1,769	884	37,715	1,014	2·69	2·77
61–65	23,926	241	133	374	187	23,739	917	3·86	3·83
66–70	12,145	15	43	58	29	12,116	734	6·06	6·08
71–75	4,944	..	15	15	7	4,937	482	9·76	9·04
76–80	1,562	..	6	6	3	1,559	190	12·19	11·35
81–85	281	..	..	..	..	281	65	23·13	23·94
86–90	24	..	..	..	..	24	8	33·33	33·33
Total	371,431	33,432	5,183	38,615	19,306	352,125	6,779		

Table VI.—*Comparative View of the Annual Rate of Mortality according to different Tables.*

	26–30.	31–35.	36–40.	41–45.	46–50.	51–55.	56–60.	61–65.	66–70.	71–75.	76–80.	81–85.
1. Gotha Life Experience (1829-58)	0·83	0·87	0·97	1·03	1·46	1·90	2·69	3·86	6·06	9·76	12·19	23·13
2. Actuaries' Table (17 English Life Offices, 1762-1840)	0·81	0·89	0·99	1·13	1·43	1·91	2·65	3·79	5·55	8·13	11·88	17·22
3. Equitable Experience (1762-1840, Morgan)	0·78	0·88	1·03	1·18	1·38	1·85	2·68	3·72	5·48	7·89	11·18	17·97
4. Friendly Societies' Experience (1836-40, Neison)	0·73	0·80	0·89	1·04	1·29	1·70	2·24	3·05	4·62	6·85	8·84	11·97
5. Ditto ditto (1846-50, Finlaison)	0·75	0·80	0·95	1·13	1·37	1·77	2·45	3·12	4·75	6·70	10·32	15·31
6. New York Mutual Society Exp. (1844-58, Homans)	0·85	0·89	0·93	0·96	1·20	1·52	2·03	3·07	4·85	7·66	13·01	18·96
7. Carlisle Table (1779-87, Heysham—Milne)	0·87	1·02	1·15	1·45	1·41	1·61	2·50	3·84	4·66	7·67	11·08	14·88
8. Hanover (1825-43) Tellkamps	1·11	1·25	1·35	1·48	1·77	2·50	3·61	5·70	7·91	9·34	12·74	18·50
9. Saxony (1840-49, Heym)	0·84	0·99	1·11	1·32	1·63	2·24	3·22	4·81	7·35	11·31	15·94	21·58
10. England, Males (1841, Farr)	0·97	1·10	1·25	1·42	1·62	1·87	2·71	3·95	5·75	8·32	11·94	16·90
11. France, Males (1817-31, Demonferrand)	0·88	0·95	0·94	1·17	1·43	1·93	2·50	4·10	5·60	9·25	12·79	18·62
12. Belgium, Males (1841-45, Quetelet)	1·50	1·58	1·72	2·16	2·43	2·47	3·15	4·24	5·83	8·65	12·86	18·01

NOTICES OF NEW WORKS.

An Essay on Life Assurance; being a popular Exposition of the Subject, and a Plea for its more general Adoption. By H. W. PORTER, B.A. Charles and Edwin Layton, Fleet Street.

IN this essay Mr. Porter points out the great advantages to be derived from the practice of life assurance, touches briefly upon the principles on which that practice is founded, describes the nature of the several institutions established for the assurance of lives and some of the various modes in which contracts of assurance are framed, combats successfully certain false and very prevalent notions with regard to the system itself, and concludes with such forcible observations as to the desirableness of persons in ordinary circumstances resorting to it, as must, we should think, have considerable influence upon any reader of them. We have good reason to believe that there are several classes of our vast population by which this essay would be read with advantage. There are many educated persons still who are quite ignorant, not only of the theory, but of the practice of assurance. Some doubt the soundness of the principles, and some the advantages of the system, whilst many are led to neglect the opportunities it affords by sheer carelessness and inattention.

Writing on this point, Mr. Porter aptly says:—"I sincerely believe that the reason why life assurance is so little resorted to, is not owing to any reluctance on the part of those who have wives and children depending upon them for support—and whose widows and orphans would be reduced to poverty by their premature decease—to make a small present pecuniary sacrifice to prevent so cruel an alternative, but because their attention has not been called to the subject, or, if it has, because they have not fully understood it or reflected sufficiently on the matter. It is hardly to be imagined that the father of a family, who works hard day by day for the support of his wife and children, and cheerfully exerts every effort for their advantage, would fail to make provision for them by the assurance of his life, supposing that he were fully cognizant of the advantages to be derived therefrom by his family, and that he clearly understood in what manner he might secure an independence for them in the event of his early death, or—if his means would not by possibility allow of his making such a provision for them as would constitute an independence—at least, a sufficient sum of money to enable them to provide for all demands at the time of his decease, to satisfy all expenses connected with his death and probable antecedent illness, and to enable them to live in comfort till such time as arrangements could be made for providing for his family, who might otherwise be left destitute, and to enable them to start fair again in the race of life. It is hardly to be imagined, I repeat, that his moral sense would fail to induce him to perform the duty so highly incumbent upon him, of making the best provision in his power for those dependent upon him, and who have clearly the same claim upon him *morally* to make provision for their support after his death, as they have *legally* upon him for their maintenance during his life."

We quite agree with Mr. Porter. It is, indeed, hardly to be imagined!

INSTITUTE OF ACTUARIES.

PROCEEDINGS OF THE INSTITUTE.

Second Ordinary Meeting, Session 1859-60.—*Monday, 2nd January,* 1860.

W. B. Hodge, Vice-President, in the Chair.

The minutes of the first ordinary meeting were read and confirmed.

The Secretary announced several donations to the library.

The following candidates, duly nominated at the last ordinary meeting, were elected members of the Institute, viz.:—

Official Associate—A. G. Ramsay.

Associates.

Charles Graham Carttar.
John James Holliday Chapman.
Sydney King.
John Thomas Minett.
William Pateman.

The Chairman announced the following as the results of the annual examinations:—

Out of five candidates for the matriculation examination, three passed, in the order of merit indicated below:

A. H. Green.
W. M. Makeham.
Sydney King.

Four candidates presented themselves for the second year's examination, and all passed, in the following order:

A. G. Finlaison.
W. C. Mullins.
C. G. Laing.
C. Bischoff, Jun.

For the third year's examination, two candidates appeared, and both passed, as follows:

W. P. Pattison.
James Terry.

These gentlemen are now entitled to certificates of competency.

Papers were read—"On a formula for calculating the value of a survivorship assurance," by M. Réboul; "On the purchase of life assurance policies as an investment," by Archibald Day, Esq.

Thanks having been voted to M. Réboul and Mr. Day, the meeting adjourned to Monday, the 30th inst.

Third Ordinary Meeting, Session 1859-60.—*Monday, 30th January,* 1860.

Charles Jellicoe, Vice-President, in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced several donations to the library.

The undermentioned gentlemen, duly nominated at the last ordinary meeting, were elected members of the Institute, viz. :—

Francis Archibald Corrie.
Alexander A. Green.

A paper was read "On some considerations suggested by the Reports of the Registrar-General; being an inquiry into the question as to how far the inordinate mortality in this country, exhibited by these Reports, is controllable by human agency." Part I. By H. W. Porter, Esq., B.A.

Thanks having been voted to Mr. Porter, the meeting adjourned to Monday, the 27th February.

Fourth Ordinary Meeting, Session 1859-60.—*Monday, 27th February,* 1860.

W. B. HODGE, Vice-President, in the Chair.

The minutes of the last ordinary meeting were read and confirmed.
The Secretary announced various donations to the library.
The undermentioned gentlemen, duly nominated at the last ordinary meeting, were elected members of the Institute, viz. :—

Official Associates.

Edward Butler. | F. Ferguson Camroux.

Associates.

Edward Plumbridge Clark. | John Edward Primrose Gage.
John Elphick Leyland.

Letters were read from Charles Babbage, Esq., M.A., and Professor Sylvester, acknowledging, with thanks, their election as honorary members of the Institute.

A paper was read, "On some considerations suggested by the Reports of the Registrar-General; being an inquiry into the question as to how far the inordinate mortality in this country, exhibited by these Reports, is controllable by human agency." Part II. By H. W. Porter, Esq., B.A.

Thanks having been voted to Mr. Porter, the meeting adjourned to 26th March, 1860.

THE

ASSURANCE MAGAZINE,

AND

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On the Rates of Interest for the use of Money in Ancient and Modern Times. (Part IV.) By WILLIAM BARWICK HODGE, *Vice-President of the Institute of Actuaries, and Fellow of the Statistical Society.*

[Read before the Institute, 30th April, 1860.]

IN the former papers upon this subject[1] which I had the honour to lay before the Institute, I described the vehement controversy carried on in earlier ages as to the propriety of allowing any interest to be taken for the use of money, and I alluded to the discussion which, after interest to a limited extent had been allowed by law, arose as to the advantages to be derived from a reduction of the legal rate.

I now propose to give some account of this latter discussion, and of the writers who took part in it. Among the foremost of them was Sir Josiah Child, whose works upon commerce were long held in high esteem, and even now retain a great reputation.

To advocate a reduction to 4 or even 3 per cent. of the legal rate of interest (then at 6 per cent.), he wrote, in 1665, a *Discourse concerning Trade.* This tract was not published until 1668, and was then without the author's name—bearing the initials "J. C." only. It opens thus:—"The prodigious increase of the Netherlanders in their domestick and foreign trade, riches, and multitude of shipping, is the envy of the present, and may be the wonder of

[1] *Assurance Mag.*, vols. vi., vii., and viii.

all future, generations. And yet the means whereby they have advanced themselves are sufficiently obvious, and, in a great measure, imitable by other nations."[1]

The writer then proceeds to enumerate the causes of their success, attributing it, principally, to their reputation for integrity in trade; their thrifty mode of living; their care to bring up their children, both sons and daughters, to have a full knowledge of "arithmetick and merchants' accompts;" the superiority of their commercial code, with their use of a public register for recording transfers and mortgages of lands; and, lastly, "the lowness of interest of money with them." "This," he says, "in my poor opinion, is the *causa causans* of all the other causes of the riches of that people; and that if the interest of money were with us reduced to the same rate as it is with them, it would, in a short time, render us as rich and considerable in trade as they now are."[2]

In support of this opinion, he urges the extraordinary increase in the prosperity of the country which followed the reduction of interest in the reign of James I. and in that of Charles II.

The proofs adduced of this increased prosperity are abundant and striking enough, and form, indeed, the most valuable portion of the book. There is, however, little attempt to show the connection between the result and the causes assigned to it, beyond the expression of the writer's conviction—"that the bringing down of interest, from 6 to 4 or 3 per cent., will necessarily, in less than 20 years' time, double the capital stock of the nation."[3]

The theory thus laid down was by no means new. Indeed, its resemblance to one propounded, with much greater ability, half a century before, by Sir Thomas Culpepper the Elder, in the *Small Tract against Usury*, already mentioned,[4] might lead us to suppose it to have been derived from that source, did not Child assure us his own *Discourse* was written before he saw Sir Thomas's tract. He spoke of it, however, with great admiration, and reprinted it with the second edition of his work in 1670, as well as with the collected edition of his writings in 1693.[5]

That the matter had already been much discussed is obvious, from the objections cited by Child and his attempts to refute them. He quotes also "Sir Henry Blount (an honourable member of His Majesty's Council of Trade), as having well said before the Lords, *at the debate*, that abatement of interest was the *unum magnum* towards the prosperity of this kingdom."[6]

[1] Page 1. [2] Page 9. [3] Page 16. [4] *Assurance Mag.* viii. 75.
[5] London; 12mo. [6] *See* Preface.

Sir Thomas Culpepper appears to have published, in 1641, a second *Tract against Usury*, to enforce the doctrines contained in the first. It is referred to in Child's *Interest of Money Mistaken*,[1] and quoted in Macpherson's *History of Commerce*;[2] but I have not seen it elsewhere mentioned.

The same writer's son, Sir Thomas Culpepper the Younger, published, in 1668, *A Discourse, shewing the many Advantages which will accrue to this Kingdom by the Abatement of Usury*. The list of promised advantages proves the extraordinary delusions then existing as to the power of legislation to increase national wealth. The reduction of interest was to "supply His Majesty's present wants; in a short time double, if not treble, the yearly fruit and product of our lands; revive our dying manufactures; plentifully relieve the poor, by setting all our heads and hands to work; prevent the fatal destruction of our timber; pay the debts of the whole gentry by timely sales; make monie so easy to be borrowed that the lender must shortly pay the broker's conveyance; rebuild London, profitably to the builder and speedily for the public; and, lastly, inviolably establish the Crown of England, by the advancement of His Majesty's revenues, by the welfare of his subjects, and by making land the overbalancing scale of wealth and power." With amusing inconsistency, the author asserts, in another part of his book, that the reduction of interest to 6 per cent. during the Commonwealth was passed "by the grandees of that junta with a view to the utter oppression of the nobility and gentry."[3] Forgetting, too, "the fatal destruction of our timber," he elsewhere asks, "Have we not in many places in this kingdom iron oare without end, with woods adjacent even to a nuisance?"[4]

Although the great majority of his contemporaries appear to have shared Child's opinions upon the subject, his *Discourse* soon called forth replies. One was published anonymously, in the same year (1668), under the title of *Interest of Money Mistaken*;[5] and, in 1669, appeared *Interest at Six per Cent. Examined, by Thomas Manley, Gent.* The former of these I have not seen; but the author seems to have pointed out, clearly enough, that the low rate of interest in Holland was not the cause, but the consequence, of the large capitals accumulated there by successful trade. "Low interest," says Manley, too, in his preface, "is both in nature and time subsequent to riches, and he who says low usury begets riches takes the effect for the cause—the child for the mother—and puts the cart before the horse."

[1] Page 10. [2] ii. 242. [3] *See* Preface. [4] Page 13.
[5] *Ency. Brit.*; 7th edit.; art., "Pol. Eco."

Child wrote a special reply to *Interest of Money Mistaken*, under the title of *Trade and Interest of Money Considered*, in the preface to which he noticed Manley's book. In this preface, which was printed with the collected edition of his works (Lond., 1693), as if it had been the general preface to the whole, occurs his celebrated aphorism—"That land and trade are twins, and ever will wax and wane together. It cannot be ill with trade but land will fall, nor ill with land but trade will feel it."

The frequent repetition of this aphorism during the discussions upon the Corn Laws, has led to a popular impression that Child was one of the early advocates of freedom in trade; but this opinion has no real foundation. Indeed, the great authority possessed for so long a period by his name in commercial matters, arose from the fact that he was deeply imbued with nearly all the narrow prejudices upon which the so-called protective system was founded.[1]

In addition to the egregious error of believing that money could be made plentiful by Act of Parliament, he looked upon the Navigation Laws as England's *Charta Maritima*,[2] proposed differential duties of 50 per cent.[3], and strenuously recommended that Ireland and the colonies should be restricted to trading with England only.[4] The ferocity with which he spoke of the "total ruin and extirpation" of our commercial rivals, the Dutch,[5] could hardly have been exceeded in the worst ages of commercial barbarism. It is true, he endeavoured to shake the popular opinions upon the subject of the balance of trade,[6] but he was unable to perceive the error of the principle upon which those opinions were founded, and his inducement to the attempt, which, after all, was but a feeble one, was merely to defend the trade to the East Indies, from which a great part of his fortune was derived, and which could only be carried on by the exportation of bullion.

That Child was an acute observer is obvious from his works, and it is certain that he had a high character for sagacity with his contemporaries; but this would hardly be inferred from his reasoning in support of his theories on the subject of interest, which is generally weak, oftentimes contradictory, and sometimes contemptible. His notion, that the power, wealth, and populousness of the Jewish nation, in ancient times, were due to the scriptural prohibition of usury, was a strange crotchet to enter the mind of a trader.[7]

In one place, he denies that there was any ground for the com-

[1] *Hist. of Com.* ii. 843. [2] *See* Preface. [3] Page 97. [4] Page 95. [5] Page 10. [6] Page 135. [7] Page 28.

plaints of the scarcity of money, alleging that people complained, partly because it was their nature to complain, and partly because they were "uneasie in matters of their religion."[1] In another, he asserts that the scarcity was entirely caused by the new trade of bankering, which enabled every man who could scrape £50 or £100 together to deposit it with a goldsmith[2]—as if goldsmiths would pay 6 per cent. upon money, merely to lock it up in their tills! It is difficult to understand how such an idea could have been adopted by anyone acquainted with the commonest operations of trade. Manley, who was full of the prejudices of the day, attempted to account for the scarcity of money by the vast consumption of foreign commodities, and instanced the great increase in the importation of wines; whereupon Child accused him of asserting "that abatement of use money brought in our drinking."[3]

The author of *Interest of Money Mistaken* having objected that a reduction of the legal rate would lead to the withdrawal of large sums of foreign capital, Child's only answer is, "So much the better, for the borrower is always the slave of the lender."[4]

Samuel Fortrey, also spoken of with great approbation by Child, published, in 1673, *England's Interest and Improvement*, recently reprinted by the Political Economy Club.[5] This writer, who was gentleman of the bedchamber to Charles II., appears to have believed, that, by merely lowering the legal rate of interest, the nation might be converted from borrowers into lenders—asserting, that, by reducing the rate below that of other nations, they would make no profit on us by lending, but rather we on them.[6]

Among the arguments against a high rate of interest urged by Culpepper the Elder, was one, that a hundred pounds, "managed at ten in the hundred, in seventy years multiplies itself to a hundred thousand pounds;". and he thence inferred, that, if a hundred thousand pounds of foreigners' money be managed here at 10 per cent., it would in threescore and ten years carry out ten millions.[7] There is rather an important mistake in this estimate, as the correct sum, according to the proportion stated, should have been one hundred millions, instead of ten millions. Child took up the same theme. He did not repeat Culpepper's error, which, however, he did not

[1] *See* Preface. [2] *Ibid.* [3] *Ibid.* [4] Page 17.

[5] *A Select Collection of Early English Tracts on Commerce*, reprinted by the Political Economy Club; London, 1856; p. 211. I take this opportunity of expressing my high sense of the usefulness of this reprint, as well as of that, even more valuable, of the *Select Tracts on Money*; London, 1856. Both volumes are ably edited by Mr. McCulloch.

[6] *Ibid.*, p. 246. [7] Child's Reprint, p. 227.

correct, speaking himself merely of the increase of £100 to £100,000 at the same rate.[1] When the accuracy of his result was called in question, he went into a very elaborate calculation, and, by taking quarterly periods of conversion, showed that money at 10 per cent. doubled itself, within a fraction, in seven years. His mode of doing this was the cumbrous one of separately calculating and adding the interest of each succeeding quarter.[2] Neither he nor his opponents seem to have perceived that there was a very startling answer to this theory.

It is clear that merchants only borrow money to carry on trade which they are unable to carry on without borrowing, and that unless they make a larger profit upon the loan than suffices to pay the interest of it, they must either leave off borrowing or become bankrupts. Merchants' profits are of course fluctuating and uncertain; and, if they were certain, it would be very difficult to ascertain their average amount. According to Adam Smith, double interest was in his time, 1776, considered in Great Britain "a good, moderate, reasonable profit."[3] Manley asserts his belief that, in 1678, all trades gained 15 per cent.,[4] which was considerably more than double the then legal rate of interest. Taking Adam Smith's estimate as correct, the borrowers at 10 per cent., referred to by Child and Culpepper, would make on the average 20 per cent. upon the money borrowed. Now, one hundred thousand pounds accumulated at that rate for seventy years, assuming quarterly periods of conversion, as in Child's calculation, would amount to nearly eighty-six thousand millions of pounds, a sum that leaves the contemptible hundred millions of accumulated debt very far behind. The answer is framed upon exactly the same principle as the assertion, and the extravagance of the one is a sufficient exposure of the absurdity of the other. The truth is, that the elaborate calculations about accumulation have nothing to do with the question; but it is surprising how slight an acquaintance with the mysteries of compound interest—and Child's knowledge upon the subject does not appear to have been deep—will enable a man to bewilder the public.

Among the controversialists of the seventeenth century respecting interest, the latest and the most celebrated was John Locke, whose opinions were published in 1691 in a letter to a member of Parliament (most probably Charles Montague, afterwards First Lord of the Treasury and Chancellor of the Exchequer), entitled,

[1] *Trade, &c., Considered*, p. 7. [2] *Ibid.*, p. 9. [3] *Wealth of Nations*, book i., ch. 9.
[4] *Interest at Six per Cent. Examined.*

Some Considerations of the Consequences of lowering the Interest of, and raising the Value of, Money.[1]

The principal object of this letter was to refute the notions of Lowndes and others, who proposed to enrich the nation by raising the nominal value of the coin. The portion of it relating to interest is said by Locke to have been written twelve years before, when his attention had been called to the subject by the discussion then going on as to the advantage of reducing interest to 4 per cent. He goes fully into the question—"Whether the price of the hire of money can be regulated by law?"—and, after stating his reasons for believing it cannot, delivers his opinion that the attempt would —1, make the difficulty of lending and borrowing greater; 2, prejudice none but widows and orphans, whose estates lie in money; 3, tend only to increase the profits of brokers and scriveners; and, 4, lead to perjury and evasion. "People," he says, "who have their estates in money, have as much right to make as much of the money as it is worth (for more they cannot) as the landlord has to let his land for as much as it will yield."[2]

The works of Locke and Bacon stand out as gigantic landmarks, showing the progress of the human intellect during the interval between them; and upon no subject had the growth of rational views been more remarkable than upon the one under consideration. The popularity of Locke's other writings, and the authority of his name, gave, no doubt, great weight to his opinions respecting interest; but the effect produced by them would probably have been much more beneficial if it had not been weakened by an inconsistency that cannot fail to create surprise.

A learned and lamented historian has made it a grave charge against Adam Smith, that he should have maintained, "in direct opposition to his own principles," that the rate of interest ought to be regulated by the State, particularly after Locke had taught "that it was as absurd to make laws fixing the price of money as to make laws fixing the price of cutlery or of broad-cloth."[3]

The opinions respecting the limitation of interest, laid down in the *Wealth of Nations*, are, undoubtedly, among the very few blemishes to be found in that wonderful work; but, strange as it may seem, it is by no means improbable those very opinions were adopted in deference to the authority of Locke. At any rate, there is considerable injustice in this attempt to elevate him at the expense of Adam Smith, who will, perhaps, be held in some measure

[1] Locke's *Works*; Lond., 1812; vol. v. [2] Page 11.
[3] Macaulay's *Hist. Eng.* iv. 631.

excused for remaining unconvinced by Locke's reasoning, when we find that reasoning failed to convince Locke himself. This eminent writer did, indeed, show, by what appears to us overpowering logic, that laws regulating interest can only be ineffective and mischievous; but he, nevertheless, concluded thus—"What then! should there (you will say) be no law at all to regulate interest? I say not so; for it is necessary there should be a stated rate:" and the reasons he gave were—1, "that where contracts have not settled it between parties, courts of justice may know what damages to allow;"[1] and, 2, "that dexterous and combining money-jobbers may not have too great a power to prey upon the necessity of borrowers."[1] The first of these reasons is quite inadequate, and the second gives up the whole question; for, with the exception of the disciples of Child and Culpepper, who had made the brilliant discovery that the way to increase the supply of a commodity was to diminish the profits of those who brought it to market, few persons ever contended for limiting the rate of interest, except from the absolute necessity of shielding needy borrowers from the grasping avarice of lenders. Dr. Johnson, indeed, once urged that the usury laws were required for the protection of creditors as well as of debtors, "for that, if there were no such check, people would be apt, from the temptation of great interest, to lend to desperate persons, by whom they would lose their money;"[2] but this opinion is more likely to be admired for its ingenuity than for its soundness.

While Locke hesitated thus in adopting the conclusions to which his arguments appeared inevitably to tend, there existed two contemporary writers who perceived the truth of those conclusions more clearly, and adhered to them more boldly. Sir Dudley North published, in the year Locke's work appeared (1691), *Discourses upon Trade, principally directed to the cases of the Interest, Coynage, Clipping, and Increase of Money.* This work is among the valuable reprints of the Political Economy Club,[3] without whose aid it would probably have been lost to the public, as the author seems, soon after its appearance, to have consented, for some reason or other, to its suppression; so that, according to his brother, who wrote his life, it was "utterly sunk, and a copy not to be had for money."[4]

That this suppression was a great evil, there can be no doubt; for no work has come down to us so well calculated to enlighten the public mind or to produce correct opinions upon the subject.

[1] Page 63. [2] Boswell; Lond., 1847; p. 502.
[3] *Early Tracts on Commerce;* Lond., 1856. [4] *Ibid.*, Preface xii.

It is alike admirable for clearness, conciseness, and comprehensiveness; and is, indeed, a perfect model in every respect.

North pointed out the true cause of the lowness of interest in Holland, and that "as plenty makes cheapness in other things, so, if there be more lenders than borrowers, interest will also fall; wherefore," he continues, "it is not low interest makes trade, but trade, increasing the stock of the nation, makes interest low." [1]

He, in some degree, disputes the facts stated by Child; for, while admitting that the rate of interest upon mortgages in Holland was as low as 3 or 4 per cent., he asserts, that "the current rate between merchant and merchant was 6 per cent." [2]

"To be a landlord," says North, "or a stocklord, is the same thing; and the state may, with as much justice, make a law that lands which heretofore have been lett for ten shillings per acre shall not now lett for above eight shillings per acre, as that money or stock from 5 per cent. shall be lett for 4 per cent." [3]

Alluding to the difference in the values of securities, he asks, "Shall any man be bound to lend a single person upon the same terms as others lend upon mortgages or joynt obligations?" adding, that the poor trader with a narrow stock, or none at all, supplies himself by buying goods of rich men at time, and thereby pays interest, not at the rate of 5, 6 or 8, but 10, 12 and more per cent., which no law can prevent; [4] concluding with the sound maxim that, all things considered, it would be found best for the nation to leave the borrowers and the lenders to make their own bargains, according to the circumstances they lie under, and, in so doing, "follow the course of the wise Hollanders so often quoted." [5]

In a postcript to the *Discourses*, the various projects for securing the advancement of trade by legislative enactments are characterised as follows:—"Thus we may labour to hedge in the cuckoo, but in vain; for no people ever yet grew rich by policies: but it is peace, industry, and freedom that bring trade and wealth, and nothing else. [6]

There is a great similarity between the reasoning of Locke and that of North, but it is in no degree probable that either of them had derived his opinions from the other. Their works were both published in the same year, although Locke's had been written so long before, and the difference in their politics would effectually prevent any communication between them. It has been truly remarked by Lord Macaulay, "that if Locke had not taken shelter from tyranny

[1] Page 518. [2] Page 518. [3] Page 522. [4] Page 520.
[5] Page 521. [6] Page 540.

in Holland, it is by no means impossible that he might have been sent to Tyburn, by a jury which Dudley North had packed.[1]

Sir William Petty, the second of the two writers alluded to, in his *Quantulumcunque* concerning money, addressed to the Marquis of Halifax, contended, that laws limiting the rate of interest were as impolitic as it would be to enact laws limiting exchange, which he defined as interest of place.[2] This work was published in 1682, after the remarks upon interest had been written by Locke, who could not, therefore, have adopted any hints from the *Quantulumcunque;* but Petty had advocated the same views at much greater length in his *Treatise on Taxes and Contributions,* published as early as 1662,[3] the fifth chapter of which is dedicated entirely to the consideration of usury. In this chapter, after giving a similar definition of exchange as local usury, Petty says, in reply to the question, What are the natural standards of usury and exchange?—"As for usury, the least that can be is the rent of so much land as the money lent will buy where the security is undoubted; but where the security is casual, then a kind of insurance must be interwoven with the simple interest, which may advance the usury very conscionably to any height below the principal itself;"[4] and he concludes, "I see no reason for endeavouring to limit usury upon time any more than usury upon place (which the practice of the world doth not), unless it be that those who make such laws were rather borrowers than lenders; but of the vanity and fruitlessness of making civil positive laws against the laws of nature I have spoken of elsewhere, and instanced in several particulars."[5]

There occurs in this *Treatise* a singular and fanciful notion as to the relation between the rent and the selling value of land, which appears very extraordinary when proceeding from a person so well versed in such matters as Petty. The number of years' purchase of the rent the fee simple is naturally worth, he estimated at as many years as one man of fifty years old, another of twenty-eight, and another of seven years old, all being alive together, might be expected to live. "Now, in England," he says, "we esteem three lives equal to one-and-twenty years; and, consequently, the value of land to be about the same number of years' purchase;" and, further on, he speaks of land selling in other countries for thirty years' purchase, "by reason of the better titles, more people, and, *perhaps, truer opinion of the value of three lives.*"[6]

[1] *Hist. Eng.* iv. 630. [2] *Select Tracts on Money*, pp. 156, 157.
[3] *See* Petty's *Tracts;* Dublin, 1769; pp. 1 to 88. [4] Chap. v. 3.
[5] Chap. v. 37. [6] Chap. iv. 19, 20, 22.

Although Petty may not have written upon the subject of interest with the force and clearness of North or Locke, he is undoubtedly entitled to the credit of having been the first English writer—and, so far as I know—the first writer of any nation who advocated the removal of all legal restrictions upon the rate.

He suggested too, in his *Quantulumcunque,* the advantage of "keeping all accompts in a way of decimal arithmetick." [1] The discovery of decimal fractions towards the close of the sixteenth century, with that of logarithms at the commencement of the seventeenth, seem speedily to have suggested to mathematicians the advantages of a decimal subdivision of all units of quantity. Edmund Wingate, described by Dr. Hutton as the clearest writer upon arithmetic in the English language, pointed out in his *Treatise of Arithmetic,*" [2] that under such an arrangement the science would be taught with much more ease and expedition; but added, in a prophetic spirit, "it is improbable such a reformation will ever be brought to pass."

North and Petty were considerably in advance of their contemporaries upon most other subjects, and they were alike remarkable for the soundness of their opinions respecting money.

They not only showed, in the clearest manner, the absurdity and mischievous tendency of the proposals for raising the nominal values of the current coins, made by persons who thought that the wealth of the nation could be increased by merely calling a sixpence a shilling, but they both attacked the laws for preventing the exportation of coin and bullion.

Petty asserted that these laws were contrary to the laws of nature, and impracticable; and boldly laid down the principle, then entirely new, that a nation was not always less wealthy for having less money.[3] "No man," said North, "is the richer for having his estate all in plate, money, &c., lying by him; but, on the contrary, he is for that reason the poorer." [4]

The numerous statutes enacted, and the various penalties imposed, by the English Legislature, to prevent the exportation of bullion and the melting of the current coin, are sufficient evidence of the importance attached to these objects and the difficulty of attaining them.[6]

These attempts arose out of the delusion that wealth consists of money alone; and that a nation, therefore, can only become rich

[1] *Select Tracts*, p. 166.
[2] *Select Tracts on Money*, p. 165.
[3] Lond., 1658; p. 233.
[4] *Early Tracts on Commerce*, p. 525.
[5] *Hist. Com.* i. 463, 464, 474, 511, 529, 530; ii. 43.

by accumulating the precious metals—a delusion lying at the root of nearly all the errors that have prevailed respecting commerce, and one still existing in the minds of many, as is evident from the opinions frequently published.

A long period elapsed before any considerable portion of mankind could be brought to perceive that the value attached to gold and silver arises principally from the facilities they afford for the exchange of commodities, and is founded upon the same principle as the value attached to canals, roads, ships, and carriages, on account of the facilities they afford for the transport of merchandise; and that to accumulate more money than is required for the purposes of exchange, would be nearly as unwise as to construct more canals, roads, and ships or carriages than can be employed; for, as these are worthless where there is nothing to transport, so money is nearly valueless where there is nothing to be exchanged.

Money, in fact, is to trade what oil is to the steam-engine. Without complete lubrication, the engine can never be worked to its full power; but all the oil in the world would be insufficient to keep it in motion if the real prime mover, the steam, were wanting.

A due regard to these considerations will show, that a nation not producing the precious metals, having once procured a sufficient supply for the purposes of exchange, can have no further occasion to import those metals except for repairing the diminution of its coinage from wear or from accidental losses, or for supplying the gradual increase of its circulating medium, required by the growth of its internal trade. The quantities necessary for these purposes must, however, always be insignificant, as compared with the total amount of its commerce.[1]

[1] The annual supply necessary to meet the demand for an increasing currency of gold and silver can hardly ever be large. If the quantity required of any article of consumption, such as sugar, were to be gradually doubled in seventy years, the importation of sugar in each year would go on increasing until it became twice as great as at first; but a constant annual importation, or balance of imports over exports, of less than 1 per cent. upon the amount of precious metals in circulation, would double that amount in seventy years, supposing no losses to accrue from waste or accidents. A celebrated writer, Mr. McCulloch, has estimated (*Ency. Brit.;* 7th edit.; art., "Money") the annual cost to Great Britain of a purely metallic currency of fifty millions sterling at three millions and a half, or 7 per cent.—allowing 6 per cent. per annum for loss of profit upon the capital sunk, and 1 per cent. for losses "from wear and tear, from fire, shipwreck, and other accidents." This estimate for a currency purely metallic is, I think, much beyond the truth; but, at any rate, it is entirely inapplicable to a mixed currency of gold and paper, such as that of Great Britain, of the cost of which it may be possible to give some idea. We know, from experience, that the loss from the wear of our gold coinage is very small. The sovereigns struck in 1819 were, in the course of twenty-five years, depreciated in value, by diminution of weight, to the extent of about two-pence each. This was at the rate of less than 4 per cent. in a century, or of 0·04 per cent. annually. The risk of the destruction of money by fire must evidently be much less than that of the destruction of immovable property, which, if not specially hazardous, may be insured against for an annual premium of 1*s.* 6*d.* (=0·075) per cent., a rate that, besides the

Importations for objects of luxury, or for use in the arts, are here left out of consideration, as not affecting the circulation.

There are, indeed, some countries, a large portion of whose foreign commerce consists in the importation of bullion in exchange for native productions; but these are among the countries which, in proportion to their population and their natural resources, are the poorest, and have the smallest external trade—countries where the remuneration of labour is at the lowest, and interest of capital at the highest, rates.

The constant flow of bullion to the East is one of the commercial phenomena that has occupied the attention from the earliest period, when the principles of trade became a subject for discussion, and can only be accounted for by the prevalence of the habit of secret hoarding, so common among oriental nations.

This habit may be in some degree attributable to the combina-

fund necessary to meet losses, covers a considerable expenditure, with, sometimes, large profits to Assurance Companies. Destruction of coin by fire, therefore, can hardly exceed 6*d.* (0·025) per cent. per annum upon the circulation; and its destruction by other modes, exclusive of shipwreck of foreign importations, is, probably, not greater. Thefts of money, although productive of losses to individuals, do not cause any diminution of the coinage, the object of those who steal money being to put it into circulation as soon as possible. Losses from shipwreck are, undoubtedly, the most important, but a small fraction only of these is chargeable to the home circulation. The enormous foreign commerce of this country gives birth to a very large transit trade in bullion, which is certainly liable to considerable losses from shipwreck; but this trade would still be carried on whatever might be the nature of the national currency, which is only fairly chargeable with the losses upon the importations of the precious metals necessary to keep it up. There is, indeed, the loss from shipwreck of coin carried coastways, but this must evidently be trivial, and may be included in the destruction from various accidents, estimated at about equal to the destruction from fire. It would be a high estimate to take the losses from shipwreck upon foreign importations at 2 per cent. for each voyage.

There appears to me to be an important fallacy in charging a metallic currency at so high a rate of interest as 6 per cent. Investments in the precious metals, so far as the safety of the principal is involved, are, undoubtedly, the most secure, and for that reason ought only to be charged at the most favourable rate which the best investments command. This rate in England has been, on the average of the last 130 years, lower than 4 per cent., and for securities immediately convertible, like money, very much less. Four per cent., then, seems to me to be the highest rate of interest that can be charged against the capital sunk in a metallic currency, the total annual cost of which as at present used in Great Britain would, I think, be estimated, with tolerable correctness, as follows:—

Annual loss from wear upon 50 millions, at 0·04 per cent. .	£20,000
Annual destruction by fire (=0·025) and other accidents (=0·025), including shipwrecks of coin carried coastways, 0·050 per cent.	25,000
Annual loss from shipwreck upon foreign importations necessary to keep up the circulation, £47,368, at 2 per cent. .	947
Annual expenses of importations, 3 per cent.	1,421
Total cost of importations	47,368
Interest on capital employed, £50,047,368, at 4 per cent. per annum	2,001,895
Making the total annual cost . . .	£2,049,263

Which is, upon a circulation of 50 millions (4·099=) £4. 2*s.* per cent.; leaving the cost, exclusive of interest upon the capital employed, at 2*s.* per cent. per annum only.

tion of timidity and faithlessness characteristic of so large a portion of the Asiatic races, such qualities being eminently calculated to engender in the mind distrust and suspicion of others; but it is, no doubt, principally due to the state of insecurity that rendered it for so many ages in all, and still renders it in some, of those countries impossible to embark capital in such extensive undertakings as can only be profitably carried on under regular and orderly governments.

Whatever the motive for the habit, it must be productive of enormous losses to the nations by whom it is practised.

Out of the erroneous ideas alluded to respecting money, grew the notion of the importance of a favourable balance of trade.

The "balance of trade" is a term for the difference in value between the exports and imports of a nation; and it was long held almost universally that prosperity in commerce consisted in having a favourable balance, or a preponderance in the value of exports, as it was supposed that an excess of imports would have to be paid for by the exportation of bullion.

Upon this theory was founded the whole system of protection, as it once existed in our own country, and as it now exists in most other countries, including bounties upon exportation, and differential and prohibitive duties upon importation, with all the long train of devices invented by men in the vain endeavour to sell to other nations without buying of them in return. Yet, as is most remarkable, the fallacy of the theory is one of the few propositions in political economy admitting of complete demonstration.

If an English merchant send a cargo of commodities to a foreign country for sale, and for the investment of the proceeds in goods to be imported into England, it is clear, if the transaction be profitable, that the value of the goods so imported must cover—1, the value of the goods exported; 2, all the expenses attendant upon the transaction; and, 3, the merchant's profit, including the interest of his capital while so employed. Now, if this be true, as it most undoubtedly is, of the transactions of each individual, it must be equally true of the aggregate of all the transactions in which the nation is engaged.[1] Thus what is called an "adverse balance of trade" may exist to a considerable extent without a single

[1] Let $E_{,}$ represent the value of the exports in the transaction referred to in the text, $I_{,}$ the value of the imports, x the expenses of the speculation, i the interest of the capital employed, and p the profit obtained beyond the interest. Then if, as has been assumed, the transaction be a profitable one, $I_{,} = E_{,} + x + i + p$, or $I_{,} - E_{,} = x + i + p$; that is to say, the surplus value of the imports is the fund out of which expenses, interest, and profits are to be paid. Now, $\overline{x + i}$ being a fixed quantity, it is evident that p, the profit,

farthing being required to liquidate it; and, so far is it from being an evil, that it is the essential condition of an extensive commerce, for without it such a commerce could not be carried on at a profit, and therefore would not be carried on at all.

It seems astonishing that conclusions so simple and obvious should have been so long neglected; but still more astonishing are the wild and extravagant assertions we constantly meet with, as to the effect of the balance of trade in causing the transference of bullion from one country to another.

Samuel Fortrey, in his *England's Interest and Improvement*, asserted that, in 1673, the trade between England and France was a clear loss to the former of £1,600,000 annually[1]—a sum that, in about 10 years, would have cleared the country of the whole of the precious metals then in circulation, if Dr. Davenant's estimate of their total amount formerly quoted be correct.[2]

The annual balance of trade in favour of this country, for the four years preceding January, 1796, was estimated by Mr. Irving, then Inspector of Imports, at £10,500,000, exclusive of the profits of the fisheries. Macpherson, who copied the statement with perfect faith, expressed, with amusing simplicity, his surprise that so large an influx should not have caused any apparent increase of the money in the country.[3] Sir Archibald Alison, in his *History of Europe* (2nd series), asserts, that in this country the balance of imports over exports "has of late years risen to thirty or forty millions a year," and that "this immense balance must, of course, be paid in cash or bills convertible."[4] In support of this assertion, the writer quotes figures from *Porter's Progress of the Nation*, which, if correct, would prove that the balance referred to amounted

increases as the excess of I_e over E_e increases. If $I_e = E_e + x + i$ only, there will be no profit; and if I_e be less than $\overline{E_e + x + i}$, there will be a loss, which will be greatly increased if I_e be less than E_e, the state of things considered so desirable by the believers in the old doctrine about the balance of trade. In the foregoing equations, $\overline{x + i}$ is supposed to be due to the English exporter; but if the expenses were paid, and the capital furnished by the foreign merchant, $\overline{x + i}$ would be deducted from the return cargo, the value of which (x being taken as the total cost of it in the foreign market) would then be $I_e - \overline{x + i} . \frac{I_e}{z} = E_e + p_1$, and the excess in value of the imports, as well as the amount of profits (p_1), would be thereby diminished. It generally happens, however, from the large capitals possessed in England and the facilities they afford, that the greater portion of the expenses and interest upon our foreign trade is payable in this country. The freights, both outwards and homewards, are, in the majority of cases, paid in England; the assurances are principally effected here, and the capital employed being for the most part English, the bulk of the interest thereon has to be included with the profit of the English merchants. All these elements of commercial superiority tend, as has been shown, to swell the excess in value of imports over exports, or the balance of trade against England, as it is often erroneously called.

[1] *Early Tracts*, p. 234.
[2] *Assurance Mag.* viii. 68, Note.
[3] *Hist. Com.* iv. 414.
[4] Vol. vi., chap. xxxv. 6.

in the course of the five years ending with 1849, to upwards of 158 millions sterling. The errors in this quotation are such as will appear incredible to those unacquainted with Sir Archibald's habitual inaccuracy, and even in those with whom it has become proverbial will excite surprise. To explain fully all his mistakes would be too tedious; I therefore print below, with totals added by myself, Porter's original table, and that published by the historian,[1] from which it will be seen that the latter has jumbled together official and declared values, in a manner that could never have been done by anyone with the slightest pretension to a knowledge of British statistics, or who had even taken the trouble to read Porter's explanation of the table printed on its opposite page. The most extraordinary thing in the case, however, is, the entire omission of all notice of the value of foreign and colonial goods re-exported, which amounted, during the five years quoted, to upwards of 96 millions, and ought, as a matter of course, to have been deducted from Sir Archibald's balance of 158 millions—an amount that, after this correction, would still be entirely untrustworthy.

[1] *Extract from Porter's Table.—Progress of the Nation*, 3rd edition, p. 356.

Years.	Official Value.			Real or declared Value of British and Irish Produce and Manufactures exported.
	Imports of Foreign and Colonial Merchandise.	Exports of Foreign and Colonial Merchandise.	Exports of British and Irish Produce and Manufactures.	
	£	£	£	£
1845	85,281,958	16,280,870	134,599,116	60,111,081
1846	75,953,875	16,296,162	132,288,345	57,786,875
1847	90,921,866	20,036,160	126,130,986	58,842,377
1848	93,547,134	18,368,113	132,617,681	52,849,445
1849	105,874,607	25,561,890	164,539,504	63,596,025
	451,579,440	96,543,195	690,175,632	293,185,803

Table published by Sir A. Alison.—Hist. Europe, vol. vi., chap. xxxv. 6.

Years.	Exports.	Imports.	Balance.
£	£	£	£
1845	60,111,081	85,281,958	25,170,877
1846	57,786,875	75,953,875	18,266,700
1847	58,849,377	90,921,586	32,072,505
1848	52,849,455	93,547,134	40,657,859
1849	63,596,025	105,874,607	42,278,682
	293,192,813	451,579,160	158,446,623

Porter's Progress of the Nation, 3rd edition, p. 356.

These enormous errors may have resulted from mere negligence, but such negligence is hardly less culpable than dishonesty; and it is impossible to speak of it with too much severity, when we reflect upon the solemn duties assumed by historians, from whose statements of facts and of opinions the great bulk of mankind, less happily situated for inquiry, deduce the principles that govern their conduct under circumstances materially affecting their happiness and security, their property, their freedom, and their lives.

I have dwelt at some length upon this instance, because it is one of a class of cases giving rise to a very common, but very foolish assertion, that "anything may be proved by figures." The public frequently find conclusions revolting to their common sense, supported apparently by an array of numbers, the accuracy of which they are unable or unwilling to test, and they impute to a most useful and valuable science defects arising solely from their own want of industry or of intelligence.

The science of statistics is as essential to the statesman as the compass to the navigator; but the science of statistics does not consist, as seems to be supposed by many British writers, and even by some British senators, in merely searching numerical statements for the purpose of picking out such figures only as may appear to favour the inquirer's opinions.

It may be supposed that a question so frequently and so earnestly discussed as the reduction of interest, could not fail to attract the attention of the Legislature.

It remained, however, unnoticed in Parliament from 1668 to 1690. On the 10th October in the latter year (2nd William and Mary), the House of Commons, without a division, resolved that leave should be given to bring in "A Bill for the reducing of Interest of Money from Six pounds to Four pounds per cent.; Sir Edward Hussey, Sir Matthew Andrews, and Mr. Papillon to prepare the same."[1] The Bill was read a first time on the 13th October,[2] but the second reading was negatived on the 26th November following, by a very narrow majority, the Yeas being 155 and the Noes 158.[3] Another attempt was made in the following year, 1691, when the House granted, by a majority of 131 to 105, leave to bring in a Bill to lessen the interest of money. The Bill was introduced on the 14th November, and read a first time. On the 8th January, 1691-2, it was read a second time, without a division, and the question—"That the Bill be committed," was carried by 169 to 153. The third reading took place on the

[1] *Commons' Journals* x. 433. [2] *Ibid.* x. 440. [3] *Ibid.* x. 484.

23rd January, after a division, the numbers being—Yeas 150, Noes 101;[1] and the Bill was sent up on the 25th to the House of Lords, where it was read a first and second time, and referred to a committee composed of sixteen earls, seven bishops, and thirteen barons. Among the last was Lord Culpepper, a near relative of the two writers of that name, whose works have been described.

No progress having been made with the measure, the House of Commons, on the 17th February, sent a message to their Lordships, reminding them of the Bill. The receipt of the message is entered in the *Lords' Journals*, but no further notice was taken of the subject, which was terminated by the prorogation of the two Houses on the 24th of the same month.[2]

Another Bill, for the same purpose, was introduced into the Lower House on the 25th February, 1697–8,[3] but it did not reach a second reading; and this was the only attempt to reduce the legal rate of interest made for a period of more than twenty years after the publication of Locke's treatise.

It may have been partly from deference to his authority that so little was done upon the subject, but the principal cause was probably the great demand for money produced by the war with France which followed the Revolution. The high rates of interest charged for loans to the Government about this period are evidence rather of the low state of its credit than of the scarcity of money. In 1662, according to Samuel Pepys, the only funds available for the purposes of the navy were obtained at 15 and even 20 per cent. from the goldsmiths,[4] who, in 1665, refused, as he informs us, to lend to the King at 10 per cent.[5] Nevertheless, the East India Company, in the latter year, were able to borrow at 4 per cent.;[6] and, in 1680, obtained money at the low rate of 3 per cent.,[7] although, by an Act of Parliament passed in 1679 (31 Charles II., c. 1), 8 per cent. was offered for advances made to the Exchequer.[8] Towards the close of the 17th century, a striking proof of the accumulation of capital that had taken place was afforded in the first of those examples of monetary excitement, amounting almost to insanity, which since that period have been phenomena of periodical recurrence in the history of English commercial finance. In the year 1680, several projects for the remunerative employment of capital were started; but the mania did not reach its height until about the year 1694, when the public were overwhelmed by projects of every possible description. To go into details upon

[1] *Commons' Journals* x. 550, 552, 616, 629. [2] *Lords' Journals* xv. 49, 53, 56, 80.
[3] *Commons Journals* xv. 23, 129, 145. [4] *Diary;* Lond., 1858; i. 374. [5] *Ibid.* ii. 250.
[6] Child's *Works*, p. 35. [7] *Hist. Commerce* ii. 597. [8] *Statutes of the Realm* v. 750.

the subject would carry me far beyond the limits of the present sketch, were I even disposed to venture upon the description of a scene already depicted by the master-hand of Lord Macaulay.[1]

The peculiarities of the time were brought on the stage (A.D. 1693) in Shadwell's posthumous comedy of *The Volunteers; or, the Stock-Jobbers*,[2] from which we may learn that the tricks and manœuvres that have sometimes characterised our money transactions in recent times were thoroughly understood and practised at a comparatively early period. The illustrious historian whose descriptions I have referred to, points out, in reference to this play, that, contrary to the general opinion, the arts of stock-jobbing arose in this country before one farthing of the national debt had been contracted.[3] The title of Shadwell's comedy is an incongruity, and the portion relating to share speculations does not seem to have formed part of the original design, but to have been an after-thought, arising out of the circumstances of the time. He attempted to throw the odium of stock-jobbing practices entirely upon the Nonconformists; but we have had sufficient experience in our own day that the contagion of credulous avarice which leads to such outbreaks is not confined to any rank, age, sex, or party.

Great as have been the losses, the suffering, and the discredit incurred by the nation upon the occasions referred to, they have not been entirely unmixed with good—some most important and valuable undertakings having arisen out of them.[4]

During the excitement of 1694 was planned and established the Bank of England, which has, since that date, been inseparably connected with the financial and commercial history of the country; and, although occasionally imperilled by injudicious management, has, upon the whole, been conducted with wonderful skill and remarkable integrity.

[1] *Hist. of Eng.* iv. 320. Those who wish for further information may also consult Macpherson (*Hist. Commerce* ii. 670).

[2] London, 1720.

[3] *Hist. Eng.* iv. 323. The etymology of the word "stock-jobber" is sufficient to show that it was not derived from the national debt. Stock originally meant a capital employed in trade or in productive industry, and to purchase shares in the stock of the East India Company, of the Bank of England, and so forth, was therefore a perfectly correct expression; but it was not usual to call taking the assignment of a mortgage or of a debt "purchasing stock." The debts of the Government, however, becoming transferable in the same manner as shares, and being dealt in by the same persons, were included in the general denomination of "stocks," and that term is now, in the money market, confined entirely to public debts, while shares in the stocks of public Companies are called merely "shares."

[4] Many years ago, I met, in a public library, with a bulky volume, consisting of the prospectuses of various projects bound up together, and labelled "Some of the bubbles of 1825." Among the projects thus described, was one that has since been productive of the greatest and most rapid advance in the social condition of mankind effected since the first dawn of civilisation; it was the plan of a Company for constructing a railway between Liverpool and Manchester.

The question of establishing a national bank had been discussed for some time in England, as was natural from the great success attained by some similar institutions upon the continent, particularly by the Bank of Amsterdam,[1] which, although founded so late as 1609, had already, by the magnitude of its transactions and the extent of its resources, excited the wonder of the age.

Sir William Petty, in his *Quantulumcunque* concerning money (1662), recommended a public bank, asserting we should thereby double our coined money, and that we had in England materials for a bank that might furnish stock enough to drive the trade of the whole commercial world.[2]

Doctor Hugh Chamberlayne, who subsequently projected the abortive Land Bank, attempted, but unsuccessfully, in 1683, to establish a bank for the circulation of bills of credit on merchandise to be pawned therein.[3]

The prejudices of a large portion of the public, however, set strongly in a contrary direction. Lord Bacon alludes to the feeling upon the subject in his time in his *Essay of Usury*, where he says, "Not that I altogether mislike banks, but they will hardly be brooked because of certain suspicions."

The public feeling, indeed, was against bankers generally, who were objects of popular dislike as money-lenders, and of popular envy for the great wealth many of them had rapidly acquired.

Strong evidence of this is to be found in an Act passed in 1671 "for raising a supply for His Majesty." The second section of this Act (22 & 23 Charles II., cap. 3), after reciting "that several persons, being goldsmiths and others, by taking or borrowing great summes of money and lending out the same again for extraordinary lucre and profit, have gained and acquired unto themselves the reputation and names of bankers," enacted, that all bankers, for every sum of £100 they possessed of personal estate, should pay fifteen shillings to the king; the same rate was ordered, by the third section, to be paid upon all money lent to the king at a higher rate of interest than 6 per cent. per annum; while the seventh section laid a tax of six shillings only upon every sum of £100 of personal estate of other kinds. Those who might advance money on security of the taxes imposed by the Act were to be allowed interest at 7 per cent., which was specially exempted from the extra charge.[4] Under this Act, then, bankers and persons who had lent the King money at a higher rate than 6 per cent., such

[1] *Hist. Com.* ii. 153. [2] *Select Tracts*, p. 165. [3] *Hist. Com.* ii. 612.
[4] *Statutes of the Realm* v. 693.

persons being principally bankers also, were taxed upon their personal estates at a rate 150 per cent. higher than the rest of the community—a most unjustifiable act of spoliation, which appears to have escaped hitherto the censure of history.

The bankers were accused of causing the scarcity of ready money by their dealings in exchanges, a charge very skilfully met and refuted, in his *England's Treasure by Forraign Trade*,[1] by Thomas Mun, one of the ablest among the early English writers upon commerce. Sir William Petty, although remarkably free from the prejudices of his day, appears to have been disposed to speak slightingly of the integrity of bankers. In answer to the question, "What is the trade of a banker?" he says, "Buying and selling of interest and exchange; who is *honest only* upon the penalty of losing a beneficial trade founded on the good opinion of the world, which is called credit."[2]

Very little opposition was offered in the Commons to the Bill for establishing the Bank of England, but it was fiercely contested in the House of Lords, where, however, it was carried by a majority of 43 to 31,[3] eight peers signing a protest against the portion relating to the incorporation of the Bank.[4] It seems very probable that this portion would have been rejected had it not formed part of a Bill of Supply, which the peers were unable to amend, and afraid, from the state of the public finances, to throw out. The Bill (6 Wm. and Mary, cap. 20) received the royal assent on the 25th April, 1694,[5] and was entitled "An Act for granting to their Majesties severall rates and duties upon tunnage of shipps and vessells, and upon beere and ale and other liquors, for securing certain recompenses and advantages in the Act mentioned to such persons as shall voluntarily advance the sum of fifteen hundred thousand pounds towards the carrying on the war against France."[6] In consequence of the title of the Act, the new establishment was called, in derision, the "Tunnage Bank."[7] Its capital was fixed at one million two hundred thousand pounds, the whole of which was to be lent to the Government at what Lord Macaulay says was "then considered the moderate interest of 8 per cent.,"[8] a further allowance being made for management of £4,000 per annum, which increased the interest to 8⅓ per cent. in all.

This accommodation was the real inducement for passing the measure, both with the executive and the two Houses of Parlia-

[1] *Select Tracts*, pp. 164 to 167. [2] *Ibid.* p. 167. [3] Macaulay iv. 501.
[4] *Lords' Journals* xv. 423. [5] *Ibid.* xv. 426. [6] *Statutes of the Realm* vi. 483.
[7] Macaulay iv. 499. [8] *Ibid.* iv. 498.

ment. That the scheme of the Bank was popular among the monied classes is evident from the fact that the whole capital was subscribed in ten days,[1] although the Government found it difficult to raise money at 8 per cent. by any other means.

This loan from the Bank of England was the first instance of the grant of perpetual redeemable annuities, in which form nearly the whole of our national debt has been contracted. It was also the foundation of our permanent funded debt; all previous loans, with the exception of some raised upon terminable annuities long since expired, having been obtained upon obligations of the nature of our unfunded debt, the principal as well as the interest being payable at fixed periods. A small portion of these obligations were Navy Bills, but they consisted principally of tallies, a species of voucher that would probably have long been forgotten but for the singular custom by which the use of them was retained in the Exchequer, down to a very recent period. They are thus described in Madox's *History of the Exchequer*:—

"These tallies were pieces of wood cut in a peculiar manner of correspondency: for example, a stick or rod of hazel (or, perhaps, of some other wood), well dried and seasoned, was cut square and uniform at each end and in the shaft. The sum of money which it bore was cut in notches in the wood by the cutter of tallies, and likewise written upon two sides of it by the writer of tallies. The tally was cleft in the middle by the Deputy Chamberlains, with a knife and mallet, through the shaft and the notches, whereby it made two halves, each half having a superscription and a half part of the notch or notches—a notch of such a largeness signified M*l*, a notch of another largeness C*l*, &c. It being thus divided or cleft, one part of it was called a 'tally,' and the other a 'counter-tally,' . . . and when these two parts came to be joined, if they were genuine, they fitted so exactly that they appeared evidently to be parts one of the other."[2]

This peculiar mode of recording money transactions was very general when the art of writing was confined to few persons; and, before the invention of that art, similar means may probably have been employed to preserve the memory of historical events.

The quipos, or knotted cords of various colours, in use by the Peruvians at the period of the discovery of America, have been supposed to contain regular annals of their empire, although that opinion was discountenanced by Dr. Robertson.[3] Lord Erskine, in one of his most celebrated speeches, the defence of Stockdale, mentions having seen in his youth a naked savage addressing the

[1] Macaulay iv. 502.

[2] *Parliamentary Paper*, No. 443 (1858), p. 85. I had recently in my possession one of these tallies, dated in 1811, which resembled more the rude contrivance of the denizens of a wilderness, than a voucher issued from the exchequer of one of the most powerful monarchs reigning over one of the most polished nations of the globe.

[3] *Hist. Amer.*, book vii.

governor of a British colony, "holding a bundle of sticks in his hand as the notes of his unlettered eloquence."[1]

The word "tally," from the French *tailler*, to cut, is evidently employed in the same manner as the word "score," derived from the Saxons, who appear to have been in the habit of making a score, or notch, at every twentieth enumeration, and hence the prevalence in the agricultural districts of the practice of counting by scores or twenties. From the word "tally" is derived the title of teller or tallier of the Exchequer, an officer whose dignity is alluded to by Pope in the couplet—

> "From him whose quills stand quiver'd at his ear,
> To him who notches sticks at Westminster."[2]

We find frequent reference to tallies in our literature. Jack Cade is made to bring forward the following, among other charges, against Lord Say:—"And whereas, before, our forefathers had no books but the score and tally, thou hast caused printing to be used."[3]

The discontinuance of the use of tallies at the Exchequer did not take place until 1826, and was rendered memorable by a very unfortunate event. Orders having been given, in 1834, for burning all the old tallies, the persons employed in doing so used the stoves of the House of Lords for the purpose, and so over-heated the flues as to cause the fire that burned down both Houses of Parliament on the 16th October in that year."[4]

Originally, tallies were merely instruments of acquittance for money due to the Crown and paid into the Exchequer; but they were also, from a very early period, issued as assignments of anticipated revenue. In such cases they were called "tallies of anticipation," or "tallies of *pro*." The latter expression was probably derived from the dealings of the Lombards with the Exchequer, *pro* being an Italian word signifying gain or benefit."[5] It is true that Mr. Chisholme, in his learned and able *Notices, &c., of the Public Debt*,[6] asserts that there is no instance of tallies of *pro* bearing interest before the reign of Charles II.; but a similar apparent contradiction regarding the loans of the Lombards to the

[1] *Erskine's Speeches* ii. 263.

[2] *Epistle to Lord Bolingbroke.*

[3] *Henry VI.*, act. iv., scene 7.

[4] The curiosity about tallies excited by this circumstance led to the publication in the *Times* of the 1st November, 1834, of a very full and accurate account of them, written by that sturdy politician and studious antiquary, William Hone.

[5] The Venetian Government having, in 1171, raised a forced loan, to which all the citizens were compelled to contribute, it was ordained that the subscribers, who were subsequently incorporated as a company and became the celebrated Bank of Venice, should receive interest *a raggione del quattro per cento di "pro."* This is said to be the earliest known instance of the use, now so general, of the expression "per cent.," to denote the ratio between two numbers.

[6] "Notices of the various Forms of the Public Debt; its Origin and Progress:" *Parliamentary Papers*, No. 443 (1858), Appendix.

Crown has been already noticed. There can be no doubt, however, that, in both cases, interest was paid in some form; it may have been included in the principal sum acknowledged, like the discount on a bill of exchange, which instrument the tally in its nature very much resembled.

The first Parliamentary sanction to the issue of negociable public securities bearing interest, was the 17 Charles II., cap. 1,[1] which provided that every person advancing money to His Majesty, or furnishing any "wares, victual, necessaries, or goods," on security of the taxes granted by the Act, should have a tally of loan struck, and be furnished with a warrant for the repayment of the amount, with interest thereon, at the rate of 6 per cent. per annum, payable half-yearly, until the discharge of the principal; and that any person entitled to money by virtue of the Act might transfer his claim by endorsement of the warrant duly registered.[2] The warrants thus became the real securities, rather than the actual tallies; but these, nevertheless, continued to be struck, from which it will be readily inferred that the tally-cutters were paid by fees."[3]

The rate of interest offered by the 17 Charles II. was increased to 7 per cent. in the 29 & 30 Charles II., cap. 1 (1677 and 1678); and both the 30 Charles II., cap. 1 (1678), and 31 Charles II., cap. 1 (1679), authorized the payment of 8 per cent. per annum for similar advances.[4] It must be understood that these Acts were only passed to facilitate the king's obtaining money in anticipation of the taxes voted, and did not in any way pledge the faith of the Legislature for the repayment of the loans to be advanced. The House of Commons was so far from considering these as made to the Government of which it formed a part, that, in the height of the contest on the subject of the Exclusion Bill, when a dissolution was impending, it passed, *nemine contradicente*, on the 7th January, 1680, resolutions declaring that whosoever should thereafter make any advances on any branches of the king's revenue, or purchase any tally of anticipation, or whosoever should pay any such tally thereafter to be struck, should be adjudged to hinder the sitting of Parliaments, and should be responsible to Parliament therefor.[5]

[1] Chisholme's *Notices*, &c., p. 90. [2] *Statutes of the Realm* v. 165.

[3] The fees upon payments by tallies appear to have been higher than 10*s*. per cent. Pepys, in his *Diary* (12th May, 1665), mentions tallies for £17,500 struck for the use of the navy, and adds, "But to see how every little fellow looks after his fees, to get what he can for everything, is a strange consideration—the king's fees, that he must pay himself for this £17,500, coming to more than £100." This virtuous indignation is in amusing contrast with the entry on the following day:—"Received my watch from the watchmaker—and a very fine one it is—given me by Briggs, the scrivener." Briggs having made the present to secure the Secretary's official influence (ii. 229).

[4] *Statutes of the Realm* v. 858, 870, 928. [5] *Commons' Journals* ix. 702.

Notwithstanding the high rates of interest paid upon tallies, we find constant mention of their being at discounts that must have afforded considerable profits to those who purchased them upon such terms. A 6 per cent. perpetual stock, bought at 10 per cent. below par, would only secure the holder interest at £6. 13*s*. 4*d*. per cent. per annum; but a tally payable in two years, and bearing interest at 6 per cent., would produce, if bought at 10 per cent. discount, more than 11 per cent. per annum. Pepys mentions that he held, in 1666, tallies yielding him 10 per cent.;[1] and that, in 1667, he was offered some as a security at 12 per cent.[2] The heavy interest paid upon tallies was not the only burden they inflicted upon the public; they were largely issued for wares, victuals, and so forth, supplied for the use of the Government; and it was justly remarked by Michael Godfrey, the first Governor of the Bank of England, in a pamphlet published in 1693, and reprinted in Francis's *History of the Bank*,[3] "that those who were to allow 15 or 20 per cent. upon their tallies, as was the case before the establishment of the Bank, would make provision accordingly in the prices they were to have for their commodities."[4]

Manley, in the preface to his *Interest at Six per Cent. Examined*, made a similar observation.

From the regulations inserted in the various Acts referred to, for securing the regular payment of tallies in their order of priority, it seems probable that there was much jobbing and corruption in that respect. The sum due upon tallies at the Revolution was only £84,888. 6*s*. 9*d*., which constituted the whole of the public debt at the time, with the exception of about £300,000 arrears due to the army and navy.[5]

In 1693, the year before the establishment of the Bank of England, the national debt had increased to the following sums[6]:—

	Unredeemed Capital.	Annual Charge.	Annual Rate per Cent.
Loans upon tallies .	£4,472,400	£302,000	£6 15 0
Navy bills . . .	1,430,439	85,826	6 10 0
	5,902,839	387,826	6 11 5
Terminable annuities		119,275	
Total annual charge . .		£507,101	

(End of Part IV.)

[1] *Diary* ii. 412. [2] *Ibid.* iii. 240. [3] 3rd edit.; Longman, London; no date.
[4] Page 243. [5] *Parliamentary Paper*, No. 443 (1858), p. 93. [6] *Ibid.* p. 2.

On an unfair suppression of due acknowledgment to the writings of Mr. Benjamin Gompertz. By PROFESSOR DE MORGAN.

IN 1839, in the *Penny Cyclopædia* (article "Mortality"), I wrote as follows, referring to Mr. Gompertz's well-known hypothesis on the law of mortality:—

"We enter into some detail of it the more readily, that it is necessary as an act of justice to Mr. Gompertz, whose ideas have been adopted by a recent writer on the subject, without anything approaching to a sufficient acknowledgment."

Mr. Gompertz gave his theory in the *Philosophical Transactions* for 1825, p. 513, in a paper *On the Nature of the Function expressive of the Law of Human Mortality.*

The writer of whom I assert that he gave a suppressive account of what Mr. Gompertz had done is Mr. T. R. Edmonds, in a work entitled *Life Tables founded upon the Discovery of a Numerical Law, &c.*; London, 1832; 8vo.

My attention has been again drawn to this matter by two things—first, the account given of Mr. Edmonds's alleged discovery by his proposer, in the list of candidates for admission into the Royal Society; secondly, the following mention of Mr. Edmonds by Dr. Farr, in a paper *On the Construction of Life Tables* (*Phil. Trans.*, 1859, part ii., p. 844):—

"I shall now notice briefly the application of this hypothesis, first suggested by Mr. Gompertz, and applied by him to the interpolation of the Northampton and other tables. Mr. Edmonds, in 1832, extended the 'Theory,' and applied it to the construction of three Life-Tables. He gave an elegant formula, similar in principle to that of Mr. Gompertz, from which the curve of a Life-Table can be deduced, upon the above hypothesis."

The first of these circumstances occasioned my pointing out to a fellow of the Royal Society what I had written in 1839, with explanation. Some correspondence followed between him and a friend of Mr. Edmonds, I having first stated my view of the case in writing. On May 31st, I was shown a letter from Mr. Edmonds's friend, recommending* a reference; and also a letter from Mr. Edmonds himself, in reply to my statement just mentioned. As the reasoning in the first paragraph of this letter showed me clearly that publicity must be the result, I declined to read any further, and expressed my intention of taking the course which I now do take.

I shall not trouble the reader with any account of what I think

* This recommendation I adopt: my referees are all who are competent to judge and who choose to read.

about the character or motive of the suppression, nor of my own motive in bringing it forward. I will put the case before him, desiring him to consider it no case at all unless, true quotation being assumed, he find it one to which he cannot imagine a sufficient answer. I will add that I have had no communication whatever with Mr. Gompertz on the subject, direct or indirect, past or present.

The account which Mr. Edmonds gave of Mr. Gompertz's theory is as follows (pp. xvii., xviii.):—

"The honour of first discovering that some connexion existed between Tables of Mortality and the algebraic expression (a^{b^x}) belongs to Mr. Gompertz: but to arrive at this single common point, his course of investigation differs so widely from mine, that appearances will be found corresponding to the reality,—that my discovery is independent of the imperfect one of Mr. Gompertz."

It is here asserted:—

1. That Mr. Gompertz discovered that *some connexion* existed, and that this—namely, that *some connexion* existed—is the *single common point*. It shall be shown that *all* the points of Mr. Edmonds's alleged discovery had been published by Mr. Gompertz; the only difference being that Mr. Edmonds takes in the period of infancy, to which Mr. Gompertz pays no attention.

2. That the two courses of investigation differ widely. If the *published* courses of investigation be intended—and no others can present "appearances" to the reader—it shall be shown that they closely agree.

3. That the discovery of Mr. Gompertz is imperfect—meaning, of course, as compared with that claimed by Mr. Edmonds. It shall be shown that there is no difference between the two, except in the introduction of the period of infancy by Mr. Edmonds, according to the method of Mr. Gompertz.

I now give the quotations which substantiate these points:—

Mr. Gompertz (p. 8).

"Art. 5.—If the average exhaustions of a man's power to avoid death were such that at the end of equal infinitely small intervals of time, he lost equal portions of his remaining power to oppose destruction which he had at the commencement of those intervals, then at the age x his power to avoid death, or [used in the sense of

Mr. Edmonds (pp. xvi., xvii.).

"The force of mortality is a simple function of the age, or time from birth, and is always of the form (ap^x) during each of the three periods of Infancy, Manhood, and Old Age. . . . Let, now, (y) represent the number Living or Surviving at any time (x). The force of mortality at that time$=ap^x=$decrement in unit of time on unit of life; the

and also] the intensity of his mortality might be denoted by aq^x, a and q being constant quantities; and if L_x be the number of living at the age x, we shall have $aL_x \times q^x.\dot{x}$ for the fluxion of the number of deaths $=-(L_x)^{\cdot}$; $\therefore abq^x=-\frac{\dot{L}_x}{L_x}$, $\therefore abq^x=-$hyp. log.* of b [misprint for q] $\times$ hyp. log. of L_x, and putting the common logarithm of $\frac{1}{b}$ [here b should be q again] $\times$ square of the hyperbolic logarithm of $10=c$, we have $c.q^x=$common logarithm of $\frac{L_x}{d}$; d being a constant quantity, and therefore L_x, or the number of persons living at the age of $x=d\overline{g}|^{q^x}$; g being put for the number whose common logarithm is c."

finite decrement of (y) at that time $=y \times ap^x$; and the true decrement, or the decrement in an infinitely small given time, $=yap^x dx$; that is, $-dy=yap^x dx$. Using (l) to signify hyperbolic logarithm, and (e) to denote the base of that system, we obtain by integration, $l\frac{g}{y}=\frac{a}{lp}p^x$ and $\frac{g}{y}=e^{\frac{a}{lp}p^x}$.

If it be assumed that $y=1$ when $x=0$, then $g=e^{\frac{a}{lp}}$, and the equation becomes $y=e^{\frac{a}{lp}} \times e^{-\frac{a}{lp}p^x}$, or $y=e^{\frac{a}{lp}(1-p^x)}$.

And calling the modulus of the common system (k), and using (λ) to signify common logarithm, the equation will finally become—

$$y=10^{\frac{k^2a}{\lambda p}(1-p^x)}."$$

Thus it appears that having a single point in common—namely, that a^{b^x} has some connexion with tables of mortality—arrived at by widely different courses of investigation, means as follows:—

Mr. Gompertz assumes that the intensity of mortality is aq^x.

Mr. Gompertz arrives at the fluxional equation

$$-\dot{L}_x=aL_x \times q^x\dot{x}.$$

Mr. Gompertz integrates this into

$$L_x=d\overline{g}|^{q^x}.$$

Mr. Edmonds assumes that the force of mortality is ap^x.

Mr. Edmonds arrives at the differential equation

$$-dy=yap^x dx.$$

Mr. Edmonds integrates this into

$$y=10^{\frac{k^2a}{\lambda p}(1-p^x)}.$$

The difference is the difference between intensity and force; between a and a; between q and p; between L_x and y; between fluxions and differentials; between d and $10^{\frac{k^2a}{\lambda p}}$; between g and $10^{-\frac{k^2a}{\lambda p}}$; and between a formula which gives dg when $x=0$, and another which gives 1.

In applying his formula, Mr. Gompertz finds it necessary to change the constants at a period depending upon the table to be verified. Thus, taking life from 10 years of age, and not consider-

* The preceding b is a superfluous constant, which is useless. It seems to have originated in the idea that the diminution of the number living is *proportional* to the intensity of mortality, and therefore represented by that intensity multiplied by a constant. But the constant a, already introduced, is sufficient.

ing infancy, he finds one set of constants to represent the Carlisle Table from 10 to 60, another from 60 to 100. Mr. Edmonds finds the same necessity, but he also takes in the period of infancy, and applies the law to the periods from birth to 8 years of age, from 12 to 55, and from 55 to the end of life, acknowledging that the periods vary with circumstances.

Mr. Gompertz considers, as appears by one of his examples, that the constant q is very slowly varying during the period at which it is near enough to uniformity for practical use. Mr. Edmonds seems to assume that during each whole period of its use, his constant q is absolutely fixed; but of this he does not give any evidence. For the logarithms of q, by the Carlisle Table, from 10 to 60, and from 60 to 100, Mr. Gompertz has 0·0126 and 0·0271. In Mr. Edmonds's book, the logarithms of p, from 12 to 55, and from 55 to the end of life, are 0·0128 and 0·0333. Mr. Edmonds calls his constants "now first discovered" (p. vi.).

I can find nothing in which Mr. Edmonds went beyond Mr. Gompertz. It is now for him to show that he not only went beyond Mr. Gompertz, but so far beyond that all which was due to Mr. Gompertz was comprised in the statement that the whole of what I have quoted gives the *single common point* that a^{b^x} has *some connexion* with tables of mortality. Should he wander from this point—which is *the* point—it will be my part to request insertion of a short exposure of irrelevancy. I will conclude by observing that I do not care to inquire whether or no what Mr. Edmonds published, as above quoted, was done independently of Mr. Gompertz, as asserted. On this point it will be well to suspend opinion until it is seen what Mr. Edmonds can say in justification of the suppressive mention which, I submit, has been fully established.

On some Considerations suggested by the Annual Reports of the Registrar-General, being an Inquiry into the Question as to how far the Inordinate Mortality in this Country, exhibited by those Reports, is controllable by Human Agency. (Part II.) By H. W. Porter, Esq., B.A., *Assistant Actuary to the Alliance Assurance Company, Fellow of the Institute of Actuaries and of the Statistical Society.*

[Read before the Institute the 27th February, 1860.]

WITH respect to the inhalation of woolly matter, Mr. Leigh, a surgeon at Manchester, and Registrar of Deaths for the Deans-

gate sub-district of that city, in which cotton-mills are so numerous, states, in one of his Reports to the Registrar-General, that in certain trades in Manchester almost every member above 40 is affected with emphysema of the lungs; and that persons who work in dust—as fustian-cutters, cotton-carders, and the like—are almost all affected with it, after working at their occupations for a few years.

This disease, which is also called "pneumatosis," is not often to be found recorded as a cause of death in the Registrar's returns, probably because the disease usually turns to bronchitis, and the deaths, which are caused primarily by emphysema, come to be registered under the head of "bronchitis."

Considerable injury also is said to be caused by the inhalation of fuliginous particles, with which the atmosphere of large towns is so generally impregnated.

It is common to find, after death, Mr. Leigh reports, the bronchial glands and pulmonary tissue perfectly black, from the presence of carbonaceous matter.

The Acts lately passed, providing for the compulsory consumption of the smoke produced in manufactories—the application of which will in course of time, no doubt, be extended all over the country, but which do not, I believe, at present apply to Manchester, or other large manufacturing towns—will gradually tend to remove this source of disease.

Some advantage is considered to have been derived by workmen engaged in occupations from which they are exposed to the inhalation of dust and other obnoxious matter, by the practice, which has lately become very general, of allowing the beard and moustache to grow, at the suggestion of a physician in Edinburgh, the late Professor Alison.

The movement was commenced, I believe, by the stone-cutters in Scotland. In many occupations, where the inhalation of metallic particles is to be apprehended, wire masks have been introduced, with the object of arresting the progress of particles which might enter the mouth, nostrils, or eyes; and magnetic guards, when the metal that is being worked is steel or iron: the intended object is, of course, to some extent effected, but the mortality among workmen engaged in these processes of manufacture shows, too clearly, that the effect is but partial, and the use of such contrivances is far from general.

I am aware that it is popularly considered that the entry, by inhalation, of particles of foreign matter into the bronchial tubes

and lungs, or, at any rate, their remaining there, is rendered impossible by reason of the ciliary action.

I am induced to refer to this matter, because I find the error perpetuated in a recently-published Magazine, edited by Mr. Thackeray, whose name carries so much weight with the public that it may be as well to enter a little into the question.

The writer of the article, which is entitled "Studies in Animal Life," after describing the nature of the cilia, states as follows, that "while the direction in which the cilia propel fluids and particles is generally towards the interior of the organism, it is sometimes *reversed;* and, instead of beating the particles inwards, the cilia energetically beat them back, if they attempt to enter. Fatal results would ensue if this were not so. Our air-passages would no longer protect the lungs from particles of sand, coal-dust, and filings, flying about the atmosphere; on the contrary, the lashing hairs which cover the surface of these passages would catch up every particle, and drive it onward into the lungs. Fortunately for us, the direction of the cilia is reversed, and they act as vigilant janitors, driving back all vagrant particles." "In vain," it is stated, owing to the direction of the cilia being reversed, as it is in man, "does the whirlwind dash a column of dust in our faces—in vain does the air, darkened with coal-dust, impetuously rush up the nostrils: the air is allowed to pass on, but the dust is inexorably driven back. Were it not so, how could miners, millers, ironworkers, and all the modern Tubal Cains contrive to live in their loaded atmospheres? In a week, their lungs would be choked up." "Perhaps," continues the writer, "you will tell me that this *is* the case: that manufacturers of iron and steel are very subject to consumption; and that there is a peculiar discoloration of the lungs which has often been observed in coal-miners, examined after death."

I shall certainly tell him that the highest medical authorities have no doubt whatever as to the possibility of the entry by inhalation of particles of foreign matter into the respiratory apparatus. It is a matter of fact that carbonaceous matter is found in the lungs after death. Mr. Queckett, the Conservator of the Museum of the Royal College of Surgeons, and Professor of Histology there—and a more competent authority I can hardly, I conceive, bring forward—has repeatedly found carbonaceous matter as the nucleus of tubercle, examples of which he has been kind enough to show me under the microscope, in which inhaled matter of this description formed clearly the nucleus of the tuberculous matter; and

there is said to be little reason to doubt that such matter not only forms the nucleus of tubercle, but that it is often the exciting cause of tuberculous deposit.

The writer of the article to which I have just alluded does not, I think, very clearly describe the action of the cilia, which is not of a nature to *prevent* the entry of foreign particles into the lungs, but, by means of the reversed action to which allusion has been made, to *expel* them from the lungs after they have reached those organs; and this, to a certain extent, no doubt they do.

Dr. Carpenter, one of the first authorities on the question of animal physiology, thus describes the cilia:—"The larynx, trachea, bronchial tubes, and air-cells of the lungs, in all air-breathing vertebrata," he says, "are lined by a mucous membrane, which is continued from the back of the mouth; and this membrane, like the gills of aquatic animals, is covered with cilia, which are in continual vibration." He explains that the purpose of this ciliary movement must be here different from that which is fulfilled by the same action on the surface of the gills of polypes, and which is stated to be to cause a constant change in the water that is in contact with their surface, and so to secure a constant supply of the air contained in the water.

Dr. Carpenter describes the cilia to be "minute vibrating hair-like filaments;" and the purpose of the ciliary movement "probably serves," he explains, "to get rid of the secretion which is being continually poured out from the surface of the mucous membrane, and which, if allowed to accumulate there, would clog up the air-cells, and in time produce suffocation."

The constant vibration of the cilia always in one direction, towards the outlet, no doubt serves to get rid of foreign particles which have effected an entrance into the lungs—not, as stated by the Magazine writer, to *prevent* their entrance. It serves to get rid of them to a certain extent, but the power of this action is probably quite insufficient to cause the ejection of the whole of the very great quantity of different dusts, and of the stony and metallic particles, which the constant exposure of persons subjected to such noxious influences might cause to enter the respiratory apparatus; and we may draw this conclusion because we know, from ocular observation, that the power of the cilia is insufficient to expel from the lungs the carbonaceous matter, which, being of a lower specific gravity, is far more likely to be driven out than the stony and metallic particles, of the presence of which in the lungs it is difficult to bring forward actual proof. Seeing, therefore, that the

cilia have not the power to eject the carbonaceous matter, we may not unfairly conclude that if the particles of the higher specific gravity alluded to are not to be found after death, their absence may be ascribed to some other cause than to that of the ciliary movement. The assumption, therefore, of the author of the Magazine article, that "although consumption may be frequent among the Sheffield workmen, the cause is not to be sought in their breathing filings, but in the sedentary and unwholesome confinement incidental to their occupation," I look upon as wholly untenable.

With reference to the peculiar discoloration of the lungs of coal-miners, which has been observed, the writer says, in some, but not in all or in many, of those engaged, and which has been observed as well in persons who were not miners, and who were not exposed to the influence of any unusual amount of coal-dust, I may refer to the reports of Mr. Leigh, the Registrar of the Deansgate sub-district of the city of Manchester, from which I have already quoted, and I would ask, whether those persons referred to as not being exposed to the influence of coal-dust to any unusual extent, were not, equally with all the inhabitants of large towns, exposed to the influence of the fuliginous particles with which the atmosphere is constantly impregnated, and evidence of the inhalation of which has already been adduced? On the other hand, it is true that evidence of the deposit of pigment in the cells of the lungs, as well as of an entire change of colour of other organs of the body, is to be found in our collections of morbid anatomy.

The ciliary action may be seen to advantage in the living oyster and mussel—in the latter particularly—in a portion taken from the gills, and viewed with a magnifying power of 200 diameters or upwards. An obvious agitation of the water, caused by the unceasing movement of the cilia, is clearly to be seen. Mr. Queckett has discovered that the cilia have a feathering action, like an oar, the object of which seems to be to prevent their motion producing two currents in opposite directions, the result of which would be to counteract altogether the effect intended to be produced. This he has observed in the mussel, and it is fair to assume that it is the same in the human subject, in which it is not easy to find the opportunity of witnessing the experiment, as the ciliary action is not visible for more than about 12 hours after death.

The ciliary motion of the human mucous membrane can, however, be seen for a much longer period on the surface of a recently extracted nasal polypus.

The ciliary action, which must be seen to be appreciated, forms altogether one of those wonderful provisions of nature of which the microscope is now every day opening up a fresh store, to delight, instruct, and astonish us.

The great and concluding argument, however, against the possibility of the inhalation of foreign matter, the writer of the article under consideration tells us, is found in an experiment by Claude Bernard on a rabbit, over the mouth of which a bladder containing powdered charcoal was tied, so that whenever the animal breathed the powder would be likely to be inhaled. The bladder was kept on constantly, except at feeding time, for many days, and yet, it is stated, when the rabbit was killed no charcoal was found in the lungs or bronchial tubes—the cilia being considered to have acted as a strainer, and to have kept the particles of the dust from the air tubes.

Now, from what has been said, it will appear that no such thing took place. If the cilia did act in the matter—which, possibly, they did to some extent—it was by expelling the noxious particles after they had entered the air tubes; but, I look upon it, other reasons could be adduced to account for the absence of the particles of charcoal from the lungs; for instance—

1st. Because the process was probably not continued sufficiently long to counteract the effect which the cilia, it is admitted, have, to some extent, in expelling from the respiratory organs particles of low specific gravity.

2nd. Because, owing to the low specific gravity of charcoal (which is, when in powder, 1·5 only, while that of stone, marble, granite, and slate, I may mention for comparison, varies from 1·7 to 2·85, and that of iron and steel is respectively 7·788 and 7·81), it would be very easily affected by the ciliary movement.

And 3rd. Because there is a peculiarity in the construction of the lungs of rabbits which might have influenced the experiment—the minuteness of the subdivisions of the lungs of those animals into small cavities or air cells being just of a nature to favour the experiment of Claude Bernard.

I may mention another fact which favours the opposite conclusion as to the inhalation of foreign bodies into the respiratory organs—namely, that experiments have shown that animals can be made phthisical by the introduction of particles of metallic mercury into the lungs.

The elaborate investigations, however, of Dr. Holland, physician to the Sheffield General Infirmary, entirely set at rest the ques-

tion of the injurious effect on the lungs of grinding in all its branches.

I shall, hereafter, more particularly allude to the inquiries of Dr. Holland.

M. Morni, in a communication to the Academy of Sciences in Paris, stated that the sword-makers on the continent die before attaining the age of 40 or 45 years. Deslandes, an eminent French writer, mentions, in his *Dictionnaire de Médecine et de Chirurgie Pratiques*, that polishers of steel scarcely reach their 36th or 40th year, and that those are considered to be old who attain the age of 45.

It is necessary to be most careful how we receive as positive facts statements put forth in popular articles on scientific subjects, lest we may be led into error. Professor Babington, with reference to the examinations in natural history at Cambridge, speaks of the inaccuracies contained in popular books on the natural sciences, on which point he thinks it necessary to caution the students of the University.

There is no doubt that many popular errors are perpetuated by means of the elementary handbooks on scientific subjects so much in vogue at the present day; and among the retarding influences with which sanitary reformers have to contend, are the prejudices founded upon errors to which those who hold them often most obstinately cling.

The evidence of the inhalation of stony and metallic substances into the respiratory organs that I have been enabled to obtain is but of a negative character.

With the exception of the carbonaceous matter that I have referred to, I have not been able to *see* any foreign particles that had been received into the lungs by inhalation.

I have taken some trouble to ascertain if any preparation existed showing the fact, and have ascertained that there is no such preparation in the Museum of the College of Surgeons in London. I found several specimens of calcareous and earthy matter—carbonate and phosphate of lime principally—which had been taken from the lungs of oxen and other animals, but this was the result of the ossification of tubercle.

I have not been able to ascertain, upon inquiry of several medical men in London, of the highest standing and experience, that they have ever actually found any stony or metallic particles in the lungs, as the result of inhalation, though no doubt seems to exist as to the fact that such bodies *do* effect an entrance, of which,

indeed, the presence of carbonaceous matter, which is repeatedly found, is sufficient evidence.

One case of a mason has been mentioned to me, in which such a discovery is stated to have been made after death; but my informant had not the facts of the case at his command, and it is very possible that the deposit in this case was the result of disease, and not of inhalation.

The absence of such appearances at the autopsy are accounted for pathologically from the fact that the irritation of their presence induces inflammation, and the particles, in consequence, become detached and are expectorated. This, it is easy to understand, may be the case to some extent; but it is scarcely conceivable that the very large quantity of foreign particles that are likely to be inhaled by persons engaged in certain occupations should be so effectually ejected in this manner as to leave no traces after death of their former existence.

It seems reasonable to suppose that, unless a *post mortem* examination were made with the special object of detecting evidences of the inhalation of foreign particles, they might, from their minuteness, escape observation; and yet, minute though they be, they might have quite sufficient power to cause most serious mischief, by the mere mechanical irritation produced by their contact with the delicate surface of the mucous membrane, in the same way that in poisoning by strychnine, though the actual presence of the poison in the nervous tissue might escape detection from the extreme minuteness of the crystals, still death by that poison may, nevertheless, be the result of the mechanical irritation of the nerves of the body by the sharp, angular, and indissoluble crystals of the strychnine. I am now assuming the truth of a theory which has been propounded to account for the way in which death from this poison is produced—viz., by the actual mechanical action of the sharp-pointed angles which result from the crystallization of strychnine, and which work their way into the substance of the nervous fibre, it being a characteristic of this poison that the crystals are of a peculiarly indissoluble nature. In fact, it is only in microscopical preparations of portions of the lungs that such particles, as from their minuteness have the power of entrance, are likely to be observed. A particle of stone or metal sufficiently minute to be inhaled, would generally occupy far less space than a particle of carbonaceous matter; and yet such a portion of the latter as is likely to be found forming the nucleus of tubercles, when magnified 250 times, appears only about the size of a moderate-sized pin's head.

Thackrah mentions that Diemerbroek relates that he had found, in dissecting the bodies of masons, heaps of sand in the lungs; and that, in dividing the pulmonary substance, he seemed to be cutting a sandy body. Thackrah, however, suggests whether the sand thus found was inhaled, or whether it was not rather a calcareous deposit formed by disease.

From what I have said as to the specimens of calcareous and earthy deposits preserved in the museum of the College of Surgeons, which were found in the lungs of animals, and which arose, in fact, from the ossification of tuberculous matter, it is not at all unlikely that the idea of Thackrah was correct.

The expectoration of calcareous matter, moreover, is not uncommon in some of the stages of phthisis, showing clearly the existence of such matter in the lungs, as the product of disease.

It is further stated, in the Magazine to which I have already referred, that "although consumption may be frequent among the Sheffield workmen, the cause is not to be sought in their breathing filings, but in the sedentary and unwholesome confinement incidental to their occupation."

Surely the writer of this article can hardly be aware of the extensive medical investigations that have been instituted into this subject! I have before referred to Dr. Holland, whose official connection with the Infirmary at Sheffield—the head-quarters of the grinding trade—afforded him the fullest opportunities of satisfactorily determining the effects of this occupation on the health of the operatives, while the fact of this physician having studied under the celebrated Laennec, rendered him most peculiarly competent to undertake such an investigation.

Dr. Holland has minutely inquired into the effect produced on the respiratory organs, by the inhalation of metallic particles, upon workmen employed in all the different branches of this trade—viz., upon scissor grinders, upon fork, needle, razor, penknife, table-knife, saw, file, and scythe grinders; and has shown, beyond the possibility of question, that such occupations are absolutely lethal in their nature. Thoracic diseases, Dr. Holland tells us, are prevalent to a great extent in Sheffield, and are more owing to the effect of dry grinding than to that which is done on wet stones.

It is in the grinding of small articles, such as needles, penknives, &c., that the greatest amount of dust is produced; and the evils entailed in these branches are accordingly greater than those resulting from the grinding of saws and scythes, which are ground on the wet stone only. The want of proper ventilation to carry off the dust is one great cause of the sufferings of the grinders,

who are constantly working in clouds of dust. Without quoting the pathological reasons which lead to the conclusion, I may mention that it seems to be considered that the mucous membrane of the air passages is the seat of the disease from which the grinders suffer—grinders' asthma—the tendency of which is to terminate in tracheal consumption.

Tubercles, however, are found in numerous cases in the lungs of grinders, and can be traced to the inhalation of dust; and many —the delicate in constitution particularly—die very early of a disease more nearly approaching to tuberculous phthisis.

The principal mortality among these operatives takes place between the ages of 21 and 35, and not many appear to survive much beyond the latter period, while the more delicate in organization die long before attaining the age of 35.

There seems to be no reason to doubt that the disease of which grinders die, whether it be bronchial or tubercular consumption, is produced from the mechanical irritation of the hard metallic particles upon the delicate surface of the mucous membrane. In the later stages of the disease, hard, black, and gritty masses are not unfrequently expectorated, which are stated to appear to be accretions of dust, as large as a pea, and even larger. Some grinders expectorate these masses for years; and the fact that extensive expectoration of this nature goes on throughout the progress of the disease, would appear to account for the very rare discovery of the accumulation after death; another reason for the non-discovery being, that the workman is not exposed to the inhalation of dust for some time previous to his death, as he is incapacitated from following his occupation for months prior to the fatal issue. The position of the grinder while at work has, no doubt, a most injurious effect, the body being bent nearly at right angles, so that the lungs have not the free play so necessary to keep them in a healthy state. The remuneration of this class of workmen is low, notwithstanding the deadly nature of their employment; and they are stated to be reckless and dissipated, so that the fatal results which ensue from their occupation are, in many cases, unnecessarily hastened.

I have previously spoken of the want of knowledge of the labouring classes as operating against any speedy amendment of their sanitary position. Dr. Holland refers to the ignorance of the Sheffield operatives as tending, in the first place, to induce them entirely to disregard the premonitory symptoms of their illness; and, in the second place, as tending to make them

consider that such evil results are necessarily inherent in their calling.

We see from these, and many similar considerations, the importance of introducing some instruction in human physiology into the ordinary routine of education, as the only means of teaching the results that must necessarily ensue if the laws of nature are disregarded, and of showing the labouring community how much it rests with themselves to guard against many of the evils from which they now suffer, and which are, to so great an extent, capable of being remedied.

A peculiar appearance is mentioned as being often observed in *post mortem* examinations of the respiratory organs of grinders: this is, a change of structure of the bronchial glands, at the bifurcation of the trachea. They seem to be converted into a black, gritty substance, the composition of which is unknown. This abnormal condition seems to be the result of the more destructive branches of the trade, such as fork-grinding. These gritty bodies are also found in the lungs, and are usually about the size of currants, but they are found much larger, and, at the bifurcation of the trachea, even as large as a hazel nut. More than a quarter of a century ago an invention was introduced by Mr. Abraham, which, it was hoped, would tend to diminish the fatal results caused by the inhalation of metallic particles. This was, a magnetic guard for the mouth; the object of which was to attract the particles of metal, and so prevent their entrance into the respiratory organs. The invention, however, does not appear to have been generally adopted, and it has long since fallen into disuse.

The dust produced in grinding is stated to consist of gritty as well as of metallic particles; the latter alone would be arrested in their course by the magnets, and the gritty particles are said to be no less injurious than the metallic. Great prejudices, however, exist among the workmen against all inventions of this kind, and it is very difficult to make them see the advantage of them, to whatever point of perfection they may be brought, and however fully they may be shown to answer the end in view.

It became, accordingly, a desideratum to introduce some arrangement, the success of which should not depend upon the men, and a contrivance was invented with this view. This was, a wooden funnel, from 10 to 12 inches square, placed above the surface of the revolving stone, on the side the farthest from the grinder, terminating in a channel immediately under the surface of the floor. Each of the grinders working in the same room has his

own funnel and channel, all of which terminate in a common channel close to the external wall, where a fan is placed, like those used for winnowing corn, so arranged as to cause a current of air to flow from the mouth of the funnels, the effect of which is to carry away all the particles thrown off from the wheels. The principle is similar to that which I have before described as having been introduced into Mr. Evans' flour mills.

This arrangement is still in use in the different works in Sheffield, but no legislative enactment compels its adoption. Notwithstanding the continued use of the invention, however, the most recent inquiries into the mortality of the manufacturers of cutlery in Sheffield and elsewhere show that the high mortality among this class of operatives still continues. Dr. Greenhow, to whom reference has previously been made, states that "the greatest excess of death-rate is in Birmingham, Ecclesall Bierlow, and Sheffield, where, especially in the two last-mentioned places, the finer kinds of metallic manufactures are made." "The pernicious influence on health of certain operations connected with the manufacture of cutlery," he adds, "has long been recognized, and it is here rendered evident by the high pulmonary death-loss among the males of Ecclesall Bierlow and Sheffield."

If, then, we suppose that the apparatus in use to carry off the dust is really and entirely effective, we must come to the conclusion that the high pulmonary death-rate is due to other causes—in a great measure, perhaps, to the bodily position of the workmen employed in the process of grinding, and to which reference has already been made. For this there seems, at present, to be no remedy, as the men explain that they cannot, in any other posture of the body than that which they habitually adopt, obtain the necessary purchase for keeping the article they are grinding in its due position on the wheel. It seems unlikely, however, that the inventive art of this country would be unequal to the production of some apparatus to prevent the injury thus accruing, should it be clearly shown to exist. To give an idea of the excessive mortality among the operatives employed in the Sheffield grinding trade, I may mention, that, in the fork-grinding branch—which is stated to be the most destructive—Dr. Holland found that, out of 1,000 deaths occurring among persons between the ages of 20 and 30, while the proportion in England and Wales was 160, among the Sheffield fork-grinders it was 475. In the next decade of ages, 30 to 40, a similar disparity was observed, the proportion in England and Wales being 136, and among the fork-grinders 410.

The mortality in this branch of the trade was, therefore, very nearly three times as great as that among the general population of the country. Carrying the inquiry a step further, we find that, between the ages of 40 and 50, the proportion for England and Wales was 126, and among the fork-grinders 115.

The sum of these three numbers, which indicate the deaths of the Sheffield workmen, it will be observed, is exactly 1,000, so that before reaching the age of 50, every operative engaged in this perilous branch of the Sheffield trade has actually been killed off, while the proportion of deaths per 1,000 among the population of England and Wales amounted only to 422. These investigations were made some years ago, but, as I have before observed, the inquiries of Dr. Greenhow show that the mortality of this class of operatives is, at the present time, most excessive, as is evidenced by the high pulmonary death-rate in Sheffield and in other places in which the manufacture of metals is carried on—the death-rate from pulmonary affections, per 100,000 males, being, for England and Wales, 569, while for Sheffield it was as high as 839, and nearly the same for Birmingham—viz., 838. In Ecclesall Bierlow it was 736.

In Sheffield the operatives are chiefly engaged as cutlers, file-makers, grinders, and other workers in steel, and as goldsmiths; in Birmingham, as brassfounders, button makers, goldsmiths, gun-smiths, iron manufacturers, tool makers, nail makers, and glass makers, and in Ecclesall Bierlow, chiefly cutlers, file makers, grinders, and other workers in steel. A very large proportion of the inhabitants of Sheffield and Ecclesall Bierlow are employed in the injurious occupation of steel working—the proportion of adult males engaged in metal manufactures in the former place being 40·9, and in the latter 43·6 per cent. Notwithstanding the improvements that have been introduced into the workshops of the Sheffield grinders to prevent the evil consequences which have been shown to result from the inhalation of the products of the grinding of metals, and particularly of the dry-grinding process, there is abundant evidence that the evil is diminished only, and not entirely prevented; and the mortality that, at the present time, still takes place annually among grinders at the principal seats of manufacture of metals is clearly too high to be the result of the "sedentary and unwholesome confinement incidental to their occupation," which the writer of the article in the Magazine to which I have referred seems to consider the cause of the frequency of phthisis among the operatives of this class. It will be evident, from the

satisfactory action of the inventions for remedying the fatal effects that result from this branch of trade, that the means of diminishing the high mortality that, at the present time, still obtains, are in our own hands; but the influence of the Government to compel the due adoption of the means at command seems to be imperatively necessary.

To show to what extent proper attention to check the inhalation of dust may tend to diminish the mortality from pulmonary complaints, which exposure to its influence creates, I may refer to some statistics of the Pentonville prison, the discipline of which enforces the separate system. Several trades are carried on by the prisoners, some of which, such as mat and rug making, produce great quantities of dust, the inhalation of which by the prisoners, it was considered, might tend to the production of the high mortality from pulmonary disease which the records of the prison showed. An investigation into the sanitary condition of the prisoners was accordingly instituted by Dr. G. Owen Rees, the principal medical officer, in whose Report on the subject, appended to the Report of the Commissioners of the prison, the result of his inquiries will be found recorded. The prisoners in the Pentonville prison, it is stated, were more subject to injury from the inhalation of dust than those in prisons where the men do not sleep in their work-rooms. The investigation into this question was made on the 14th March, 1846, when the prison was full.

The proportion of men employed in dusty, as distinguished from those engaged in non-dusty occupations, was, at that date, as 1 to 2·38—so that, if the chances of death from consumption were equal for men employed in both ways, it would have been expected that more than twice as many cases would have occurred among men employed in non-dusty trades; but 8 men employed in dusty trades appeared among the casualties, which, were the chances of immunity equal, would give 19·04 for the number of deaths and free pardons among the non-dusty trades.

It is clearly necessary to include the free pardons on medical grounds as deaths from pulmonary disease, in all these comparisons, as, had not the pardons been granted, it is quite clear that death from such disease would have happened within the prison.

The result of the inquiry of Dr. Rees, however, he reports, did not show this to be the case, for only 9 of such men suffered, showing that the chance of escape from the disease in the prison was twice as great for those employed in the non-dusty trades.

Acting on the information thus obtained, means were taken to

lessen the evils shown to result from dusty occupations. The winding of worsted was prohibited, the coir used in mat-making was ordered to be picked and beaten before being given to the prisoners to work up; and, probably, other measures, not detailed in the Report, were adopted to diminish the evils that were shown to have been the consequence of the inhalation of dust; and the result became apparent by the end of the year 1846, when the deaths by consumption scarcely exceeded the returns for the general population at the same ages—being, including free pardons, ·47 per cent. It therefore appeared that consumption in the Pentonville prison was an accidental, rather than a necessary, accompaniment of the system of discipline carried on there. Dr. Rees reports that the result of his investigation satisfied him that the admission of dust, of an irritating character, into the minute structure of the lungs, and particularly in occupations that require workmen to remain in an atmosphere of dust, leads to an excess of mortality by consumption; and that such is the case whether the dust created consists of metallic or of vegetable particles.

The result of this inquiry is important in two ways:—

1st. As showing that prison discipline, even when carried on upon the separate system, is not necessarily the cause of the tubercular cachexia, which has always been considered to be one of the evils of confinement in prisons; the improvements adopted with reference to the prevention of the inhalation of dust by the men, and the great attention devoted to ventilation, drainage, and the sanitary arrangements of the prison generally, under the able and zealous directions of Dr. Rees, having had the effect of so reducing the mortality of the prison, that, including all free pardons as deaths, the total mortality came but little to exceed that of the general population—the excess being 1·73 per cent. per annum—showing most satisfactorily the efficacy of the introduction of the sanitary measures recommended:

And 2nd. As evidencing how much we hold in our hands the means of diminishing the mortality from phthisis and other cognate diseases, which cut off, in the very prime of life, so large a proportion of the population of this country.

In our inquiries into the subject of pulmonary disease, we must not fail to consider the high mortality from this cause in the British army on home service. This question has recently begun to attract attention.

The disclosures in the Report of the Commissioners appointed in May, 1857, to investigate the sanitary condition of the army,

were more startling in their nature than the public were prepared to expect.

It appears that one of the greatest enemies the soldier has to fear, far more than the sword or the rifle of the enemy, is the disease we are considering—phthisis.

It will be impossible for me, within the limits of this paper, to do more than give a slight outline of the causes tending to produce the effect observed.

Notwithstanding the circumstance of the soldier being, in fact, a selected life on entering the army, and that he is proved to be then free from any bodily defect or infirmity—notwithstanding that he is placed in a much better position than the rest of the community of the same social standing with himself, as to food, lodging, and clothing; that he has the best medical attendance upon the slightest suspicion of illness; and that, altogether, his position is one which might naturally be imagined must conduce to a high state of bodily health and to a lengthened existence—notwithstanding all this, the result of the inquiry is stated to be that the mortality in the army is very much greater than among any class of civilians.

At a meeting held lately at the Hanover Square Rooms, on the subject of the sanitary condition of the army, it was stated, by Dr. Gourlay, that the life led by the Guards in town unfits them entirely for campaigning; and that, upon the occasion of the Crimean war, after one night in the trenches, a great proportion of the newly-arrived soldiers were on the sick list.

It was stated further, that while the death-rate in the Prussian army was only 6 in 1,000, that in the British army was no less than 33 in 1,000, or nearly six times as great, and that 70 per 1,000 were invalided—a severe loss, even in a pecuniary point of view, to the country, when it is considered that each soldier, after he has learnt his duties, has cost the nation the sum of £100.

I am not aware what authority Dr. Gourlay had for this statement; as it appears, from the evidence of Colonel Tulloch before the Commission, that the mortality in the Grenadier Guards was 21·5 per 1,000—an enormously high rate as compared with that of the civil population, which was stated to be only 9·6 per 1,000 at the same ages—viz., 20 to 25.

The principal causes for this high rate appear, from the evidence before the Commission, to be—

1st. Defective arrangements connected with night duty.

2nd. Want of exercise and suitable employment.

3rd. Intemperance and vice.

4th. Bad barrack accommodation.

5th. Improper food and cooking.

Now, without going into any discussion of these five heads, there can be no doubt that the evil results traceable from such causes as these are within our power to remedy, and it is, accordingly, very satisfactory to learn that the Army Medical Department is now being reorganized by the Director-General, with the object of carrying out the recommendations contained in the Report of the Royal Commissioners for the improvement of the sanitary condition of the army.

A new statistical nomenclature, we are told, has been adopted for the army returns—viz., that which was drawn up at the Statistical Congress held in Paris in 1855, and which is identical with that of the Registrar-General; so that, in future, we shall be in a better condition to compare the mortality of our army with that of the armies of foreign powers, as well as with that of the civil population of this country.

One great cause of the maintenance of the high rate of mortality from phthisis, is, no doubt, the intermarriage of persons in whom the hereditary taint of this disease exists.

There is a theory put forth by Walker, that if one parent be afflicted with pulmonic disease, it is an even chance that any one child that may be the result of the marriage of two persons so situated should derive the organs of the chest from the parent so afflicted; and it has been suggested that this theory might account for the very capricious selection that this disease makes in the different members of the same family for its attacks.

Be this as it may, we know perfectly well that the hereditary transmission of this malady is an undoubted fact; and Assurance Companies, whose experience is so large on such points, very generally feel called upon to reject applicants for life assurance, whose apparent health may be undoubted, when several cases of phthisis have occurred among the members of the family.

Some Companies, indeed, think that it is safer to reject the case when the mother or father, or even one brother or sister, has died from this disease, if the applicant be young—say under 45.

In Walker's work on "Intermarriage," another theory is broached—viz., that, under certain restrictions, the male gives to the progeny the external or locomotive organs, and the female the internal or vital organs.

Arguing from the analogy of breeding among animals, this is

more than a mere theory; the fact is well known to breeders of stock. So much, indeed, do certain known laws with respect to propagation prevail, and so thoroughly have they been made the subject of scientific investigation, that a cattle-breeder can produce, within certain limits, almost any class of animal he desires.

If we recognise the truth of this second theory, the necessity for extreme caution, on the part of Life Assurance Companies, in accepting the lives of individuals whose mothers have died from any hereditary disease, becomes clearly apparent. Still, arguing from what has been proved to be the case with respect to animals, Assurance Companies might, with reason, carry their inquiries into the family history of lives proposed for assurance much further than they have yet thought of doing.

It would, perhaps, startle a person desirous of assuring his life, to ask him whether his mother had been married previously to her marriage with his father, and if so, whether her former husband died of phthisis; and yet this would not be by any means a ridiculous inquiry.

It is a fact quite beyond dispute, and one to which public attention was first drawn by Lord Morton in the *Philosophical Transactions*, that the female retains traces of the influence of a prior impregnation, and transmits the same to a second, third, and even fourth progeny.

This has been clearly proved to be the case with horses, mules, sheep, dogs, pigs, and fowls; and what is most extraordinary in the matter is, that it is not necessary that a prior impregnation, in order to influence a subsequent one, should have resulted in the production of progeny.

In illustration of this, I may mention two authentic cases communicated to the Newcastle Farmers' Club by Mr. Reginald Orton, a surgeon at Sunderland; the first being the case just referred to as having been brought before the public by Lord Morton, who allowed a quagga he possessed to breed with a thorough-bred chestnut mare. The produce was a quagga mule, having the stripes, and, in many respects, the characteristics of the male parent. The next season the same mare was allowed to breed with a black Arab horse, and the result was, to the astonishment of the owner, a foal bearing strongly the marks of the quagga; and this effect continued to be visible through three successive foals produced by horses.

The second case is recorded in the *Transactions of the Royal Society*, and is perfectly analogous to the first. It had reference

to a mare, the property of Sir Gore Ousley, which was allowed to breed with a zebra. The result was an animal bearing the zebra stripes. The mare next bred with a blood-horse, and produced a striped foal. The following year the same thing happened again.

A precisely similar result has been obtained in the case of other animals.

I mentioned that it had been shown, by experiments with animals, that a prior impregnation need not necessarily have resulted in the production of progeny, in order that evidences of it may be transmitted to a subsequent generation. Of course, with human beings, if there were any children of a first marriage, and phthisis showed itself in them, the case would be stronger; but the result, according to this theory, would be the same in the case of a *primipara*, as of a widow who had borne children.

With respect to animals, the fact under consideration is undoubted; and if we may fairly argue by analogy that the same holds for the human species, there is, probably, some reason to believe that we may, in this way, account for some of those sad but inscrutable cases which we all of us see happen occasionally among our friends and relations, in which phthisis comes on suddenly and unexpectedly, without our being able to trace either any hereditary tendency to the disease, or to find any reason to account satisfactorily for the origin of the affliction.

Should our reasoning be correct, the desirableness of marriage with a widow whose husband has died of phthisis or other hereditary disease, may, at least, be questioned; and, according to what has been said, whether she had borne children to her first husband or not; and possibly the attention of those engaged in the conduct of life assurance business may not inappropriately be directed to the subject.

With the view to confirm the truth of some of the foregoing opinions as to the effect of occupation on phthisis, I applied to the authorities of the Hospital for Consumption and Diseases of the Chest, at Brompton, for some statistics on the subject; and, through the kindness of the medical committee of the hospital, and of the resident medical officer, Mr. Vertue Edwards, I have been furnished with a list of the occupations of 557 male and 318 female in-patients who died of phthisis, or some disease of the chest, in the hospital in the ten years from January, 1850, to December, 1859. It is necessary to confine our attention to the in-patients, as the results of the cases of the out-patients cannot be ascertained. Although the sum of these numbers, which amounts to 875, forms

but a small portion of the numbers recorded under similar heads in the Registrar's Reports, the list is sufficiently extensive to show which are the conditions of life in London that tend to the production of the particular class of diseases treated in this hospital.

In considering any records of this nature, we must take into account the situation, apart from the cause of the disease, in which the persons who are likely to become inmates of such an institution are placed.

We are at first struck by the number of domestic servants, both male and female—the latter particularly—who have died in the hospital, and we are not aware of any particular predisposing cause to phthisis in their case; indeed, in comparison with almost all the other classes in the list, the condition of domestic servants, as a class, contrasts very favourably; yet we find this class contributing the greatest number of deaths—viz., males 78, and females 192, or 14 and 60 per cent. respectively of the whole cases under observation.

On reflection, however, we see that this is the class, more than all others, likely to be sent to the hospital. Their illness comes on under the very eyes of their employers, who are, for the most part, willing, as well as able, to gain admission for their servants.

We may, therefore, consider that the highest classes on this list are those of labourers, among males, and of needlewomen, milliners and dressmakers, and other similar workers for shops, among females.

These classes contribute respectively 14 and 20 per cent. of the whole number of cases; next in order follow governesses and teachers.

This is just what we might naturally expect, that those who fare the hardest and have the greatest struggle to gain a livelihood, are among the first to succumb to the great malady of the country; and, further, in proportion to their poverty will they be the most likely classes to be left to die in a hospital, while the classes a little above them in the social scale will be more likely to be removed by their friends when their cases are found to be incurable.

The other unhealthy occupations as tested by these returns, arranged in order of unhealthiness, are, among males, those of clerks, carpenters, shoemakers, tailors, porters, drapers' assistants, painters, gardeners, soldiers, sailors, bricklayers, engineers, cabinet makers, bakers, policemen, and compositors; and, among females, governesses, teachers and schoolmistresses, laundresses and ironers.

With some slight exceptions, these, too, follow very much the order that we might expect.

We are not surprised to find clerks, shoemakers, tailors, and drapers' assistants, high in the list, seeing that their employment is of a sedentary nature; that these classes have very little opportunity for active exercise in the open air, and that the three first classes are not only engaged in sedentary employment, but such as, in addition, necessitates stooping very much over their work—the clerks over their desks, against which they most unwisely press their chests, to the very serious injury of the thoracic viscera, and the tailors and shoemakers over their stitching; the injury to the latter, besides, is increased by the pressure of the last against the chest, to which I have before referred.

The long hours during which the drapers' assistants work—the close, hot, crowded, gas-laden shops in which their lives are passed—the constant standing posture, in combination with the almost total impossibility of taking out-door exercise in the daytime—render this class particularly prone to phthisis. The mortality from this cause, in the large metropolitan drapers' shops, is said to be very great. The want of daylight in the work-rooms in which a considerable portion of the labouring classes perform their daily avocations, is stated to conduce to this disease.

It is satisfactory to see that there is just beginning to be some slight improvement in respect to the long hours during which these shops are open; and some large establishments have, in a very praiseworthy manner, instituted libraries and reading-rooms on their premises for the use of those employed.

The number of deaths recorded in the hospital returns, particularly among persons following other callings than the four classes I have enumerated, is too small for us to deduce much from it, even by comparison with the number of persons employed in the different trades, and which the Census returns of 1851 give us the means of knowing.

By combining together, however, the numbers of deaths stated to have happened in the hospital from phthisis, among persons working in metals—such as engineers, watchmakers, smiths of all kinds, cutlers, optical instrument makers, and the like—we find the total number proportionately very high; in fact, this class compares, for unhealthiness of occupation, with clerks, who stand almost at the head of the list—each class producing, in fact, 5½ per cent. of the whole mortality from this disease; and thus we are enabled, to some extent, to confirm, from the statistics of the

hospital, the opinion as to the injurious effect upon the lungs of the inhalation of metallic particles.

No report has yet been issued by the authorities of the hospital of the result of the last ten years' experience to which I have just been alluding, in respect of which certain information has been furnished to me.

An elaborate medical report for the first six years, however—viz., from the establishment of the hospital in September, 1842, to the 31st of December, 1848—has been issued, and to this I have been kindly allowed access.

About the same number of in-patients are included in this period as in that embracing the last ten years; the exact number was 888, and consisted of 542 males and 346 females.

The out-patients numbered in all 10,051, but those treated for phthisis amounted only to 3,470—viz., males 2,137, and females 1,333.

The result of the medical inquiry based upon the facts recorded in the above cases, showed that there was a greater liability to phthisis in males than in females. I mention this, as the result is stated to be not in accordance with the opinion entertained on this subject by writers of authority in this country or on the continent.

The fact that the female population in London, during the period under observation, exceeded that of the males, was not lost sight of in the investigation.

The intensity of the disease, judging from the hospital statistics, appears to be between the ages of 25 and 35 for males, and at ages a little younger for females. The unhealthy occupations seem to follow very much the same order as those in the more recent period, and the results tend to confirm, to a very great extent, our preconceived views on the subject; and the more satisfactorily, as the number of cases under observation which were classified under the heads of the occupations of the patients that died of phthisis embraced 4,358 cases—viz., males 2,679, and females 1,679—or about five times as many as those that I have considered previously. Among the males we find the class of labourers, as before, at the head of the list; next come clerks, warehousemen and shopmen, mechanics, servants, tailors, carpenters, shoemakers, coachmen and cabmen, printers and compositors, painters and glaziers, weavers and glovers, bakers, and butchers; and among the females we find, as before, a very large and probably undue proportion of servants; then follow, as before, needlewomen, and those engaged in similar

occupations, laundresses and governesses. Of course, we cannot accurately compare the relative mortality from any particular disease among persons following different occupations, unless we know the respective numbers engaged in those occupations.

An investigation of this nature would lead us further into the details of the question than is contemplated in this paper.

The question of the contagious nature, or otherwise, of phthisis, as bearing upon the production of this disease, is not an unimportant one. The difference of opinion, however, on this point appears to be very great, and the evidence on the subject to be very contradictory. As long, however, as the question admits of a doubt, it would certainly be well to be on the safe side, and, at all events, to avoid needless exposure to contagion.

The medical report of the hospital contains the results of a most interesting investigation into the question of hereditary predisposition to phthisis.

The information which is the result of such inquiries as these cannot but be of great interest to those connected with the business of life assurance, and of the highest importance to the Companies themselves, if they wish, not as a matter of course, to reject applicants for life assurance in whom an hereditary taint is suspected; in fact, if they desire to act upon principles which are the result of actual inquiries, instead of treating the cases brought before them in an arbitrary and unscientific manner, as they are now very much obliged to do.

With reference to the transmission of the disease, we learn from the report that it has been observed that one or more of several children of the same individuals may exhibit traces of the disease of their parents, and yet the other children of the family, though not apparently diseased themselves, may, on becoming parents, transmit to their offspring the elements of the disease, which only requires some exciting cause for its development. Thus a diseased parent in one generation will have offspring to all appearance healthy, and in the next generation an apparently healthy parent will be found to have diseased children.

The result of the inquiry of the medical officers of the Brompton hospital, as to the proportion of cases on the books in which hereditary taint could be traced, showed that out of 1,010 cases, comprising 669 males and 341 females, 122 males and 124 females, forming 18 and 36 per cent. respectively of the whole, or 24½ per cent. of males and females combined, were born of phthisical parents; or, in other words, 1 in every 4 patients, nearly, of

the 1,010 cases under observation was traced to be hereditarily predisposed to the disease. It appears, too, that females are more likely to inherit the disease than males, in the proportion of 2 to 1. With the view to show how very large is the comparative proportion who appear to inherit phthisis from their parents—and, accordingly, to show how peculiarly severe the power of the hereditary transmission of this disease is—a comparison was instituted with the similar power in cases of insanity, which is also known to be, unhappily, too frequently transmitted through successive generations; and the result was, that 11·9 per cent. males and 13·4 per cent. females—or, combined, 12½ per cent. of the cases under observation—were born of insane parents. The probability, therefore, of the hereditary transmission of phthisis, as compared with that of insanity, is, taking both sexes together, as 2 to 1; and while the probability of inheriting phthisis, as just shown, is twice as great among females as among males, that of a similar transmission of insanity is only 1½ per cent. greater.

(*To be continued.*)

CORRESPONDENCE.

ON A METHOD OF USING THE "TABLE OF QUARTER SQUARES."

To the Editor of the Assurance Magazine.

SIR,—Allow me to call your attention, and that of your readers, to a method of using the "Table of Quarter Squares" computed by me and published in 1855,* which, although not pointed out in the introduction to that work, is one which will extend the usefulness of the table, and, in certain cases, materially diminish labour.

A constant factor is of frequent occurrence in some of the calculations of an actuary; for my present purpose it will be sufficient to instance the expression for the value of a policy when the premium is just due and not paid, viz.—

$$1-\frac{1+a_{m+n}}{1+a_m}=1-(1+a_{m+n})\left(\frac{1}{1+a_m}\right),$$

in which the term $\frac{1}{1+a_m}$ is constant, whatever may be the value of the other factor. Supposing a computer to be about to form a table of the values of a policy for a series of years—say from 1 to n years—it will manifestly be advantageous if he can save one reference to the table in each calculation.

* *Table of Quarter Squares of all Integer Numbers up to* 100,000, *by which the Product of Two Factors may be found by the aid of Addition and Subtraction alone.* C. & E. Layton, Fleet Street.

This saving can be effected, where one of the factors is a constant; as may be seen by reference to the equation [2], at p. xiv. of that work, where it is shown that

$$ab=2\left(\frac{a^2}{4}+\frac{b^2}{4}-\frac{(a-b)^2}{4}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [2]$$

But this expression requires the tabular results in every case to be multiplied by 2; and the object of the present communication is to show that where a constant factor occurs, as in the instance above quoted, the trouble of doubling the tabular results may be saved. For, transforming the above expression [2], by doubling the factor a, we obtain

$$(2a+b)^2=4a^2+4ab+b^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [A]$$
$$(2a-b)^2=4a^2-4ab+b^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [B]$$

By addition of [A] and [B], we obtain

$$4a^2+4ab+b^2+(2a-b)^2=8a^2+2b^2;$$
$$\text{hence } 4ab=4a^2+b^2-(2a-b)^2,$$
$$\text{and } ab=a^2+\frac{b^2}{4}-\frac{(2a-b)^2}{4};$$

and since $a^2=\frac{(2a)^2}{4}$, the expression may be further transformed into

$$ab=\frac{(2a)^2}{4}+\frac{b^2}{4}-\frac{(2a-b)^2}{4}.$$

It will, therefore, be obvious, that if, before entering the table of quarter squares, we double the factor a, we shall obtain precisely the same result as by the use of the expression [2].

I am aware that, in an isolated case, it is immaterial which of these methods be adopted; in general, in such a case, it will be preferable to use the other formula stated in the work, viz.—

$$ab=\frac{(a+b)^2}{4}-\frac{(a-b)^2}{4} \quad . \quad . \quad . \quad . \quad . \quad . \quad [1]$$

It may be said that, by the process here recommended, three tabular entries are necessary, while by the use of the formula [1] two tabular entries only are needed; but it must be borne in mind, that in the operation by the former of these modes we have only to take the difference of the factor, while in the latter both the sum and difference of the factors must be found. In isolated cases, the amount of work by either formula is about equal; but with a series of quantities to be multiplied into a constant factor, the computer will, I think, find it more convenient to adopt the method now suggested. It may further be remarked, that the process here pointed out is quite as simple, if not more so, than if done by logarithms. The amount of work is not greater, and we obtain a result directly in natural numbers.

In illustration of the above remarks, I append an example of the values of a policy taken out at age 25, at the end of 1, 2, 3, &c., years, the premium being just due and not paid (Carl. 3 per cent.). For the sake of comparison, I place in juxtaposition the logarithmic process. The constant is supposed to be written on a card, and moved onward for each operation,

and for this reason the constant figures do not appear except at the head of each column.

Here the constant factor is $\frac{1}{1+a_{25}} = \frac{1}{21{\cdot}666} = {\cdot}0461575$, and this quantity being doubled, for the reason before pointed out, $= {\cdot}092315$.

$2.\left(\frac{1}{1+a_{25}}\right)$	(1) Constant factor, ·092315.	Quarter square of (1), 213052.	Colog. $(1+a_{25})$, 8·66424.
$1+a_{26}$	21·442	11494	1·33127
Difference	70873	Ar. Co. $\bar{1}874425$	9·99551 = ·98971
		·98971	
$1+a_{27} =$	21·212	11249	1·32658
	71103	$\bar{1}873609$	9·99082 = ·97909
		·97910	
$1+a_{28} =$	20·981	11005	1·32183
	71334	$\bar{1}872787$	9·98607 = ·96844
		·96844	
$1+a_{29} =$	20·761	10776	1·31725
	71554	$\bar{1}872001$	9·98149 = ·95828
		·95829	
$1+a_{30} =$	20·556	10564	1·31294
	71759	$\bar{1}871266$	9·97718 = ·94882
		·94882	
$1+a_{31} =$	20·348	10351	1·30852
	71967	$\bar{1}870519$	9·97276 = ·93921
		·93922	
$1+a_{32} =$	20·134	10134	1·30393
	72181	$\bar{1}869748$	9·96817 = ·92933
		·92934	
$1+a_{33} =$	19·910	9910	1·29907
	72405	$\bar{1}868938$	9·96331 = ·91900
		·91900	
$1+a_{34} =$	19·675	9678	1·29392
	72640	$\bar{1}868086$	9·95816 = ·90816
		·90816	
$1+a_{35} =$	19·433	9441	1·28854
	72882	$\bar{1}867205$	9·95278 = ·89698
		·89698	

The foregoing results, subtracted from unity, give the value of a policy at the end of 1, 2, 3, &c., years.

I have the honour to be, Sir,
Your obedient Servant,

Eagle Life Office,
26th March, 1860.

SAMUEL L. LAUNDY.

INSTITUTE OF ACTUARIES.

PROCEEDINGS OF THE INSTITUTE.

Fifth Ordinary Meeting, Session 1859-60.—*Monday, 26th March*, 1860.

CHARLES JELLICOE, Vice-President, in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced various donations to the library.

It was announced to the meeting that the following Report had been received and adopted by the Council, and that the Syllabus appended would be in future acted upon :—

"The Examiners of the Institute have the honour to report to the Council, that they have for some time past, in the discharge of the duties entrusted to them, been sensible that the existing Syllabus required modification, in consequence partly of the time which has elapsed since it was formed, and partly of the experience gained whilst it has been in operation; that they have accordingly had several meetings to discuss such modifications as appeared to be desirable; and, having decided upon them, beg leave to submit a copy of the Syllabus in its amended form, with a recommendation that the same be adopted and published."

A. H. BAILEY. H. W. PORTER, B.A.
SAMUEL BROWN. T. B. SPRAGUE, M.A.
W. B. HODGE. ROBERT TUCKER.
C. JELLICOE.
} *Examiners.*

SYLLABUS.

MATRICULATION EXAMINATION.

	Questions.
Vulgar fractions, Decimal fractions, Logarithms, Evolution	7
Equations, simple and quadratic	4
Series, arithmetical and geometrical, Permutations and combinations, Binomial theorem	8
Finite differences	2
Geometry—First four books of Euclid	4
Total	25

Maximum number of marks 500, of which 250 must be obtained to enable the Candidate to pass.

SECOND YEAR'S EXAMINATION.

	Questions.
Theory of logarithms Elements of the theory of probabilities Compound interest and annuities certain	6
Tables of mortality	4
Construction of auxiliary tables Annuities and assurances on lives Annuities and assurances on survivorships Miscellaneous questions	15
Total	25

Maximum number of marks 750, of which 375 must be obtained to enable the Candidate to pass.

THIRD YEAR'S EXAMINATION.

	Questions.
Life Assurance Finance.—Construction and graduation of tables of mortality; existing tables of mortality—the mode of their construction and their respective merits; methods of determining the surplus in an Assurance Company and of distributing it amongst the assured	5
Legal Principles.—Acts of Parliament; Charters of Incorporation; Deeds of Settlement; Partnerships, limited and unlimited—powers and duties of persons constituting them; Policy considered as a Contract; Probates and Letters of Administration; Assignments; personal representatives; Bankruptcies	5
Statistics.—Methods for the arrangement and collection of data; tests of accuracy; preparation of abstracts and reports; general system of the country's finance; funded and unfunded debt, and fiscal arrangements; taxation	5
Currency, Banking, and Investments.—Currency, metallic and paper; nature of banking—Bank of England, Private and Joint Stock Banks; Bills of Exchange; comparative value of securities; high and low prices; fluctuations in the rate of interest	5
Miscellaneous.—Book-keeping; auditing; valuation of marketable securities; and approximate calculations	5
	25

Maximum number of marks 1000, of which 500 must be obtained to enable the Candidate to pass.

A paper was read, "On the Life Assurance Companies of Germany; their business and position in 1858." By Herr Rath G. Hopf, of Gotha, Corresponding Member.

Thanks having been voted to Herr Hopf, the meeting adjourned to 30th April, 1860.

Sixth Ordinary Meeting, Session 1859-60.—*Monday,* 30*th April,* 1860.

WM. BARWICK HODGE, Vice-President, in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Honorary Secretary read a letter from Mr. A. G. Finlaison, announcing the decease of his father, Mr. John Finlaison, the President of the Institute; and Mr. Hodge, after expressing the regret with which that intelligence had been received, reported that Mr. Jellicoe had been unanimously elected by the Council to fill the office until the annual general meeting in June. Mr. Jellicoe briefly expressed his acknowledgments for the honour conferred upon him, and the business of the meeting was then proceeded with.

The Secretary announced various donations to the library.

Frederick Bigg, Esq., duly nominated at the last ordinary meeting, was unanimously elected a Fellow of the Institute.

Mr. Hodge then read Part 4 of his paper, "On the rate of interest for the use of money in ancient and modern times."

A vote of thanks was very cordially given to Mr. Hodge, and the meeting adjourned to the 26th November, 1860.

Annual General Meeting, Saturday, 2nd June, 1860.

CHARLES JELLICOE, President of the Institute, in the Chair.

The circular convening the meeting having been read,

The minutes of the last ordinary meeting were read and confirmed.

The Report of the Council was then read by Mr. J. Hill Williams, one of the honorary secretaries, and was as follows :—

"The Council have the satisfaction of reporting, that, notwithstanding the great changes which have taken place of late as regards the various assurance institutions established in the metropolis, the number of members on the roll of the Institute suffers no decrease of importance. At the present time it appears there are 46 Fellows, 20 Official Associates, and 81 Associates—147 members in all—a number differing but little from that reported of late years as constituting the whole body.

"The receipts during the year amount to £612. 12*s*. 7*d*., and the payments to £392. 2*s*. 8*d*.; a further small purchase of Consols has been made, and the assets now consist of £220. 10*s*. 11*d*. in cash, and £192. 8*s*. 6*d*. invested in (£198. 16*s*. 2*d*.) Stock, making together the sum of £412. 19*s*. 5*d*. This includes the donation so obligingly made by Mr. Gompertz towards an extension of the library.

"The following papers have been read during the session, viz., 'On the rationale of certain actuarial estimates,' by Charles Jellicoe; 'On a formula for calculating the value of a survivorship assurance,' by M. Réboul; 'On the purchase of life assurance policies as an investment,' by Archibald Day; 'On some considerations suggested by the Reports of the Registrar-General, being an inquiry into the question how far the inordinate mortality in this country, exhibited by these Reports, is controllable by human agency,' by H. W. Porter, B.A.; 'On the Life Assurance Companies of Germany, their business and position in the year 1858,' by Herr Rath G. Hopf; 'On the rate of interest for the use of money in ancient and modern times' (Part IV.), by William Barwick Hodge, Vice-President.

"Some of these have already appeared, and the remainder will shortly be published, as usual, in the *Journal* of the Institute.

"The members have been informed that the prize offered last year by the Institute for an essay on the methods of distributing the surplus among the persons assured in a Life Assurance Company, has not been awarded. The Council have therefore determined on repeating their offer of a similar prize, and again invite the associates to compete for it. The subject of the essay and the regulations to be the same as before.

"The members have also been made aware of the heavy loss which the Institute has sustained in the death of its President, the late Mr. Finlaison. The office of president was tendered to that gentleman at the formation of the Institute, in accordance with the unanimously-expressed wishes of its founders, and was accepted by him, without hesitation, at a time when its capabilities were unknown and its success uncertain.

"Throughout its career the late President manifested a lively interest in its proceedings, and never failed, when taking part in them, to exhibit the utmost kindness and courtesy to its members. The Council forbear to touch upon the events of a life of great activity and usefulness, whether regarded from a public or a private point of view. They trust that a more fitting occa-

sion for doing so will shortly present itself, and meanwhile content themselves with this brief record of their respectful remembrance and unfeigned regret.

"In accordance with Article 28 of the Laws and Constitution of the Institute, it was necessary to appoint a successor to Mr. Finlaison, for the period intervening between the time of the vacancy occurring and that of the annual general meeting; and Mr. Jellicoe having, at the request of a majority of the members of the Council, allowed himself to be put in nomination for the office, was balloted for at a special meeting of the Council on the 28th April last, and found to be unanimously elected. Mr. Jellicoe has, accordingly, exercised the functions of the presidentship until the present time, and, as the members will observe, is now recommended for re-election."

The abstract of the receipts and payments of the Institute, for the financial year ended 31st March last, was then read (*see* p. 120).

On the motion of the Chairman, the Report was unanimously adopted.

The election of a President, Vice-Presidents, and Officers, for the year ensuing, was then proceeded with.

Mr. Bailey and Mr. Cutliffe were appointed scrutineers.

On the result of the ballot being obtained, the following was declared to be the list, viz. :—

President.

CHARLES JELLICOE.

Vice Presidents.

SAMUEL BROWN. | PETER HARDY, F.R.S.
WILLIAM BARWICK HODGE. | ROBERT TUCKER.

Treasurer.

JOHN LAURENCE.

Honorary Secretaries.

JOHN REDDISH. | J. HILL WILLIAMS.

The following gentlemen were also unanimously elected Auditors of the Institute :—Edward Cutbush, John Coles, and James Terry.

It was resolved unanimously—"That the best thanks of the meeting be given to the President and Council for their services during the past year."

Mr. Jellicoe said—"Gentlemen, on behalf of the Council I have to express our acknowledgments for your kindness in adopting so readily the resolution which has just been passed ; and on behalf of the Vice-Presidents, the other officers and myself, to thank you for the honour you have done us in this day's election, and to assure you that while we are sensible of the compliment you have paid us, we are not forgetful of the responsibilities which it involves. We have laboured now for some years under the auspices of our late President, whose loss we so much regret. In the language of our laws, we have striven 'to elevate the attainments and to promote the efficiency of those engaged in our pursuits; and we have sought to extend and improve the data and methods of that science which has its origin in the application of the doctrine of probabilities to the affairs of life.' That our efforts have not been wholly unavailing I think any impartial person must acknowledge, who regards for a moment the number and variety of the subjects discussed in our *Journal*, and who can appreciate the clear light which has there been thrown upon many questions heretofore hidden in the deepest obscurity. The mathematical theories applicable to our science have been developed and extended, and the ideas of almost every writer upon them collected and examined. The manifold processes required in our daily practice have been in like manner reviewed—improvements introduced into them where improvement was needed, and new methods substituted where the old ones were found to be radically defective—and thus have been brought about a precision and uniformity of practice which, it must be admitted, were a few years ago altogether unknown. I will not dwell upon the advantages to be derived from the bringing into one focus such a mass of information on the subject of assurance generally, both at home and abroad, and from the collection of so much valuable and original data having reference to it, nor upon the utility of the discussions which serve to throw no

little light upon many questions of public interest and importance; neither will I now detain the meeting by any lengthened allusion to the very satisfactory progress which the educational section of the Institute has exhibited, and to the wholesome and purifying influence which it is now exercising, and which it evidently promises to exercise in time to come—an influence more needed, as it would seem, in our pursuit than in almost any other. Suffice it to say, that the general result of our efforts is such as fully to justify them, and to encourage us to persevere with increased energy in the course upon which we have entered. I need hardly say there is much to be done: we have to inquire into the desirableness of adopting the modification so ingeniously suggested by Mr. Farren, and so ably illustrated by Mr. Younger, of what we may, perhaps, without impropriety, refer to as our peculiar 'calculus,' and to determine whether the advantages accruing are sufficient to justify an alteration, which, while it undoubtedly calls into exercise a much more refined method of determining our averages, will still leave them to be dealt with as are now those found in the more ordinary way, and which will apparently render it necessary to recalculate almost the whole of the immense collection of our tabulated values. We have also to ascertain whether any material change will appear in our mortality tables after they shall embody the results of a more prolonged experience, or whether they will remain unaffected by the events which have yet to be registered. It is surmised, as I need hardly mention, that the rates which they now exhibit may be more or less modified by the character of the withdrawals occurring in later years. It is, too, becoming more and more important to ascertain whether the rate of human mortality in the United States and elsewhere, amongst persons assured, corresponds with that found to obtain in this country: a vast amount of risk is being incurred on the assumption that it does, but there exists at present little or no data to warrant such an assumption. I am glad, however, to be able to state that an investigation, with a view to the elucidation of this matter, is now going forward in New York, under the able superintendence of gentlemen connected with the native Companies in the States. Some difference of opinion still exists as to the most accurate mode of dividing the surplus amongst the persons assured in a Life Assurance Company. It is very desirable that any doubts on this subject should be set at rest, and that, as far as possible, an uniformity of practice should obtain in regard to it. Amongst the purely statistical investigations which we have already touched upon are those connected with the questions of the decimal system of weights, measures, and coinage, and direct taxation. The arguments in relation to the former seem to be nearly exhausted; nevertheless, the somewhat unreasonable opposition made to the proposed reform calls for further efforts, and it is to be hoped that additional advocates, or at least expounders, will not be wanting amongst the members of our Society. As regards the question of taxation, it is very desirable that further investigations should be made, to determine whether it be true or not that trade is paralysed by the imposition of such duties as those of the customs and excise, and whether it be true or not that a vast means of employment for the population is thereby destroyed, and a needless poverty created and enforced. On the other hand, there remains abundant room for discussing what is the true measure of liability in a system of direct taxation, and for illustrating, with the aid of statistical records, the merits or demerits by which such a system may be found to be characterised. Such are a few of the subjects which may hereafter profitably engage our attention, and I trust I may be allowed to express the hope that the younger members of the Institute will assist in the investigation of them. I would ask them to bear in mind that unless the habits which these labours require are formed in early life, they become irksome and distasteful in later years; that if they desire to exercise their best faculties then, it is needful that they exert some energy now; and that the Institute must of necessity look to them at no very distant day to maintain and improve whatever reputation or distinction it may then have acquired from the persevering labours of their predecessors."

The meeting then adjourned.

INSTITUTE OF ACTUARIES.

Dr. *Abstract of Receipts and Payments for the Year ending 31st March,* 1860. **Cr.**

RECEIPTS.

			£	s.	d.
March 31, 1859.	To balance brought forward		64	10	8
Special subscriptions, viz.—25 at £3 3 0		£78 15 0			
1 „ 1 1 0		1 1 0			
1 „ 1 0 0		1 0 0			
			80	16	0
Subscriptions for 1858–9 (arrears)			5	5	0
Subscriptions due for 1859–60—					
46 Fellows.....	34 Town ..at £3 3 0	£107 2 0			
	12 Country „ 2 2 0 ..	25 4 0			
20 Official Associates..	18 Town .. „ 3 3 0 ..	56 14 0			
	2 Country „ 2 2 0 ..	4 4 0			
82 Associates.....	64 Town .. „ 2 2 0 ..	134 8 0			
	18 Country „ 1 1 0 ..	18 18 0			
		346 10 0			
Less Subscriptions of Members not paid, viz.—					
4 Associates, Town ....at £2 2 0..	£8 8 0				
2 „ Country .. „ 1 1 0..	2 2 0				
		10 10 0			
			336	0	0
Guarantee Society			90	8	1
Examination Fee			5	5	0
Dividends on Messenger Legacy.....			5	4	0
Sundries			25	3	10
			£612	12	7
March 31, 1860.	To Balance brought down		£220	9	11

Note.—The Assets of the Institute, on the 31st March, 1860, consisted of £198. 16*s.* 2*d.*, 3 per cent. Consols, say 179 10 1
Cash 220 9 11
Books in Library, say 300 0 0
Total £700 0 0

PAYMENTS.

1859.	£	s.	d.	£	s.	d.
Rent	75	0	0			
Salaries	98	8	7			
Journal (5 quarters)	119	3	0			
Library	8	6	10			
Stationery and Printing.....	16	19	0			
Postage and Receipt Stamps	6	15	0			
Firing and Lighting	12	17	7			
Institute Meetings.....	14	0	8			
Advertising Examinations (2 years)	4	18	0			
Miscellaneous.....	18	1	4			
Total of general expenses.....				374	10	0
Purchase of £18. 12*s.* 3*d.* 3 per cent. Consols (Dividends from Messenger Legacy)				17	12	8
March 31, 1860. Balance carried forward.....				220	9	11
				£612	12	7

Examined and approved:—

12, ST. JAMES'S SQUARE, LONDON,
18*th May*, 1860.

C. CHILD,
EDWARD CUTBUSH,
CHAS. WATKINS, } *Auditors.*

THE

ASSURANCE MAGAZINE,

AND

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On the Construction of Life Tables, illustrated by a new Life Table of the Healthy Districts of England. By W. Farr, Esq., M.D., F.R.S.*

THE *Transactions of the Royal Society* contain the first Life Table. It was constructed by Halley, who discovered its remarkable properties, and illustrated some of its applications. The Breslau observations did not supply Halley with the data to frame an accurate table, for reasons which will be immediately apparent; but the conception is full of ingenuity, and the form is one of the great inventions which adorn the annals of the Royal Society.

Tables have since been made correctly representing the vitality of certain classes of the population; and the form has been extended so as to facilitate the solution of various questions.

In deducing the English Life Tables from the national returns, I have had occasion to try various methods of construction; and I now propose to describe briefly the nature of the Life Table, to lay down a simple method of construction, to describe an extension of its form, and to illustrate this by a new table representing the vitality of the healthiest part of the population of England.

The Life Table is an instrument of investigation; it may be called a *biometer*, for it gives the exact measure of the duration of life under given circumstances. Such a table has to be constructed for each district and for each profession, to determine their degrees of salubrity. To multiply these constructions, then, it is necessary

* Extracted, with permission, from the *Philosophical Transactions*, 1860.

to lay down rules, which, while they involve a minimum amount of arithmetical labour, will yield results as correct as can be obtained in the present state of our observations.

I. GENERAL DESCRIPTION OF A LIFE TABLE. (*See* Table C.)*

A Life Table represents a *generation of men* passing through *time;* and time under this aspect, dating from birth, is called age. In the first column of a Life Table, *age* is expressed in *years*, commencing at 0 (birth), and proceeding to 100 or 110 years, the extreme limit of observed lifetime.

If we could trace a given number of children, say 100,000, from the date of birth, and write the numbers down that die in the first year, living therefore less than one year, against 0 in the Table, and on succeeding lines the numbers that die in the second, third, and every subsequent year of age until the whole generation had passed away, these numbers would form a *Table of Mortality*, showing at what ages 100,000 lives become extinct.

Again, if the 100,000 children were followed, and the numbers living on the first, on the second, and on every subsequent birthday until none was left, the column of numbers would constitute a *Table of Survivorship*. So if, of 100,000 children born at a given point of time, the numbers dying (d_x) in each subsequent year were written in one column, and the numbers surviving (l_x) at the end of each year in another column, the two primary columns of the Life Table would be formed.

It is evident that if one of these columns is known, the other may be immediately deduced from it; for if, of 100,000 children born, 10,295 die in the first year of age, 3,005 in the second year of age, it follows that the numbers living at the end of one year must be 89,705, at the end of two years 86,700. Upon adding the column (d_x) from the bottom up to the number against any age (x), the sum will represent the whole of the numbers *dying after that age;* and, consequently, the numbers *living at that age*, as shown in the collateral column (l_x).

The 100,000 children born at the same moment, and counted *annually* to determine the numbers *living* at *the end of every year*, would, by our Table, completely pass away in less than 107 years. If another generation of 100,000, born a year afterwards, were followed, the numbers dying in the various years of age would not be very different, the circumstances remaining the same; and the numbers of those entering each year of age would vary inconsider-

* The tables and plates referred to in this and the following pages will be found, with the concluding portion of the paper, in the next number.

ably from those of the first series. If 100,000 children again were born at annual intervals, and were subject to an invariable law of mortality, they would form a community of which the numbers living at each age would be represented by the successive numbers (l_x) in the Life Table. The sum of these numbers, by the new Table of Healthy Districts, would be 4,951,908. The births are here assumed to take place simultaneously, at annual intervals; immediately before the births, therefore, in such a community, its population would be 4,851,908, to which it would fall progressively from 4,951,908 by 100,000 successive deaths in the year. The average number constantly living would be some number between 4,951,908 and 4,851,908; and it would be very nearly the mean of these limiting numbers.

In the ordinary course of nature, the births in a community take place in remittent succession; and if it is assumed that the 100,000 births occur at equal intervals over every year, it is evident that, at any given date, a certain number will be found living at all the intermediate points of age between 0 to 1 year, 1 to 2, 2 to 3, and all the remaining years of age. The population in the above instance would be found, by enumeration, to be nearly 4,899,665.

The annual *births* would be 100,000 in such a community. The annual deaths would also be 100,000; and by taking out the deaths at each year of age, from the parish registers of a single year, the second column (d_x) of the Life Table would be found. By adding this column of deaths up, and entering the sum of the numbers year by year against every year of age (x), the third column (l_x) of the Life Table would be obtained; for it has been already shown that the numbers attaining any age x are equal to the numbers dying at that age and all the subsequent ages. From the registers of the deaths, a table of the numbers of the *population living* in a parish *so constituted* could be immediately determined without any enumeration. Its deviations from the truth would be accidental; and they would be set right by taking the mean of many years. So, also, from a simultaneous enumeration of the *numbers living in each year of age*, the two columns d_x and l_x of the life could be constructed without reference to any registry of the deaths at different ages.

The *mean age at death* in such a community would express the mean lifetime, or the expectation of life at birth; and the product of the number expressing the annual births multiplied into the mean age at death would give the numbers of the population.

The facts which a Life Table expresses in numbers may be

represented by the lines of a figure; age (x) being indicated by the abscissas measured from 0, the *numbers living* (l) at each age by the ordinates of a curve line, and the numbers living between any two ages by the plane surface within the two ordinates, the curve line, and the corresponding portion of the abscissa. The relative numbers living at the ages 20 and 21 are seen in the two lines of Plate XLII., fig. 1, over the ages 20 and 21; if the deaths in the intervening year all occurred immediately after the age 20 was attained, the numbers living would also be represented by the parallelogram having its two sides equal to the ordinate over 21, and for its base the portion of the abscissa between 20 and 21; but if all the deaths occurred only the instant before the age 21 was attained, the height of the parallelogram would be represented by the ordinate over the age of 20. The deaths occur at intervals between the two ages, so the numbers living, and the *lifetime* which is passed between the two ages, are correctly represented by the curvilinear area.

The deaths in each year of age are called the *decrements of life.* They are represented by the differences in the lengths of the successive ordinates. Thus, by cutting off a small portion of the ordinate at the age 20, the ordinate at the age 21 is obtained; this small portion, shown in Plate XLII., represents the decrement of life in that year of age. It will be observed that the decrements vary at every year of age; and this is more evident when they are exhibited on the larger scale of Plate XLII., fig. 2. The decrement in the first year is large; in the first five years the decrements of life are considerable; at the age of 10 to 15 they fall to their minimum, slowly increase to the age of 56, increase more rapidly until the maximum is attained at the age of 75, then decline gradually to 85, and, after that, more rapidly, until every life is extinct at the age 107 by this table.

II. PRINCIPLES OF CONSTRUCTION—THE FUNDAMENTAL COLUMN l_x.

The conditions of the hypothesis upon which the preceding reasoning rests are never precisely realised in nature—in the first place, the number of births fluctuates, increases, or decreases, from year to year; and the deaths fluctuate still more, rarely equalling the births in number. Immigration and emigration interfere. Under these circumstances, tables such as those which Halley, Price, and others, made from the observations on the *deaths alone* are never accurate, and require correction to give approximate

results. If it be assumed that the law of mortality remains invariable, and that migration does not interfere, then the nature of the correction to be applied to a table framed from the deaths alone will become immediately apparent by an example. The births increase in England. Let the annual births in a portion of the community be doubled in sixty years—thus, be 50,000 in 1796, and 100,000 in 1856; then the deaths of persons of the age of 60 in 1856 must be doubled to obtain the deaths which would have happened at that age if the annual births sixty years before these deaths had been 100,000. If the births have been accurately registered, formulæ for correcting the ordinary table drawn up from the deaths at different ages will be suggested by the above considerations.

I now proceed to describe another method which has been adopted in framing the Table C, and is applicable wherever (1) the number of annual births, (2) the numbers of the population living at definite periods of age, (3) the deaths at the corresponding ages during a certain number of years, in any community, are ascertained by observation. This method is not open to the previous objections.

The aim is to obtain equations which will describe the curve lines (Plate XLII., fig. 1) of the Life Table in the most direct way; and these equations may be deduced from the determined rate of mortality at certain intervals of age.

The relative numbers living at two ages, 20 and 21, can evidently be found from an equation which expresses the relation of the average numbers living and dying between those ages during a given time. This can be determined very nearly; for, although the ages of the living are not ascertained with exact precision at the census, still, by taking all the numbers living at the ages 15, 16, 17 years, up to 24 and under 25, together, the aggregate represents very nearly the numbers living in that decenniad of life. The deaths at the same ages are obtained with at least equal accuracy from the registers of deaths. By this process, and by extending the observations over five or more years, a number of facts is obtained sufficiently great to yield average results; and it may be assumed that the ratio of the living at the ages 15–25* to the dying in a year at the same ages, 15–25, represents the annual rate of mortality at the exact age 20. So also the mortality rate at the ages 30, 40, 50, and other ages, may be determined. As observations grow more exact, and the facts are multiplied, the

* By this, 15 and *under* 25 years of age is understood, and so in all similar cases.

intervals of age may be diminished to 5 years, and ultimately to 1 year.

In determining the *rate* of *mortality*, a given number of persons living a year is considered equivalent to twice that number living half a year, or to half the number living two years.

Thus, if nd represent the deaths in n years out of a number amounting, on an average, to P during the same years, then $\frac{nd}{n\mathrm{P}}=m=$the rate of mortality, or the proportions of death in a *year* (always taken as the unit of time) out of *one year* of *lifetime*. It is found, from all the observations hitherto made on a large scale, that the rate of mortality varies at every interval of age; but at the same age it may, for the present purpose, be considered invariable under similar circumstances.

m_x therefore varies in every moment of age; but I have employed it to express the mean annual rate of mortality during the year following the year of age x, $\therefore \frac{d_x}{\mathrm{P}_x}=m_x$, where d_x indicates the deaths, P_x the year of lifetime, after the year of age x. The m_x is the expression of the force of the causes that induce death, of the death-force, *vis mortalis*; and its reciprocal $\frac{1}{m_x}=u_x$ measures the forces that sustain life, the *vis vitalis*.

The vital force, under natural circumstances, may, by one hypothesis, be sufficient to sustain a whole generation alive for seventy or eighty years, and then suddenly collapse. The Life Table, if this hypothesis were true, would be represented by the *parallelogram* in which the curve of the Life Table is inscribed (Plate XLII., fig. 1).

By the hypothesis of Demoivre,* the rate of mortality is such, that, at the age of 20, 1 in 66 living at the beginning dies before the end of the year, leaving 65, 64, 63, 62, 61, to enter on each year of age until, at the age of 86, all are dead.

Upon this hypothesis the relative numbers living up to the age 86 form an arithmetical progression; and the deaths in the equal times are equal out of the diminishing numbers living. The rate of mortality increases on this hypothesis, as age advances, in the same ratio as $n-\frac{1}{2}:1$; where n is the difference between the actual age x and 86. It is called the complement of life. The Life Table upon this hypothesis has equal decrements, and might be represented on Plate XLII., fig. 1, by drawing a diagonal line

* See *Treatise of Annuities on Lives*; Preface to 2nd edition.

through the parallelogram. Its deviation from the true curve on this scale is evident; but it is also evident that a series of straight lines, which would nearly represent the true curve, may be drawn from point to point of all the ordinates.

If the causes of death act with equal intensity at all ages, they may be represented by any simple external cause destroying an equal *proportion* of the numbers living in equal intervals of time. Thus, if 1,600 men were distributed equally over ground where they were exposed to certain dangers, represented by successive discharges of musketry, which, at every discharge, shot down one-half of the numbers remaining, they would be reduced successively from 1,600 to 800, to 400, to 200, to 100, to 50, and so on *ad infinitum*, if a fraction of a living man could be conceived; the numbers living at each year of age in a Life Table would not decrease at *these rates*, but they *would decrease* at a constant rate if the dangers at every stage of life remained *constant* and equally *great*. The numbers of the living at successive ages would be in geometrical progression, and would be represented by the ordinates of the logarithmic curve.

The law of mortality can only be derived from observation, and it is found to be less simple than either of these hypotheses implies. It can, however, be represented nearly by equations at different periods of age. Upon inspecting Table A, it will be seen that, at the age 55–65, which may be represented by the exact age 60, the mortality is such, that 2,162 women die in a year out of a number equal to 100,000 living a year; and the mortality, which is the ratio of the dying to the living in a unit of time, here set down as a year, is, therefore, $m = \cdot 02162$. Again: the mortality at the age of 70 is ·04992, at the age of 80 it is ·11866, and at the age of 90 it is ·26711. The mortality increases rapidly, and is more than doubled every ten years. The four numbers differ little from the terms of a geometrical progression, the logarithms of which have a constant difference. Let the rate at which the mortality increases be r, and $r^{10} = 2 \cdot 3116$, and the first term (m) be ·02177; then a series of numbers will be formed differing little from those which express the value of m at decennial intervals of age.

Values of m at the precise age x.—*Females.*

Age (x).	60.	70.	80.	90.
By observation .	·02162	·04992	·11866	·26711
By hypothesis .	·02177	·05083	·11633	·26891

Note.—It may be assumed that m at 60 is the mean value of m in its range from $m_{59\frac{1}{2}}$ to $m_{60\frac{1}{2}}$; and so in other cases.

The *annual rate* of the increase of m from the age of 55 to 95 is $r=1{\cdot}0874$; and if m is the mortality at any age after 55, then $m_z=mr^z=$the mortality at z years after the age at which m is taken. The common logarithm of r is $=\lambda r=\cdot03639$.

The mortality (m) of males at corresponding ages is higher than the mortality of females; but the rate of increase as age advances is nearly the same.

The value of m for females at the age of 20 is ·00765, and the mortality increases at the rate of nearly one-seventh part every ten years. The exact value of r is 1·0149, and $\lambda r=\cdot006423$.

Values of m.—*Females.*

Age.	20.	30.	40.	50.
By observation .	·00765	·00894	·00998	·01192
By hypothesis .	·00760	·00882	·01022	·01185

By these observations, in the healthy districts the mortality (m) of men at the ages 15 to 45 is lower than the mortality of women at the same ages; yet, during that period, the rate of increase r is nearly the same for the two sexes. From the age of 40 to 50, and 50 to 60, the mortality of males increases at a rate intermediate between the rates of manhood and mature age.

Limits of Ages.		
	Females.	
15 to 55 or 20 to 50	$r=1{\cdot}0149$	$\lambda r=\cdot00642$
55 to 95 or 60 to 90	$r=1{\cdot}0874$	$\lambda r=\cdot03639$
	Males.	
15 to 45 or 20 to 40	$r=1{\cdot}0148$	$\lambda r=\cdot00640$
55 to 95 or 60 to 90	$r=1{\cdot}0874$	$\lambda r=\cdot03640$

The subjoined Table exhibits the series of values for m derived from the hypothesis of two constant rates, and from direct observation. The values of r for females may be evidently applied to males in every period, except in the ten years of age, 40 to 50.

Mortality (m) *of males and females,* (1) *derived from observation, and* (2) *from the hypothesis that* m *increases at the preceding rates.*

Precise Age.	Annual Mortality to 100 constantly living at each Age (m).			
	Males.		Females.	
	By observation.	By hypothesis.	By observation.	By hypothesis.
20	·691	·696	·765	·760
30	·818	·807	·894	·882
40	·928	·935	·998	1·022
50	1·273	1·083	1·192	1·185
60	2·294	2·329	2·162	2·177
70	5·486	5·385	4·992	5·033
80	12·817	12·451	11·866	11·633
90	28·350	28·785	26·711	26·891
100	40·000?	66·550?	45·000?	62·160?

The observations on the numbers living and dying of the age of 95 and upwards are exceedingly uncertain; and it is probable that many of the persons believed to be 100, &c., are really persons five or ten years younger; so that these values of m_x, by the hypothetical method, are probably as correct as the direct numbers.

I shall now notice briefly the application of this hypothesis, first suggested by Mr. Gompertz, and applied by him to the interpolation of the Northampton and other Tables.* Mr. Edmonds, in 1832, extended the "Theory," and applied it to the construction of three Life Tables.† He gave an elegant formula, similar in principle to that of Mr. Gompertz, from which the curve of a Life Table can be deduced, upon the above hypothesis.

In the equation $\frac{s}{t}=v$, where s indicates space, t time, v velocity, the units of measure must be fixed before numbers can be inserted in the general expression; and then v will express, in the measure that has been applied to space, the number of such units of space described in *one* unit of time. Here v is a ratio—it is the rate at which the body moves; and, in the same manner, m, in the equation $\frac{d}{l}=m$, is the *rate of dying*—that is, as I shall express it, the *mortality;* or it is the ratio of the dying to the living in a given unit of time, the time during which the deaths occur being of precisely the same duration as the time during which the living are under observation—

l (living during 1 year) : d (dying during a year) :: 1 (year of life) : m.

If for l the number 100,000 is substituted, it is assumed that immediately a death occurs another life is substituted; and as the time is a year, then 760 will represent the value of d at the age 20, according to the preceding Table; $\therefore m=\cdot 00760$. If the *time,* instead of *one year,* be the *thousandth part* of one year, then $m=\cdot 0000076$; and if the time be infinitely short, m will be infinitely small: m is a ratio; the quantity of life existing during the time is represented by 1, and the quantity of life destroyed by a fraction, m. Whether the life inheres in the first organic molecule after conception, in the infant, or in the man, the vital action has a certain force of continuance, which is constantly varying; and the amount of this *force* that is *extinguished* at a given instant of time will be represented by the force of mortality—namely, by

* *Philosophical Transactions,* 1825; paper by B. Gompertz, Esq., F.R.S.

† *Life Tables founded on the Discovery of a Numerical Law regulating the Existence of every Human Being, &c.* By T. R. Edmonds, B.A., 1832.

m at that instant. Then let the age $x=z+a$, where a represents the number of years up to the age at which a given rate (r) of increase of m begins, then $z=x-a$; and the mortality at any instant of age, in an instant of time at the end of z years or parts of years, will be mr^z. Now, let y represent the living at that precise age, then the decrement of y in an infinitely short time will be $-dy=ymr^z dx$; the dy being negative, as it is taken in a direction opposite to that in which the ordinate y of the curve is assumed to be drawn. Transferring y to the other side of the equation, this becomes $-\frac{dy}{y}=mr^z dz$; and integrating both sides, we have ($\lambda_e y$ being put for the hyperbolic logarithm of y, and $\lambda_e c$ for the difference between the constants of the two integrals)—

$$\lambda_e c-\lambda_e y=\lambda_e \frac{c}{y}=\frac{mr^z}{\lambda_e r}; \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\therefore \quad \lambda_e y=y_e c-\frac{mr^z}{\lambda_e r}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and

$$\lambda_e c=\lambda_e y+\frac{mr^z}{\lambda_e r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

When z is made zero, let $y=1$; then $\lambda_e y$ will also disappear, and $\lambda_e c=\frac{m}{\lambda_e r}$. Upon substituting this value of $\lambda_e c$ in equation (2), it becomes—

$$\lambda_e y=\frac{m}{\lambda_e r}-\frac{mr^z}{\lambda_e r}=\frac{m}{\lambda_e r}(1-r^z). \quad . \quad . \quad . \quad (4)$$

Upon passing to the numbers, equation (4) becomes

$$y=\varepsilon^{\frac{m}{\lambda_e r}(1-r^z)}$$

=the value of y (taken as 1 at the origin) at the end of z years.

Let λ denote the common logarithm with the base 10, then $\lambda_e y=\frac{\lambda y}{k}$, where k is the modulus of the common system of logarithms; as also

$$\lambda_e c=\frac{km}{\lambda r}, \quad \text{and} \quad \frac{mr^z}{\lambda_e r}=\frac{kmr^z}{\lambda r}.$$

Equation (2) becomes, after the required substitutions,

$$\frac{\lambda y}{k}=\frac{km}{\lambda r}-\frac{kmr^z}{\lambda r}$$

and

$$\lambda y=\frac{k^2 m}{\lambda r}(1-r^z); \quad . \quad . \quad . \quad . \quad . \quad (5)$$

so the equation becomes finally

$$y=10^{\frac{\lambda^2 m}{\lambda r}(1-r^z)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This is the form given by Mr. Edmonds, and is convenient for use.

By making z successively 1, 2, 3, ... up to any number less than the number of years of age within which r remains constant, the number l_x being known, the number living at any other age within that range will be obtained by multiplying l_x by the corresponding value of y. Thus, if y_{10} is the value of y. when $z=10$ in equation (6), then, putting l_{20} for the numbers living at the age 20, the living at the age 30 will be $y_{10} \times l_{20}=l_{30}$.

This hypothesis does not express the facts deduced from the observations exactly. If m_z could be expressed exactly over more than 20 years by $m_z=m_0 r^z$, the first differences (δ^1) of the logarithms in the series following would, in a certain number of cases, be equal.

Females in Healthy Districts of England.

Precise Age.	Annual Rate of Mortality.	Logarithms of the Annual Mortality.	First Decennial Differences of λm_x.	Second Decennial Differences of λm_x.
x.	m*.	λm.	δ^1.	δ^2.
20	·00765	$\bar{3}$·8835	·0677	—·0197
30	·00894	$\bar{3}$·9512	·0480	·0290
40	·00998	$\bar{3}$·9992	·0770	·1817
50	·01192	$\bar{2}$·0762	·2587	·1047
60	·02162	$\bar{2}$·3349	·3634	·0126
70	·04992	$\bar{2}$·6983	·3760	—·0236
80	·11866	$\bar{1}$·0743	·3524	—·1259
90	·26711	$\bar{1}$·4267	·2265	
100	·45000	$\bar{1}$·6532		

The inequalities in the second differences vary in every separate class of observations, but there is generally a tendency in the first and in the second differences to increase, over a certain extent of the series. The error of the hypothesis is slight if the rate of increase (r), of which λ·00677 is the logarithm in the case in hand, is only assumed to remain uniform for the ten years 20 to 30, or for the one year 20 to 21. Now, let the number living at the age

* Here, at the age 20, m is the mean mortality that rules over the age 19½ to 20½ years of exact time.

20 be represented by l_{20}, and the number living at the age 21 by l_{21}; then put $\frac{l_{21}}{l_{20}}=p_{20}$. Here it is evident that if l_{20} and p_{20} be known, l_{21} is determined immediately by the equation $l_{21}=l_{20}\times p_{20}$. But p_{20} is the value of y in the equation $y_1=10^{\frac{k^2m}{\lambda r}(1-r^z)}$, when z is put $=1$. Taking the numbers from Table A, we have $m=\cdot00765$ at the precise age $20=(19\frac{1}{2}+20\frac{1}{2})\frac{1}{2}$; and $\lambda m=\bar{3}\cdot8835130$, $\lambda r=\cdot0067728$, and $\therefore r=1\cdot015717$; k is put for the modulus of the common logarithms, $\therefore \lambda k^2=\bar{1}\cdot2755686$; $k(\lambda r)$ is the complement of the *logarithm* of (λr).

λk^2	$\bar{1}\cdot2755686$
λm	$\bar{3}\cdot8835130$
$k(\lambda r)$	$2\cdot1692317$
$\lambda(1-r)$	$\bar{2}\cdot1963697$
$-\cdot0033472$	$\bar{3}\cdot5246830$
$\bar{1}\cdot9966528$	

As the factor $(1-r)$ is negative, it makes the exponent of 10 negative; and, upon taking the complement of this, the logarithm of y is found to be $\bar{1}\cdot9966528$. This is also the logarithm of $p_{20}=\cdot99232$, and it enables us to pass, in the construction of a Life Table, from the living at the age of 20 to the living at 21. If we obtain the several values p_x at every year of age, the whole of the Life Table can be constructed.

It will be found that p_x is always a fraction, and it does not differ very much from $1-m_x$; but while m_x* shows the *deaths* in a year out of a *unit* of *life* (which may consist of any *number* of individual *lives* constantly kept up), p_x shows how much out of a *unit of the same life* at the beginning of a year, the dead not being replaced, *survives a year* after the age x; and $1-p_x$ is the amount of loss which occurs in the same year out of a unit of life at its commencement. Thus, as $p_{20}=\cdot99232$, it follows that $1-p_{20}=\cdot00768$. In the same year of age, 20 to 21, the mortality is $m_{20}=\cdot00771$, or $\cdot00003$ more than $(1-p_{20})$. If the unit of life is made 100,000 living at the age 20, then 99232 will survive, and 768 will die in the ensuing year of age. But if it is assumed that the deaths take place at equal intervals, it may also be assumed that the number of lives (100,000) being constantly sustained, the accession of 768 new lives takes place at equal intervals, conse-

* m serves to indicate the mean mortality in the year following the exact age x.

quently that they are under observation half a year on an average, giving the equivalent of $\frac{768}{2}=384$ years of lifetime at the age 20 to 21. Now, out of this number (384), at that age *three* die when the mortality is m_{20}. This accounts for the difference of ·00768 and ·00771, the former occurring in a year out of a unit of life of which the waste is not replaced.

From these considerations, it may be inferred that, if m_x is known, p_x may be deduced from it, upon the hypothesis of equal decrements through the year, by the formula $p_x=\frac{1-\frac{1}{2}m_x}{1+\frac{1}{2}m_x}=\frac{2-m_x}{2+m_x}$. Thus, m_{20} being ·0077072, we have $\frac{\cdot 9961464}{1\cdot 0038536}=\cdot 99232$,* as before. The λp_{20}, by the previous method, is $\bar{1}\cdot 9966528$, and by this method it is the same. By either of the methods the value of p_x may be deduced for the subsequent ages, and p_{20}, p_{30}, p_{40} . . . . . p_{90}, p_{100} will be obtained. These values are here given, and it will be seen that the results by the two methods are nearly identical at all ages, except the last two, when the observations themselves become less exact.

Females.

Age (x).	$\lambda p_x=\lambda y_1=10^{\frac{x^2 m}{\lambda r}(1-r)}$.	$\lambda p_x=\lambda\left(\frac{1-\frac{1}{2}m}{1+\frac{1}{2}m}\right)$.
20	$\bar{1}\cdot 9966528$	$\bar{1}\cdot 9966527$
30	·9960967	·9960967
40	·9956263	·9956264
50	·9946669	·9946676
60	·9902049	·9902073
70	·9773538	·9773557
80	·9463182	·9462643
90	·8809176	·8801776

It will be observed that the fraction $p=\frac{1-\frac{1}{2}m}{1+\frac{1}{2}m}$ approximates to $1-m$ as m becomes less; for, upon developing it into a series, $p=1-m+\frac{1}{2}m^2-\frac{1}{4}m^3+\frac{1}{8}m^4$ . . . . And taking m infinitely small, the terms after the first two may be neglected.

* m at the precise age 20 is nearly ·00765. The increase in this mortality from the age 20 to $20\frac{1}{2}$, the middle of the year of age 20 to 21, is obtained by adding $\frac{1}{2}\lambda r$, as above given, to $\lambda m_{19\frac{1}{2}}$, that is, to the log. of $(m_{19\frac{1}{2}}+m_{20\frac{1}{2}})\frac{1}{2}$; $\therefore m_{20}=\cdot 0077072$

$$\begin{array}{rl} \lambda m_{19\frac{1}{2}} & \bar{3}\cdot 8835130 \\ \frac{1}{2}\lambda r & 0\cdot 0033864 \\ \hline \lambda m_{20} & \bar{3}\cdot 8868994 \end{array}$$

The values of $m_0, m_1 \ldots . m_5$ may be obtained by the method already described; but it rarely happens that the population living at each year of age is accurately enumerated at the census; and, besides inaccuracies of statement, the numbers living at each of the early years of age fluctuate considerably, so that the numbers of children living of each year of age in 1851 do not represent the average numbers living of those ages in the five years 1849 to 1853 for instance.

The following method is less exceptionable. It may be assumed *for this purpose,* (1) that the births registered in the year 1848 represent the births in that year; (2) that the births are equally distributed over the years in which they occur; and, consequently, (3) that the *mean date* of all *the births* in the two years 1848, 1849, was immediately before January 1, 1849. The *half* of the births in those two years will consequently represent pretty accurately the number of births out of which the deaths of children *under one year* of age happened in the year 1849; and the deaths and survivors can be followed by this method year by year, as is evident in the annexed scheme:—

Age.	
0	$\frac{1}{2}$ (births 1848, 1849)=mean annual births of which the mean date is January 1, 1849.
	minus deaths under age 1 in 1849
1	=surviving on January 1, 1850.
	minus deaths age (1 to 2) in 1850
2	=surviving on January 1, 1851.
	minus deaths age (2 to 3) in 1851
3	=surviving on January 1, 1852.
	minus deaths age (3 to 4) in 1852
4	=surviving on January 1, 1853.
	minus deaths age (4 to 5) in 1853
5	=surviving on January 1, 1854.

By commencing with the mean number of births in the years 1849, 1850, and deducting the deaths, a similar series may be obtained; and thus a succession of similar series may be deduced, the mean of which will supply the ordinary series $l_0, l_1, l_2, l_3, l_4, l_5$ of a Life Table.

These series are liable to various disturbances. If all the births are not registered, the *rate* of mortality is overstated; if all the deaths are not registered, or if the children are carried off as emigrants, the decrements of life are understated. The annual number of births fluctuates, and now increases, in England; they are in excess also in the early months of the year. Several of the disturbances are slight, and some of them are in opposite directions.

The results can also be, and have been, checked by the results of the other method. The values of m_7 and m_{12} are deduced by dividing the annual deaths at the ages 5 to 10 and 10 to 15 by the mean population at those ages. The interpolation of the series λp_x from λp_3 to λp_{20} succeeds, taking λp_3, λp_7, λp_{12}, and λp_{20}, as the fixed points of the series, and λp_{12} being adjusted to allow for the turn of the curve.

The Tables A, B, and C supply the data from which the Life Table of Healthy English Districts was deduced. One or two arithmetical examples of the application of the method adopted in the earlier ages are also supplied.

III. INTERPOLATION.

We have therefore determined the values of λp_x at certain ages. The values of λp_x at the intervening ages may be determined by changing the value of r and making z successively 1, 2 . . . . 10 in the formula (p. 131). They may also be interpolated for every year of age by the method of finite differences; and, upon the whole, this method is preferable to any other. The logarithms of p_x are required, and to them it will be convenient to apply the interpolation directly. Any number of differences beyond four becomes cumbersome, and it will be therefore sufficient to give the general formula, which can be employed in deriving the first of either four or three orders of differences.

Investigation of Formulæ—Intervals equal.

Let any numbers of a series be so related that u_n, the nth from the first, u_0, is determined by the equation (1)—

$$u_n = u_0 \underset{-}{+} \frac{n}{1}\delta^1 \underset{+}{+} \frac{n(n-1)}{1.2}\delta^2 \underset{-}{+} \frac{n(n-1)(n-2)}{1.2.3}\delta^3 \underset{+}{+} \frac{n(n-1)(n-2)(n-3)}{1.2.3.4}\delta^4. \quad . \quad (1)$$

δ^1, δ^2, δ^3, and δ^4,* the first differences of the four orders, are unknown; they can all be determined from any five values of u_n. Now, let n be successively $1x$, $2x$, $3x$, $4x$; then the coefficients of u_0, u_{1x}, u_{2x}, u_{3x}, u_{4x}, can be found to give the values of δ^1, δ^2, δ^3, and δ^4, in four equations. But when x is ten or more, the coefficients become large and the numerical calculation laborious; it is, therefore, well to obtain the numerical values of δ^4, δ^3, δ^2, δ^1, in succession. Thus, if the series is ascending or descending, the following are convenient forms—the upper rows of signs are used in the *ascending*, the lower rows in the *descending*, series:—

* It will be borne in mind that these imply first differences, or $\delta^1 u_0$, $\delta^2 u_0$, $\delta^3 u_0$, $\delta^4 u_0$.

$$\delta^4 = \frac{\overset{+}{\underset{+}{}} u_{4x} \overset{-}{\underset{-}{}} 4u_{3x} \overset{+}{\underset{+}{}} 6u_{2x} \overset{-}{\underset{-}{}} 4u_x \overset{+}{\underset{+}{}} u_0}{x^4} \quad \ldots \ldots \ldots \ldots \quad (2)$$

$$\delta^3 = \frac{\overset{+}{\underset{-}{}} u_{3x} \overset{-}{\underset{+}{}} 3u_{2x} \overset{+}{\underset{-}{}} 3u_x \overset{-}{\underset{+}{}} u_0}{x^3} \underset{+}{-} \tfrac{3}{2}(x-1)\delta^4. \quad \ldots \ldots \ldots \ldots \quad (3)$$

$$\delta^2 = \frac{\overset{+}{\underset{+}{}} u_{2x} \overset{-}{\underset{-}{}} 2u_x \overset{+}{\underset{+}{}} u_0}{x^2} \underset{+}{-} (x-1)\delta^3 \underset{-}{-} \frac{(7x^2-18x+11)}{12}\delta^4. \quad \ldots \ldots \quad (4)$$

$$\delta^1 = \frac{\overset{+}{\underset{-}{}} u_x \overset{-}{\underset{+}{}} u_0}{x} \underset{+}{-} \frac{x-1}{2}\delta^2 \underset{-}{-} \frac{(x^2-3x+2)}{6}\delta^3 \underset{+}{-} \frac{(x^3-6x^2+11x-6)}{24}\delta^4. \quad . \quad (5)$$

It is necessary to be careful in deducing the successive values of δ from the values preceding; and, before commencing their use, their accuracy should be tested by inserting them in the checking equation—

$$u_{4x} = u_0 \underset{-}{+} \frac{4x}{1}\delta^1 \underset{+}{+} \frac{4x(4x-1)}{1.2}\delta^2 \underset{-}{+} \frac{4x(4x-1)(4x-2)}{1.2.3}\delta^3$$
$$\underset{+}{+} \frac{4x(4x-1)(4x-2)(4x-3)}{1.2.3.4}\delta^4. \quad \ldots \ldots \quad (6)$$

x may be any number. If only four terms are given, δ^3 is assumed to be constant; and δ^4 being 0, all the terms into which it enters disappear. The above formulæ, if this is borne in mind, are applicable when δ^4, δ^3, or δ^2, are assumed to be constant, and serve, therefore, to supply the differences, when there are one, two, three, or four orders, by the most expeditious method.

In constructing the Life Table, x was made 10 from the age of 20, and on inserting the numbers, the equations (2, 3, 4, 5, 6) became

$$\delta^4 = \frac{\overset{+}{\underset{+}{}} u_{40} \overset{-}{\underset{-}{}} 4u_{30} \overset{+}{\underset{+}{}} 6u_{20} \overset{-}{\underset{-}{}} 4u_{10} \overset{+}{\underset{+}{}} u_0}{10{,}000} \quad \ldots \ldots \ldots \quad (7)$$

$$\delta^3 = \frac{\overset{+}{\underset{-}{}} u_{30} \overset{-}{\underset{+}{}} 3u_{20} \overset{+}{\underset{-}{}} 3u_{10} \overset{-}{\underset{+}{}} u_0}{1{,}000} \underset{+}{-} 13\tfrac{1}{2}\delta^4. \quad \ldots \ldots \ldots \quad (8)$$

$$\delta^2 = \frac{\overset{+}{\underset{+}{}} u_{20} \overset{-}{\underset{-}{}} 2u_{10} \overset{+}{\underset{+}{}} u_0}{100} \underset{+}{-} 9\delta^3 \underset{-}{-} 44\tfrac{1}{4}\delta^4. \quad \ldots \ldots \ldots \quad (9)$$

$$\delta^1 = \frac{\overset{+}{\underset{-}{}} u_{10} \overset{-}{\underset{+}{}} u_0}{10} \underset{+}{-} 4\tfrac{1}{2}\delta^2 \underset{-}{-} 12\delta^3 \underset{+}{-} 21\delta^4. \quad \ldots \ldots \ldots \quad (10)$$

The checking equation is—

$$u_{40} = \begin{smallmatrix}+\\+\end{smallmatrix} u_0 \begin{smallmatrix}+\\-\end{smallmatrix} 40\delta^1 \begin{smallmatrix}+\\+\end{smallmatrix} 780\delta^2 \begin{smallmatrix}+\\-\end{smallmatrix} 9880\delta^3 \begin{smallmatrix}+\\+\end{smallmatrix} 91390\delta^4. \quad . \quad . \quad (11)$$

If three orders of differences are used, the checking equation is—

$$u_{30} = \begin{smallmatrix}+\\+\end{smallmatrix} u_0 \begin{smallmatrix}+\\-\end{smallmatrix} 30\delta^1 \begin{smallmatrix}+\\+\end{smallmatrix} 435\delta^2 \begin{smallmatrix}+\\-\end{smallmatrix} 4060\delta^3. \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

After adding or subtracting any constant to or from a series of numbers, the differences remain the same; and if consecutive terms are multiplied or divided by the same factor, the differences are multiplied or divided by that factor. Thus $(b+a)-(c+a)=b-c$, and $ab-ac=a(b-c)$. Advantage is taken of these properties to reduce any one of the terms in the equations to *zero*.

Thus, let the logarithms to be interpolated be the following—values of p_{20}, p_{30}, p_{40}, and p_{50}, taken from the column headed *males*, Table B, then they may, among other ways, be interpolated as follows:—

As $\bar{1}\cdot9969724$ is the contracted expression of $(\cdot9969724-1)$, we have—

Age.			
20	$\bar{1}\cdot9969724=-\cdot0030276$	(1) Multiplying each term by 10,000,000—that is, striking out the decimal point and the two adjoining ciphers—and (2) then subtracting from each 30,276, the values of $u_x=\lambda p_x$ to be operated on become	$u^0=-00000$
30	$\bar{1}\cdot9964260=-\cdot0035740$		$u_{10}=-\ 5464$
40	$\bar{1}\cdot9959051=-\cdot0040949$		$u_{20}=-10673$
50	$\bar{1}\cdot9943048=-\cdot0056952$		$u_{30}=-26676$

By inserting these values with their negative signs in the equations, and taking the upper signs, the three differences are found—that is,

$$\delta^3=-11\cdot049; \quad \delta^2=101\cdot991; \quad \text{and } \delta^1=-872\cdot7715.$$

The differences are now divided by 10,000,000—that is, ciphers are added to their left-hand side, so that the above decimal point may be moved seven places in that direction—and the operation may be thus commenced. By adding the differences successively to each other and to $\lambda p_{20}=\bar{1}\cdot9969724$, the successive values are found of λp_{21}, λp_{22}, λp_{23} . . . . λp_{50}, up to and including λp_{58}, for males, where the series joins naturally the subsequent series, commencing at λp_{59}.

δ^3.	δ^2.	δ^1.	λp_x.
—·000,0011,0490	·000,0101,9910	—·000,0872,7715	$\bar{1}$·996,9724,0000
(constant)	·000,0090,9420	—·000,0770,7805	$\bar{1}$·996,8951,2285
		—·000,0679,8385	$\bar{1}$·996,8180,4480
			$\bar{1}$·996,7500,6095

In the actual operation the δ^3 is *subtracted* from δ^2, δ^2 from δ^1, and δ^1 from λp_x; it is, therefore, convenient to substitute for their present values the complements of δ^3 and δ^1, as thus all the series become additive.

As $\lambda l_{20} + \lambda p_{20} = \lambda l_{21}$, and $\lambda l_{21} + \lambda p_{21} = \lambda l_{22}$, and, generally, $\lambda l_x + \lambda p_x = \lambda l_{x+1}$, it is evident that the λp_x is the *first difference* of the series λl_x; and the whole series, λl_x, from λl_{20} to λl_{58}, may be formed as in the subjoined example, where δ^3 becomes δ^4, δ^2 becomes δ^3, and so on.

Healthy Districts—Males.

δ^4 (constant)

9·999,9988,9510

Age.	δ^3.	δ^2.	$\delta^1 = \lambda p_x$.	$u_x = \lambda l_x$.
20	0·000,0101,9910	9·999,9127,2285	9·996,9724,0000	4·584,1951,2769
21	0·000,0090,9420	9·999,9229,2195	9·996,8851,2285	4·581,1675,2769
22	0·000,0079,8930	9·999,9320,1615	9·996,8080,4480	4·578,0526,5054
23			9·996,7400,6095	4·574,8606,9534
24				4·571,6007,5629

Note.—The four last figures in the decimal portion of the series λp_x and in λl_x may, in practice, be omitted.

The corresponding values of λp_x, in the column headed "Females," Table B, are interpolated in the same way; and the λp_{60}, λp_{70}, λp_{80}, and λp_{90}, are interpolated by the same methods, the series being continued backwards to λp_{57} and forwards to λp_{105}; the actual observations of age after the age of 90 furnishing results less liable than those thus obtained, which bring a generation of 100,000 to their last end in 107 years. The successive values of λp_x, in the period from the age of 3 to the age of 19 inclusive, are derived from λp_3, λp_7, λp_{12}, and λp_{20}, which represent u_0, u_4, u_9, and u_{17}. As the terms of the series are here at unequal distances, the first differences cannot be derived from the preceding formulæ. The δ can in this and similar cases be derived from the proper equations by substituting figures for letters. But three literal equations supply formulæ for finding the three first differences from any four terms of series of the kind which have been dis-

cussed; u_0, which has a troublesome coefficient, can always be reduced to *zero*, and is, therefore, omitted. The first given term being u_0, let the second, u_x, be the xth from u_0, and u_y be the yth, u_z the zth from u_0; here $x<y<z$; then the following equations give the differences*:—

$$=\frac{6\left\{(y-x)\frac{u_z}{z}-(z-x)\frac{u_y}{y}+(z-y)\frac{u_x}{x}\right\}}{(y-x)\{(z-1)(z-2)-(y-1)(y-2)\}-(z-y)\{(y-1)(y-2)-(x-1)(x-2)\}} \quad (13)$$

$$=\frac{2}{y-x}\left\{\frac{u_y}{y}-\frac{u_x}{x}-\{(y-1)(y-2)-(x-1)(x-2)\}\frac{\delta^3}{6}\right\} \quad (14)$$

$$=\frac{u_x}{x}-(x-1)\frac{\delta^2}{2}-(x-1)(x-2)\frac{\delta^3}{6} \quad (15)$$

By making $y=2x$, and $z=3x$, these equations assume the same forms as equations (3), (4), (5), with the term δ^4 struck out.

Putting $x=4$, $y=9$, and $z=17$, the three preceding equations become those which were actually used in constructing the series p_3 to p_{19}: u_0 is reduced to zero and is not used.

$$\delta^3=\frac{45u_{17}-221u_9+306u_4}{13260} \quad (16)$$

$$\delta^2=\frac{4u_9-9u_4-300\delta^3}{90} \quad (17)$$

$$\delta^1=\frac{u_4-6\delta^2-4\delta^3}{4} \quad (18)$$

Checking equation.

$$u_{17}=u_0+17\delta^1+136\delta^2+680\delta^3 \quad (19)$$

* *A useful Table in applying the above formulæ.*

x.	$(x-1)(x-2)$.	x.	$(x-1)(x-2)$.	x.	$(x-1)(x-2)$.
20	342	34	1056	47	2070
21	380	35	1122	48	2162
22	420	36	1190	49	2256
23	462	37	1260	50	2352
24	506	38	1332	51	2450
25	552	39	1406	52	2550
26	600	40	1482	53	2652
27	650	41	1560	54	2756
28	702	42	1640	55	2862
29	756	43	1722	56	2970
30	812	44	1806	57	3080
31	870	45	1892	58	3192
32	930	46	1980	59	3306
33	992				

Table of first differences in the Life Table of Healthy Districts of England.

Age x.	λl_x.	$\lambda p_x = \delta^1$.	δ^2.	δ^3.	δ^4.
			MALES.		
3	4·631,5849,0000	9·993,2422,0000	0·001,2416,1260,934	9·999,8012,4393,666	0·000,0141,9648,567
20	4·584,1951,2769	9·996,9724,0000	9·999,9127,2285	0·000,0101,9910	9·999,9988,9510
59	4·403,7768,0454	9·990,6137,0980	9·998,9756,9020	9·999,9704,0800	9·999,9843,4520
60	4·394,3905,1434	9·989,5894,0000	9·998,9460,9820	9·999,9547,5320	9·999,9843,4520
	Note.—The last series p_x was carried backwards from λp_{60} to λp_{59}.				
			FEMALES.		
3	4·623,2586,0000	9·993,2928,0000	0·001,2164,1598,794	9·999,7874,2556,561	0·000,0170,4566,365
20	4·570,6868.3846	9·996,6528,0000	9·999,9241,5455	0·000,0060,2930	9·999,9994,2530
57	4·405,2189,6826	9·992,9332,3725	9·999,0836,2675	0·000,0123,2100	9·999,9838,1950
60	4·381,2818,8126	9·990,2049,0000	9·999,0720,4825	9·999,9637,7950	9·999,9838,1950
	Note.—The last series p_x was carried backwards from λp_{60} to λp_{57}.				

(20)

A series of the form $v^x l_x + v^{x+1} l_{x+1} + v^{x+2} l_{x+2}$ is required in rendering the Life Table applicable to the solution of questions in annuities and life insurance.

The logarithms of the series are obtained by making the first term of the new series $\lambda(v^x l_x)$, and the first term of the first order of differences $\lambda(vp_x) = \lambda v + \lambda p_x = \delta^1$; the δ^2, δ^3, and δ^4 of the original series remaining unchanged. Taking the interest of money at 3 per cent., $v = \frac{1}{1 \cdot 03}$; and $\lambda v = \bar{1} \cdot 9871627,753$.

The derivation of the new series from this value of λv, and from the above Table (males), is shown in the annexed example. Any value of v^x may be introduced in the same way.

$$\delta^4 = 9 \cdot 9999988,951$$

Age.	δ^3.	δ^2.	$\lambda(vp_x) = \delta^1$.	$u_0 = \lambda(l_x v^x)$.
20	0·0000101,991	9·9999127,2285	9·9841351,7530	4·3274506
	·0000090,942	·9999229,2195	·9840478,9815	·3115858
		·9999320,1615	·9839708,2010	·2956337
			·9839028,3625	·2796045
				·2635074

In describing the first English Life Table, I ventured to express the belief that the chances of life may ultimately be calculated by Mr. Babbage's machine.* Mr. Babbage's conception has been realized in the original and ingeniously-constructed machine of the Messrs. Scheutz, which was favourably reported upon by a com-

* Letter to the Registrar-General, in Appendix (p. 352) to his *Fifth Annual Report*, year 1843.

mittee of the Royal Society. The first differences to be inserted in the machine can be immediately deduced from those given above; and we may hope ere long to see the logarithms of Life Tables, for single and for joint lives, printed from types cast in moulds stamped by the machine now in the course of construction by the Messrs. Donkin, for Her Majesty's Government, at the instance of the Registrar-General.

(*To be continued.*)

On the Clearing of the London Bankers. By Sir John W. Lubbock, Bart., F.R.S., *formerly Treasurer of the Royal Society, and Vice-Chancellor of the University of London.**

Atque equidem, extremo ni jam sub fine laborum
Vela traham, et terris festinem advertere proram;
Forsitan et pingues hortos quæ cura colendi
Ornaret, canerem, biferíque rosaria Pæsti:
Quóque modo potis gauderent intyba rivis;
Et virides apio ripæ, tortúsque per herbam
Cresceret in ventrem cucumis: nec sera comantem
Narcissum, aut flexi tacuissem vimen acanthi,
Pallentésque hederas, et amantes litora myrtos.

The operation of the clearing has the effect of enabling all the payments from one bank to another to be performed without the passing of bank-notes; and the result is, that if any bank has to receive from the clearing, say, £50,000, the account of that bank at the Bank of England is better by £50,000 at 9 o'clock the next morning; and if a bank has to pay into the clearing £50,000, the amount of that bank is worse by £50,000.

The following description of the mode of conducting the clearing is taken from Mr. Babbage's *Treatise on the Economy of Machinery and Manufactures* (second edition, page 124):—

"In London all checks paid in to bankers pass through what is technically called '*The Clearing House.*' In a large room in Lombard-street, about thirty clerks from the several London bankers take their stations, in alphabetical order, at desks placed round the room; each having a small open box by his side, and the name of the firm to which he belongs in large characters on the wall above his head. From time to time, other clerks from every house enter the room, and, passing along, drop into the box the checks due by that firm to the house from which this distributor is sent. The clerk at the table enters the amount of the several checks in a book previously prepared, under the name of each bank to which it is due.

* The object of this paper is one in the attainment of which the readers of this journal are probably little interested; but the paper itself affords so remarkable an instance of the application of the doctrine of probabilities to the ordinary affairs of life, that we have not hesitated to insert it.—Ed. *A. M.*

"Four o'clock in the afternoon is the latest hour to which the boxes are open to receive checks; and at a few minutes before that time some signs of increased activity begin to appear in this previously quiet and business-like scene. Numerous clerks then arrive, anxious to distribute, up to the latest possible moment, the checks which have been paid into the houses of their employers.

"At four o'clock all the boxes are removed, and each clerk adds up the amount of the checks put into his box and due from his own to other houses. He also receives another book from his own house, containing the amounts of the checks which their distributing clerk has put into the box of every other banker. Having compared these, he writes out the balances due to or from his own house, opposite the names of each of the other banks; and having verified this by a comparison with the similar list made by the clerks of those houses, he sends to his own bank the general balance resulting from this sheet, the amount of which, if it is due from that to other houses, is sent back in bank-notes.

"At five o'clock the *Inspector* takes his seat, when each clerk, who has upon the result of all the transactions a balance to pay to various other houses, pays it to the inspector, who gives a ticket for the amount. The clerks of those houses to whom money is due, then receive the several sums from the inspector, who takes from them a ticket for the amount. Thus the whole of these payments are made by a double system of balance, a very small quantity of bank-notes passing from hand to hand, and scarcely any coin.

"It is difficult to form a satisfactory estimate of the sums which daily pass through this operation: they fluctuate from two millions to, perhaps, fifteen. About two millions and a half may possibly be considered as something like an average, requiring for its adjustment perhaps £200,000 in bank-notes and £20 in specie. By an agreement between the different bankers, all checks which have the name of any firm written across them must pass through the Clearing House; consequently, if any such check should be lost, the firm on which it is drawn would refuse to pay it at the counter; a circumstance which adds greatly to the convenience of commerce.

"The advantage of this system is such, that two meetings a day have been recently established—one at twelve, the other at three o'clock; but the payment of balances takes place once only, at five o'clock.

"If all the private banks kept accounts with the Bank of England, it would be possible to carry on the whole of these transactions with a still smaller quantity of circulating medium."

Since this was written, the system of paying in bank-notes has been completely done away with, and the amounts are settled at the Bank of England, by an entry to the debit or credit of each banker, as the case may be. There are at present 33 clearing bankers.

The details given by Mr. Babbage are not quite accurate. It is the duty of the inspector to be always present during the hours of business, to maintain order.

To calculate the probability of the concurrence of any number of independent events, the probabilities of each separately being given, is the most elementary question in that science which is called the

Theory of Probability, and is treated of in all books upon that subject.—*See* Bethune and Lubbock, *on Probability*, p. 9.* De Morgan's *Essay on Probabilities*, p. 30.

It is only necessary, in order to understand what follows, to bear in mind—

1. That the probability of the concurrence of any number of independent events is equal to the product of the probabilities of each considered separately.
2. When the number of trials is infinite, the number of times an event happens is to the number of times another independent event happens, in the ratio of their simple probabilities.

If any event in the clearing happens n times in m trials or days, I shall consider the probability of the event to be properly represented by $\frac{n}{m}$, so if I find that my house has to receive from the clearing 200 times in 400 clearings, I shall estimate the probability of that event happening on any day in future to be $\frac{200}{400}$ or $\frac{1}{2}$, and so in any other case. Strictly speaking, this probability being unknown, and only to be derived from experience, belongs to what De Morgan calls inverse probabilities; such are the probabilities which occur in questions relating to insurances on lives.

I do not pretend in these pages to give a treatise on probability, but only to show the application of the most elementary principles of that science to questions which arise in the clearing.

If a be the probability of any bank having to receive on a given day, and b the probability of that bank having to pay in, as one of these events must happen, $a+b=1$.

This is the only principle which can be predicated with certainty without recourse to observation, but it is upon the numerical values of these quantities a and b that this inquiry is based.

I think it will be admitted that $a=b$, and therefore $a=\frac{1}{2}$; at any rate, I find this to be about the case at my own bank, and I have no doubt the same obtains with all other banks, but any banker can verify it in his own case at once by reference to his pass-book.

The probability of any number of bankers having to receive on a given day is, therefore, nearly the same as the probability if thirty

* This treatise was published by the Society for the Diffusion of Useful Knowledge. When this work was written, no work on "Probability" had appeared since the time of Simpson and De Moivre. If anyone shall pretend that this work was written by De Morgan, I can produce the letter of my lamented friend with which he furnished our manuscript to Mr. Coates.

shillings are tossed that the same number of them comes heads. But as two events are equally impossible, namely, that all thirty banks either bring in or take out, these events must be excluded and the other numbers must be increased in the ratio of 1 to $1-(\frac{1}{2})^{29}$.

The next principle will, I think, also be admitted; and, at any rate, I find it in my own clearing that the probability of having to receive any given sum decreases with enormous rapidity; so that, for example, if the chance, in case of having to pay in or receive any sum between nothing and £50,000 is $\frac{1}{2}$, the chance of having to pay in or take out any sum between £50,000 and £100,000 is probably less than $\frac{1}{4}$, and so on in a rapidly decreasing ratio.

It is probable, further, that the probability decreases on each side zero in the same ratio, and that for a given bank the chance of paying in £5,000, or any given sum, is precisely the same as that of taking it out.

The next question which arises, and which is of great importance, is to obtain the accurate value of the probability of a banker paying in or taking out any particular sum. Any banker can obtain this without difficulty from his own pass-book with the Bank of England, and I think it may fairly be taken for granted that the probability for any sum within certain limits varies directly with the amount of the deposits. So that if the probability, that if a bank whose deposits are half a million takes out, it takes out a sum between 0 and £10,000, be $\frac{1}{2}$, the probability that if a bank whose deposits are a million takes out, it takes out a sum between 0 and £20,000, is also $\frac{1}{2}$, and so on. Thus this quantity, which may be called p, will vary for every bank, and as I know nothing of the phenomena which the clearing presents of any bank but my own, I am thrown upon the necessity of making the best conjectures I can from my own pass-book with the Bank of England.

The deposits of the joint-stock banks are not quite 50 millions; I conjecture the deposits of the private clearing banks to be about 50 millions. In order, therefore, to simplify the calculation, I will suppose the thirty banks to have each deposits to the amount of three millions each. I conjecture the value of p to be about $\frac{1}{2}$ for such a bank, and for any sum not exceeding £50,000.

The probability that any event will happen amongst these thirty banks is given very nearly by the corresponding term in the development of $(\frac{1}{2}+\frac{1}{2})^{30}$, so that the chance of

29 banks taking out is $30(\frac{1}{2})^{30}$ very nearly.
28 " " $\frac{30}{1}\frac{29}{2}(\frac{1}{2})^{30}$ "

Upon the supposition, therefore, that the business of each bank is so conducted that it is an even chance on any day whether any given bank has to pay or to receive in the clearing, the following table shows the chance of any given event happening on any given day, or the number of times that the event will happen in 1,000,000,000,000 days, on the average of a great many years. The upper line is given to complete the table; but as one bank at least must bring in, it is necessary to exclude the two events of no bank bringing in and no bank taking out, and the numbers in the table should be increased in the ratio of 1 to $1-(\frac{1}{2})^{29}$, that is, they should be multiplied by 1·000000002, by which, of course, they would not be sensibly altered.

No. of Bankers.	No. of Days, or Probability.		No. of Bankers.
	·0000000009313	0·9691000	
29	·0000000279397	2·4462213	29
28	·0000004051250	3·6075893	28
27	·0000037811700	4·5776260	27
26	·0000255229000	5·4069298	26
25	·0001327190000	6·1229331	25
24	·0005529960000	6·7427218	24
23	·0018959800000	7·2778350	23
22	·0054509600000	7·7364728	22
21	·0133246000000	8·1246530	21
20	·0279816000000	8·4468723	20
19	·0508756000000	8·7065096	19
18	·0805530000000	8·9060820	18
17	·1115350000000	9·0474111	17
16	·1354350000000	9·1317320	16
15	·1444640000000	9·1597609	15

The table reads thus—the extreme columns to the right and left contain the number of banks. The chance of fifteen banks having to pay in the clearing, on any given day, is ·14446. The third column gives the logarithms of the numbers in the second column.

The following table shows the number of days, in the year of 313 days, that any given event will happen, on an average :—

No. of Bankers.	No. of Days.	No. of Bankers.	No. of Days.	No. of Bankers.	No. of Days.
29		19	15·93	9	4·17
28		18	25·21	8	1·71
27		17	34·91	7	·59
26	·01	16	42·39	6	·17
25	·04	15	45·21	5	·04
24	·17	14	42·39	4	·01
23	·59	13	34·91	3	
22	1·71	12	25·21	2	
21	4·17	11	15·93	1	
20	8·76	10	8·76		

The table reads thus—in a year of 313 days, fifteen banks on an average have to pay on forty-five days, on the average of a great number of years. Of course, in order to ascertain what will take place in ten years, or 3,130 days, it is only necessary to move the decimal point one place to the right. So that in ten years seventeen banks on an average have to pay on 349 days.

I have already said, that I have little doubt that the value of a does not differ sensibly from $\frac{1}{2}$, and, if so, the values given in the table above cannot differ much from the truth. The records of the clearing would furnish the means of determining the exact value of a if there were any advantage in so doing.

The value of p is subject to greater difficulty; it may be different for different banks, and so may the value of a; but the conditions of the problem are such as to render it evident that if a is less than $\frac{1}{2}$ for one bank it is probably greater than $\frac{1}{2}$ for another, and unless its average value for all the thirty banks differs widely from $\frac{1}{2}$, my results will not be affected. The same remark applies also to my value of p; it may, and no doubt it does, differ very much for every bank, varying with their mode of conducting the business, and with the magnitude of their deposits. But it is the average value with which we are mainly concerned, and unless its value differs considerably on the average of all the banks from the value which I shall assign to it, my results will still hold good.

One thing I hold to be certain with regard to p, which is, that its value decreases rapidly; so that, if its value for any given bank and for any given b pounds is p, the value for the b following pounds is far less than p.

The greatest difficulty we have to contend with in determining the value of p, arises from the possibility of its varying with the state of the times, that is, with the state of the money-market; and it is, probably, some unknown function of the rate of discount. I do not think it can vary with the rate of discount; it may, perhaps, be higher in times of panic than at others, but I do not think so. It may, of course, be very much higher for days when the dividends are being paid, and when the clearing is affected by any large operation. It would be difficult to determine the value of p for any sum within small limits, and for any given bank, say, for £1,000, but, of course, the wider the limits the less danger of error.

I think, for a bank holding £3,000,000 deposits, and for a sum not exceeding £50,000, the value of p does not differ materially from $\frac{1}{2}$. Of course, if I knew the returns of the clearing-house I could tell its value, and for any given bank I could easily ascertain

its value from their bank pass-book, but with only my own experience to guide me in this difficult determination, I give this value with much diffidence. I wish the reader to bear in mind, that while I am groping my way in the dark, a few minutes' examination of the clearing-house returns, properly tabulated, would give a clear insight into this and all similar questions.

Upon this hypothesis, the chances of the Bank having to pay more than £50,000 is, of course, also $\frac{1}{2}$, and the chance of fifteen banks taking out each £50,000 is the probability given in the following table, multiplied by $(\frac{1}{2})^{15}$. So that the probabilities given in the table are enormously reduced, and the probability of fifteen banks having to pay, which is

·14446

an event which would happen 14,446 times in 100,000 trials, upon the supposition that, if all the fifteen banks have to pay at all, they have to pay at least £50,000, becomes very much reduced.

Mr. Farley has kindly examined the pass-book of my bank with the Bank of England, and, from the results Mr. Farley obtained, I conjecture the following numbers to apply to a bank whose deposits are about £3,000,000:—

	Probability.	Logarithm of Probability.	Probability.	Logarithm of Probability.	
£					£
250,000	·01383	8·1408155	·01383	8·1408155	250,000
200,000	·01330	8·1237822	·02713	8·4334498	200,000
150,000	·03298	8·5182338	·06011	8·7789467	150,000
100,000	·07074	8·8496938	·13085	9·1167737	100,000
50,000	·14841	9·1714464	·27926	9·4460037	50,000
0,000	·22074	9·3438903	..	..	0,000
0,000	·22074	9·3438903	..	..	0,000
50,000	·14841	9·1714464	·27926	9·4460037	50,000
100,000	·07074	8·8496938	·13085	9·1167737	100,000
150,000	·03298	8·5182338	·06011	8·7789467	150,000
200,000	·01330	8·1237822	·02713	8·4334498	200,000
250,000	·01383	8·1408155	·01383	8·1408155	250,000

The first column gives the sum in thousands.

The second column gives the number of times the event happens in 100,000 trials.

The third column gives the logarithm of the number in the second column.

The table reads thus:—A bank with £3,000,000 deposits has to receive between £50,000 and £100,000 14,841 times in 100,000 trials or clearings. The chance of a bank holding £3,000,000

deposits having to pay above £50,000 is ·27926, the logarithm of which is 9·4460037.

I apprehend this law or curvature of p will be similar for all banks, or nearly so; but, of course, for banks whose operations are greater, their deposits being also greater, the figures in the first column will be proportionably greater.

The numbers in the third column are nearly in a geometrical ratio, of which the common ratio is $\frac{1}{2}$, confirming the opinion given in p. 146, of the rapidity with which these numbers decrease on each side zero.

It is evident, that the chances of large sums being taken out of the clearing diminish, for two reasons: the one is, the great improbability of the number of bankers who have to receive greatly exceeding the number of those who have to pay; and secondly, that for each bank the chance of having a moderate sum to pay greatly exceeds that of having a large sum to pay.

Supposing the values of the probability to be approximately correct, it is easy to find the chance of the clearing amounting to any given sum on a given day.

At present the London bankers are compelled to keep very large sums of money unemployed, in order to provide for the possible results of each day's operations. It is, of course, impossible to ascertain the amount of notes kept in the tills of the several banks, but the returns presented to Parliament give, for each week, the total amounts of the balances kept by the bankers with the Bank of England, up to the end of 1857.

A large amount might, therefore, safely be employed by the bankers collectively, and a considerable profit obtained, if the clearing could be worked out of a common fund, so as to assimilate the position of the banks to what it would be if they were all united in one establishment.

In 1839, the actual transfer of money, or *difference of the sides*, was not, upon the average, more than £20,000, and now, probably, £500,000 would be sufficient to provide for this payment. Although, however, the average difference is so small, the uncertainty is so great, that the London bankers are compelled to keep at the Bank of England a balance which varies between £2,500,000 and £4,000,000, and, probably, never falls below the smaller amount.

It is somewhat remarkable, that during the panic at the end of 1857 the balances of the London bankers exceeded £6,000,000.

It is probable also, that independent of the actual profit, the

collateral advantages of a combination of the bankers, and the working the payments in the clearing out of a joint account, instead of thirty-three separate accounts at the Bank of England, would be by no means inconsiderable. Although the London bankers enjoy the confidence of the public, still they would, if united, be able more effectually to provide against any panic, and to prevent the recurrence of a state of events such as that which in November, 1857, endangered the whole of our monetary system.

As my career is drawing to a close, or, in the words of the poet, as my bark is nearing the shore, I must leave the details of such a plan to be worked out by younger heads.

On some Considerations suggested by the Annual Reports of the Registrar-General, being an Inquiry into the Question as to how far the Inordinate Mortality in this Country, exhibited by those Reports, is controllable by Human Agency. (*Part II.*) *By* H. W. Porter, Esq., B.A., *Assistant Actuary to the Alliance Assurance Company, Fellow of the Institute of Actuaries and of the Statistical Society.*

(Concluded from page 112.)

As regards the hereditary transmission of phthisis, another investigation of the statistics of the Hospital for Consumption at Brompton showed, that where one parent only was affected with pulmonary disease, the fathers being so affected transmitted the disease to their sons in 63 out of 106 cases, being 59·4 per cent. of the whole number observed; and to their daughters in 47 cases only out of 108, being 43½ per cent.: while the mothers being phthisical transmitted the disease to their sons in 43 cases, being 40·6 per cent.; and to their daughters in 61 cases, being 56½ per cent. of the cases under observation. Judging, therefore, from these figures, it would probably appear, if a large number of cases were registered, that the power of transmission of disease by phthisical fathers to their sons, and by phthisical mothers to their daughters, is about the same.

Very similar—in fact, almost identical—results are shown by a similar comparison of the statistics of insanity.

From the consideration of such facts as these, it appears that we hold in our hands the power of checking the increase of diseases of an hereditary character by discouraging marriages under certain circumstances—the extreme inexpediency of which, in some cases, seems clearly apparent; and Life Assurance Companies, by

attending closely to such subjects, may deduce valuable rules to enable them to deal with proposals for the assurance of lives in which a possible tendency may exist to phthisical or other hereditary disease.

With respect to the mortality from fever, without going very much into the question—the length of this paper having already, I fear, exceeded its fair limits—I may mention that there is abundant evidence to show that we hold in our own hands the means of checking, to a very great extent, the loss of life from this cause. Mr. Simon, in a recent report to the Board of Health, speaking of fever, quotes an expression of the late M. Baudens, one of the physicians of the French army in the Crimea, in support of this statement—"*On pourrait le faire naître et mourir à volonté.*"

In a former paper read before this Institute, and in some communications to the *Assurance Magazine,* in reference to the efficacy of sanitary improvements as respects the health of the labouring community, I had occasion to refer to the fact of its having been shown by Dr. Southwood Smith, that the result of such improvements as had been introduced and carried out by the Metropolitan Association for the Improvement of the Dwellings of the Industrious Classes actually exceeded belief.

The improvement of the sanitary condition of the districts under observation had been attended with marked results; and it was shown that it was quite within our means to hold in check not only fever, but other contagious diseases.

Similar results were shown to have attended a like scheme originated by Lord Shaftesbury; and the working of the Common Lodging-houses Act had been equally satisfactory.* In fact, all the evidence derived from the reports on these subjects tends to show how much we hold in our own hands the key to the improvement of the health and longevity, to say nothing of the comfort and happiness, of the working classes, which form the great bulk of the entire community.

It is painful to see how very little, comparatively, has been done in this respect; and the great proportion of what has been effected is due almost entirely to the philanthropic exertions of a few individuals—what has been done by the Government having only been tardily conceded at the earnest and repeated solicitations of a few sanitary reformers, among whom Lord Shaftesbury stands pre-eminent.

Hygienic reform is now, no doubt, beginning to produce satis-

* *Assurance Magazine*, vol. iv., pp. 112, 260; vol. vi., p. 110.

factory results. In a recent report, Sir Benjamin Hall's Act is stated to be working with success. The medical inspectors, it is said, "have done quietly a great deal of good work, and it is probable have saved many lives and *prevented* much sickness."

The alarming excess in the returns of deaths from small-pox has lately given rise to much public discussion as to the cause of this increase.

Dr. Collinson and Mr. Norway, in a recent letter to the *Times*, stated that it was the conviction of a number of medical men that small-pox is likely again to appear as a scourge in this country unless efficient means are taken for its prevention; adding that the alarming increase of the disease was due to the inefficacy of vaccination as at present pursued, and attributing such inefficacy to the fact that the virus now used for vaccination is the same as that which was employed by Jenner at the time of his great discovery, and transmitted since through countless human bodies—that it has never been renewed from the cow since that period—and accordingly, that it is not now of sufficient efficacy to prevent the infection of small-pox. These gentlemen suggest that a fresh supply of virus from the cow itself should be obtained, as Jenner originally procured it. They consider, moreover, that in a number of the poor infant population lurks the poison of inherited syphilis, scrofula, and cancer, or other degenerated states of the constitution; and that, in addition to this, owing to bad drainage and ventilation, overcrowding in their dwellings, and the peculiar diet of the children of the poor, cutaneous diseases are common; and that, accordingly, the transmission of the cow-pox matter through such bodies as these must have had the effect of deteriorating the virus.

Now this seems plausible enough; but, unfortunately, as happens in all medical discussions, a large body of the profession take an opposite view, and maintain that there is no more reason why the strength of the virus should be diminished and its power to ward off infection lessened, than that the disease itself—or, in fact, any other disease—should wear itself out, or be less virulent than it used to be ages ago, by reason of its subsequent transmission through the bodies of whole generations of men. This is as far as regards the first part of our statement: with reference to the second, I will appeal to the gentlemen present whether many do not, in their own families or in those of their friends or acquaintances, know of cases where it is easy to trace disease and weakly constitution to the fact of the victim having been vaccinated from an unhealthy subject; and if this be the case, how much more

likely—nay, how certain it is—that a constant transmission of the virus through generations of the sickly infant population of the poor must tend to the effect above referred to.

We must bear in mind, when we read in the Reports of the Registrar of the increase of any disease, that it is the impoverished lower classes that contribute the bulk of the cases reported in his department; and as a considerable prejudice against vaccination exists amongst this class, and as, owing to this and other causes, the Act regulating vaccination is very much evaded, the increase of this disease among the lower orders need not surprise us. Complaint is made that the public vaccinators are not so careful as they might be, and, it is added, that the difficulty of procuring lymph is much complained of. In Paris, a regular establishment—not a Government one—has for many years been kept up, from which a constantly fresh supply of lymph is always procurable, a number of calves affected with cow-pox being maintained, from which establishment, conducted by Dr. Mangeant, of No. 8, Faubourg St. Denis, a supply of pure lymph may always be procured; and it can be forwarded to all parts of the world, in hermetically sealed tubes, at 5 francs each.

It is a curious fact that it is found necessary to transmit the virus through seven or eight calves before it is safe to use it upon the human subject, otherwise the disease would be transmitted in so severe a form that the result would be highly dangerous, if not fatal.

The number of deaths recorded by the Registrar under the head of small-pox varies considerably from year to year.

In 1851 and 1852, for example, the deaths from this cause, in England, were 6,997 and 7,320, more than double the numbers recorded in the two following years, 1853 and 1854, which were 3,151 and 2,808. In the years 1855 and 1856, the numbers were still less—viz., 2,525 and 2,277 only. In the year 1857, the deaths from small-pox appear again to be inclined to increase, being 3,936. The complete returns for 1858 and 1859 are not yet issued.

In the last weekly report as to the mortality in London for the year ending 31st December, 1859, we are told that "the mortality from small-pox increases slowly but steadily"; and we hear that the disease has been raging as an epidemic in Paris, and has been observed to have the characteristic of attacking, not the young alone, but persons of advanced age who have not taken the precaution of being re-vaccinated.

It would seem, therefore, from these figures, that this disease may be an epidemic, as many other diseases are at particular times.

It is a curious fact, and a satisfactory thing to know, that, while a serious epidemic of any kind is raging, the deaths from other epidemic diseases are always observed to be less numerous. From the fact that there does not appear to be a regular constant increase in the mortality from small-pox, but that it varies in its intensity in particular years, as other epidemics do, it would seem that it is not the lymph that is in fault, but that the disease is epidemic at certain periods, in conformity with some law of which we are at present ignorant; and that the power of vaccination, as now practised, is not sufficient to overcome the disease.

There is no lack of scientific investigation constantly being made into this subject; but the pursuit of such inquiries has, until a certain point of information is attained, a tendency to confuse the mind, and probably to shake the public confidence, to some slight extent, as to the power of vaccination to destroy the disease entirely, which, theoretically, it is supposed it should do in the course of time, if satisfactorily and completely carried out.

Vaccination is, no doubt, often imperfect in its results. At times there is a great scarcity of lymph, so that many cases arise in which the system is not properly saturated with the disease, though the vaccinator may be enabled to certify that the operation had been duly performed. I say nothing of those cases in which the manual work has been carelessly or inefficiently done, as in such cases cow-pox is not produced; but there are, I believe, great causes of complaint in this respect, and possibly some more stringent legislative enactment on the subject, or, at least, some more effectual method of seeing that the present Act is duly carried out, may be the result of public attention being called to the subject.

The necessity is, happily, now beginning to be recognised for providing special carriages for the conveyance to the hospitals of small-pox and fever patients. Too many are still conveyed in the common street cabs, by which course these diseases are, no doubt, much disseminated.

The Board of Guardians of the Greenwich Union have lately established a conveyance of their own for the above purpose; and so recently as at the December Middlesex Sessions, the Assistant-Judge, Mr. Bodkin, called the attention of the Court to the practice of removing to the hospitals, in street cabs, prisoners labouring under small-pox. The Court immediately gave orders for the discontinuance of the practice. It is believed that the disease is

propagated to an alarming extent by means of street cabs employed for the conveyance of patients.

In consequence of the recent increase in the returns of deaths from small-pox, the attention of the Government has been seriously called to the matter, and the necessity has been recognised of endeavouring to increase the efficacy of vaccination; the result of which has been the issue, by the Privy Council, of certain special instructions to the different parish authorities on the subject, embodying some minute directions as to the mode of performing the operation of vaccination; and medical inspectors under the Public Health Act have been appointed to inquire into the present state of vaccination, and to communicate the views of the Government to the proper authorities, for the safety of the public, to the end that unprotected persons may be immediately vaccinated; and generally to take stringent steps to enforce the Act, which has been heretofore so very much evaded.

The question of the necessity, or otherwise, of re-vaccination after the lapse of a certain number of years, is one that we may hope will now be set at rest; and if it be determined that the necessity exists, steps will, no doubt, be taken to render the performance of vaccination imperative after the lapse of a certain number of years.

In connection with this subject, I may refer to a fact bearing on the statistics of the results of vaccination in cattle, which has lately appeared in the newspapers. It seems, that of three Cattle Assurance Companies in Holland, one made a practice of having all the cattle they assured vaccinated, as a safeguard against pneumonia; another had vaccination performed as soon as the disease had broken out in the animals' stalls; and the last took no precaution of the kind. It is stated that the first Company in a certain period lost 6 per cent. of the assured cattle, the second 11 per cent., and the third 40 per cent.

It does not, of course, follow that the circumstances were identical in which the cattle assured by the different Companies were placed; but the numbers of the losses differ so widely, that, allowing a large margin for this reason, it is not too much to assert that the vaccination of the cattle must have had some prophylactic effect.

I am not aware that any general system of vaccinating cattle exists in this country, though it has long been the practice to vaccinate kennels of hounds, and other valuable dogs, against distemper; but I think it has been done in an amateur kind of way, and not from any positive knowledge of its efficacy.

We now come to the consideration of the mortality among the infant population.

In this class the sacrifice of life is fearful. It appears from the last published Annual Report of the Registrar-General, that, in the year 1857, no less than 57,285 male and 45,942 female children—making a total of 103,227, or about 1 in 6 of the whole number of children born alive in England, and comprising very nearly one quarter of the whole number of deaths registered in the year—died before attaining the age of a twelve-month.

I had intended entering into an investigation of the causes of this high mortality, but the subject is so extensive that I feel called upon, owing to the length of time I have already trespassed upon the attention of the members, to refrain from doing so on the present occasion. I shall only state that an inspection of the Registrar's Reports clearly shows that a large proportion of the deaths of infants is to be attributed to accidental causes quite under human control—probably one of the greatest of which is the absence of physical power in the mothers to supply the requisite nutriment to their offspring, owing, generally, to the want of proper food for themselves. Of course, the humbler classes cannot afford the expensive process of bringing up their children by hand, and so they die of debility and atrophy, the Registrar records—but of *starvation*, if we do not shrink from applying the correct term, to the reproach of the age in which we live.

I have before referred to the too early employment of children in factories as affecting the question of education; the consideration also arises as to its effect on their health and longevity.

There can be no doubt, I believe, that a great amount of disease and of undue mortality is created by the working of young children in factories before they are old and strong enough to bear the strain upon them of the long hours of labour imposed by the manufacturers, as well as by the unhealthiness engendered by the processes of manufacture—some evils arising from which, it is true, are necessarily inherent in the requirements attending on the law of supply and demand, but others, it is no less true, are remediable, if not entirely, at least partially; and it is by such means as these, amongst others, that the mortality, as recorded in the annual returns, is unduly swelled, to the discredit and opprobium of our legislative system.

With respect to the hours of labour, the Factory Act does not allow of young persons working after six o'clock p.m.; but the

reports of convictions for infringements of the Act in this respect show that the law is systematically evaded.

There is some difficulty in directly tracing the deaths of factory operatives to the local circumstances by which they are really caused, and in ascertaining what proportion of such deaths may be due to influences which might be controlled, though it is perfectly well known that the number is far from insignificant; but there is no difficulty in ascertaining the loss of life from accidents connected with the machinery used in these factories, as we shall presently see.

Many classes of disease, the inordinate mortality from which, it is submitted, may be to a great extent lessened, are fostered and encouraged, nay, more, perpetuated to a fearful extent—for the hereditary nature of phthisis is now perfectly well known—by the unhappy position in which great masses of our labouring community are placed, and numerous fatal accidents are caused, owing, in some degree, to the apathy of employers, but partly, it must be admitted, to the want of forethought and knowledge of the workmen themselves; for while we find, on the one hand, that employers are reluctant, in many cases, to incur the trouble and expense of improvements in which their own interest is not immediately apparent, with the view to the prevention of accidents, and, what is of infinitely more importance, of disease, so we find, on the other hand, an equal degree of reluctance on the part of the operatives to avail themselves of those precautionary measures within their reach, which the humanity of the employers in many cases leads them to introduce for the advantage of their workmen.

It is painful to read the accounts of the accidents that are of constant recurrence in factories; some due to neglect on the part of the masters, and others to the inherent recklessness of the men, but which more stringent regulations might check, particularly if a proper system of overlooking were adopted with a view to prevent the workmen, before it is too late, disobeying the rules devised for their advantage.

There is no lack of inspection to see that the hands are constantly duly attending to their work, but a very great want of the other sort of overlooking.

Probably, an inspector, in addition to the usual overlookers of the works, placed on each floor of a factory where machinery is in use, whose sole duty it should be to see that the men did not act in opposition to the regulations of the place—which the millowners are now content with merely having placarded on the walls—would fully answer the required end; and this can hardly be considered

to be asking too much of the rich millowners, when we consider the sad detriment to life and limb that now unhappily arises daily under the present system.

The mistaken leniency of the unpaid magistracy is considered to tend to encourage this sort of neglect.

It is true, that clauses 10 and 71 of the Factory Act of 1844 preclude "the occupier of a factory, or the father, son, or brother of the occupier of a factory, being a justice of the peace," from deciding upon cases in which his own pecuniary interests are involved; but even of this most proper restriction, the National Association of Factory Occupiers—a combination organized for the express purpose of opposing the existing law—are said to be straining every effort to obtain a repeal: it being considered, by the parties more immediately interested, to imply "an unwarrantable suspicion upon the honourable conduct of that portion of the magistracy who are engaged in manufactures."

The power of the rich millowners is immense; and it is easy to see how, by a combination among them, they may avert from themselves for a time the inconvenience of what they term "meddling legislation."

This Association, some time ago, issued a pamphlet, written by Miss Martineau, in defence of the factory occupiers, and which, while purporting to be a justification of their conduct, was, in point of fact, a tirade against the Government, on the subject of the legislative enactments devised with the sole object of checking some of the evils which are, more or less, necessarily inherent in the conduct of manufactures upon a large scale.

The pamphlet in question, moreover, accuses Mr. Charles Dickens for unfairly calling the attention of the public, through the medium of his valuable weekly series, to the large number of these accidents.

Mr. Dickens would appear to be about the last person in the world to be so attacked.

He has evidenced throughout his works—all written, I conscientiously believe, in a purely philanthropic spirit—an anxious desire to benefit the working-classes; and such attacks as these, therefore, only make it more than ever apparent how weak the cause of the millowners is; and we are reminded of the instructions to counsel, in the case of a trial in which the defendant had but a poor defence to make—"No case—bully the opposite counsel."

It should be remembered, that it is almost entirely by means of popular articles in such works as these, with which, fortunately,

the present generation is well supplied at a cheap rate, that the general body of the people come to be cognizant of and to understand these questions, which so nearly affect their well-being.

The great mass of the people, within whose means these serial writings are, do not belong to the Statistical Society or to the Institute of Actuaries; and writers of eminence who take the trouble to get up the statistics of such subjects and to put them into a popular shape, as Mr. Dickens does so well, are most highly to be commended; and that this sort of writing is of rather an invidious nature, as tending to place the author at issue with some of the *Dives* of the community, the controversy to which I have alluded very clearly shows.

With reference to the alleged unfairness on the part of this writer in dealing with the statistics of the question, it is not difficult to reconcile the discrepancies that appeared to exist between his statements and those of the factory occupiers as to the number of accidents in a given time. Without troubling you with the figures, I may mention that the differences in question appear to have arisen from Mr. Dickens quoting the gross number of accidents that happened in the period of observation, while the opposite party referred to such accidents only as were caused by the shafts alone.

A few remarks on the subject of these accidents may not, perhaps, inappropriately be introduced here. The question is so far important, that when an erroneous opinion on any subject is believed to exist, it is desirable to remove it, if possible. Now, as in the education question, to which I have alluded, certain notions have been referred to as prevailing that will not bear the test of investigation: so in this matter, I believe, a very general opinion obtains, that the accidents, so very constantly occurring in factories, are almost invariably due to the fact that occupiers object to incurring the expense of securely fencing off their machinery, in conformity with the Act of Parliament—that they fail to provide sufficient means for the purpose generally of preventing the recurrence of such accidents as may fairly be considered to be controllable—and that to secure themselves an immunity from the pecuniary penalties to which they thereby lay themselves open, the National Association of Factory Occupiers has been instituted.

Now this is not altogether the case. A great difference of opinion prevails, even among scientific men, as to the fencing off the machinery used in mills; and this being so, the occupiers must not have all the onus of deciding the matter thrown upon them,

and particularly as the Act of Parliament which regulates the question does not seem to be sufficiently stringent.

The terms of the Factory Act of 1844 require that all machinery in certain positions and under certain circumstances and conditions, shall be "securely fenced."

It seems difficult to determine in what the *secure fencing* of machinery actually consists, and on this point the millowners and the inspectors of factories have long been at issue.

The Act of Parliament, from its vagueness, is much at fault; and it will be the object of any new legislation on this subject to remedy the evil complained of in this respect.

The permanent casing of the machinery with wood or iron, insisted on in many cases by the inspectors, is considered by the millowners to be likely, in some cases, to *increase* the tendency to accidents; and this opinion is confirmed, it seems, by high engineering authority, Mr. Fairbairn having stated that "he did not see how it was possible to fence off the horizontal shafts of mills driving machinery, without incurring greater evils, and probably more danger, than at present exists from their being left entirely open."

Vertical as well as horizontal shafts should be fenced off when within seven feet of the floor, and the sources of danger from excess of fencing are said to arise from the necessity of suspending the fencing for shafts, which entails a risk of the suspenders giving way, and from the danger of the hands employed tampering with the shafts and pulleys.

Now it certainly seems strange, that, with all the mechanical skill at our command, we cannot obviate the first two of these difficulties; and the constant employment, as previously suggested, of an extra overlooker to act as inspector, would certainly prevent the last.

The Acts for the regulation of factories, as regards the safety of the operatives employed in them, were passed in 1833 and 1844, but were not enforced until the year 1853, when the great number of accidents constantly occurring was brought under the notice of the then Secretary of State, Lord Palmerston.

From the reports of the inspectors of factories for the year ending 31st October, 1859, it appears that the total number of accidents *arising from machinery* was 3,939, of which number 64 resulted in death, and 1,100 were of a serious character—a large proportion of these, viz., 545, rendering amputation of part of the body necessary.

Though these numbers may not appear large as compared with the total number of persons affected by the factory law, which has been estimated to be upwards of 500,000—and though the inspectors, who are, of course, competent judges of the matter, do not express themselves to be dissatisfied with these numbers—it is clear that a great sacrifice of life, and a terrible amount of bodily mutilation, takes place annually in factories from causes undoubtedly within human control; and the inspectors admit that "the number of serious accidents might be reduced if the excellent example set by many millowners, in fencing dangerous machinery whenever practicable, were more generally followed."

Mr. Fairbairn's opinion has been already quoted as against the safety of fencing horizontal shafts; but, on the other hand, it appears from the reports of the inspectors that in all the districts, except that of Lancashire, where the great opposition to the Factory Acts exists, a considerable amount of horizontal shafting, under 7 feet from the floor, *has been* properly cased; and that, in the west of England, "the experiment had been tried on a sufficiently large scale, and for a sufficiently long period, to prove the fallacy of the apprehensions that were expressed as to the practicability of fencing securely horizontal shafts."

That the factory operatives are far from being satisfied with the manner in which the Acts are administered, is evidenced from the circumstance, that, as recently as the end of last Session, a deputation from the operatives of Lancashire, in which district I mentioned the greatest opposition to the Acts of Parliament was to be found, had an interview with the Home Secretary, Sir G. C. Lewis.

The operatives did not seek, it was stated, any new legislation on the subject, but asked for more efficient inspection, to put a stop to the systematic violation of the law. It seems that there never was a time at which a better feeling existed between masters and men than exists at present, and that a large number even of the employers are anxious that the legal requirements of the Acts regulating factories should be strictly enforced. It was stated also, that the operatives themselves were not free from blame in conniving at violations of the law which they knew were going on. The violation was clearly on the increase, and this was said to be owing chiefly to the want of due inspection—the number of factories having increased 50 per cent. since the Government inspectors were first appointed. The number of these inspectors was originally only four, and of these one has recently retired.

It is stated that the Act under which their appointment was originally made contains no power to allow of vacancies arising in even this small number being filled up, unless such vacancies arise by death.

It seems clear that, if the existing Act is so faulty, that it can with impunity be evaded as it is at present, no more stringent method of carrying it into effect is likely to prevail; and that, therefore, some new and more satisfactory legislation on the subject is called for, to remedy, in the first place, the admitted defects, and to provide, in the second, for such additional amount of inspection as the great increase in the number of factories since the matter was last settled may seem to demand, as well as to bring under Government inspection any manufactories that are at present exempt from its operations, such as paper mills.

Sufficient, no doubt, has been said on this head to show that some, at least, of the mortality recorded in the Reports of the Registrar-General might be checked by proper legislative enactments stringently enforced.

But however much millowners and proprietors of manufactories, and of mines—for in these again the sacrifice of life and limb is fearful—may be to blame, and some amount of censure they will scarcely contrive to escape—and however cruel the deaths and permanent bodily injuries that arise from such accidents, whether preventable or not, may be—the loss of life and injury to bodily health from such causes as those to which I have referred—and I have said nothing about the injury caused by insufficient ventilation, or by the evils created by the processes of manufacture—the loss of life and injury to health, I repeat, from such causes is trifling in comparison with that arising from the neglect of sanitary measures, not in mills alone, but in all kinds of works and localities, for the prevention of the undue increase of mortality, and of the propagation of the most terrible bodily and mental diseases which are shown to be so universally prevalent; for, apart from the deaths from accidental causes, the early and laborious employment prematurely imposed upon young children injures their health, stints their growth, and renders them unfit for such labour as, when they had become adults, they should be properly able to endure.

A very great number of deaths from accidental causes takes place annually in mines.

The average annual number of deaths from accidents from collieries, for example, in England, is stated to be about 1,000,

out of a population engaged in coal-mining of about 250,000, in addition to an enormous number of accidents. Of this number about one-fourth arise from explosion. Now a great proportion of these accidents—probably the whole number caused by explosions—may be considered to be preventable; and that this is the fact may be fairly assumed from the circumstance of the death-rate in different mines varying considerably, according to the precautions taken in each.

Surely, if this be the case, the proprietors of mines should be forced to adopt proper precautionary measures to protect the lives of the miners, and should, if they failed to do so, be held legally responsible for their neglect!

In the course of a paper on the subject of colliery accidents, read recently at the Society of Arts, by Mr. P. H. Holland, one of the Government Medical Inspectors under the Burial Acts, it was suggested, that, as a means of checking these accidents, no one should be allowed to work in a colliery without having his life assured at the expense of the proprietor of the mine, against death by accident, to a sufficient amount to secure his family from destitution in the event of his death.

The proprietor would be called upon to pay an extra premium should his mine be considered by the Accidental Death Office to be in a dangerous state, owing to neglect of due precautions; it would thus be the interest of the proprietor to bring his mine into the best possible condition, in point of safety. It is added that an increase in the price of coal would provide for the payment of the ordinary but not for the extra premium, as the one, of course, would be a uniform charge in all mines.

This is all very well on paper; but we know that, in practice, the premium for such an assurance would virtually come out of the wages of the men—and the premium would, moreover, probably, be so high, as to preclude the possibility of carrying out the views of Mr. Holland. It was suggested, whether an assurance scheme of this nature might not be made legally compulsory, though Mr. Holland stated that it was doubtful if assurances could be enforced by law.

Of course, any idea of the sort is not likely to be entertained in practice—it would be well for the Assurance Offices were it otherwise—but a law to render the actual proprietors of the mines liable to be tried for manslaughter, in case of a fatal accident caused by the neglect of those precautions which the law might require to be adopted, and a strict enforcement of the punishment for that

crime, might probably tend to diminish the extreme mortality recorded in the reports of the inspectors.

It is scarcely necessary to remark, that a large proportion of the casualties registered as "accidents" eventuate in the death of the victims; but the deaths occurring some time after the accidents are accordingly not set down to the primary cause, but to the immediate cause of death, whatever that may chance to be.

The total number of deaths from violence recorded in the year 1857, was no less than 15,027, and even this large number does not include the deaths from drowning at sea.

I had intended to refer to some of the minor causes which tend unduly to increase the mortality of the country, but, owing to the length to which this paper has already extended, I feel called upon to defer carrying out my intention.

It may be as well for me to mention, that the statistics which I have quoted in the course of this paper have reference almost entirely to the year 1857—the last Report of the Registrar-General, the 20th, not going beyond that date.

It is worthy of remark that while the deaths from one large class of diseases alone have been shown to be upwards of half a quarter of a million, those from "old age" are only 26,847. Probably, in a less artificial state of society than that in which we live, so manifest a disproportron would not exist; for in a natural state, I presume, there can be no doubt it is to "old age" that we should expect to find the great mass of the population owed their death. "Death by old age," Mr. Simon says, in a recent report to the Board of Health, "is, physiologically speaking, the only normal death of man."

Do we not at the present day—as the poet records, nearly two thousand years ago, the presumptuous race of man in primeval times had done—attempt to violate all natural laws, and so draw down upon ourselves consumption, and a new train of fevers before unknown, and thus accelerate the slow approaching necessity of death?

> "Macies et nova Febrium
> Terris incubuit cohors;
> Semotique prius tarda necessitas
> Leti corripuit gradum."

It would be a most interesting inquiry to follow the course taken by Thackrah, 30 years ago, in his round of inspection of the different manufacturing processes throughout the kingdom, with the view to see to what extent the sanitary state of the workmen em-

ployed in the different manufactures had been improved since his time.

Judging from the limited extent to which I have been able to carry my inquiries on this subject, I fear that but small progress has been made; and yet his work *ought* to have opened the eyes of the public to the matter.

An investigation of this nature could easily be carried out by Assurance Companies in combination with the visits of inspection of their agencies, which such Companies have in all parts of the kingdom. The expense of an enquiry of this nature would be, in fact, almost nothing, as it would, at most, but necessitate a few hours extra stay in the different towns which required to be visited; and I feel satisfied that such an investigation, even if conducted at some expense to the Office undertaking it, would more than repay the outlay, by the important information that would be derived as to the actual effect of certain occupations on health and longevity, and of which those engaged in life assurance operations have at present but a vague, and perhaps incorrect, idea.

In the foregoing remarks, I have endeavoured to show that, under the five principal heads, viz., phthisis, fever, small-pox, infantile diseases, and accidental death, an unnecessary amount of mortality takes place annually from causes greatly within human control.

These causes are as follows, viz.—

With reference to Phthisis and diseases of the Respiratory Organs.

1. The unhealthy nature of certain employments.

2. The bad arrangements, as respects ventilation, in manufactures and works of all kinds in which large bodies of the labouring classes are employed.

3. The quantity of drink taken by this class of people.

4. The apathy shown to the position in which they are placed as to sanitary matters, and the prejudices on the part of the operatives generally against all suggestions for the removal of the causes of the evils that injure them, in many conditions of life, particularly in the case of printers, millers, stonemasons, and persons employed in the manufacture of metals.

5. The defective sanitary arrangements as regards the Army.

6. The propagation of the disease by the marriage of those hereditarily affected with phthisis.

With reference to Fever.

I have referred to facts which prove that the means of checking fevers, and other contagious diseases, depend entirely upon sanitary

arrangements, and that we, in fact, hold in our own hands the power of prevention of this class of diseases.

With reference to Small-pox.

I have endeavoured to show that this complaint does not, perhaps, differ from other epidemics; but that, probably, more stringent legislative enactments as to vaccination, or, at least, a more effectual method of carrying out the present Act, would have some effect in diminishing the number of deaths from this cause.

With reference to deaths from Infantile Diseases.

I have said enough to show that the mortality under this head is excessive, and that a considerable portion of it might be prevented.

With reference to deaths from Accidental Causes.

There is, I think, no reason to doubt that a great proportion of the very high mortality under this head arises from causes clearly within human control.

These causes are principally as follows, viz.—

1. Fractures and contusions.
2. Drowning.
3. Burns and scalds.
4. Hanging and suffocation.
5. Wounds and violent deaths, not classified.
6. Poison.

It will be admitted, I apprehend, without question, from the foregoing considerations, that an enormous unnecessary mortality takes place annually in this country. Without venturing myself to assess even an approximate number, I may state that the Registrar-General speaks of 100,000 deaths, in round numbers, as being of a preventable character.

Notwithstanding the evident unnecessary sacrifice of life in this country, there is no doubt that a gradual progressive increase in the mean duration of life has been maintained for some centuries past—with one exception, the 17th century—but it is probably only within our own time that any considerable increase in the longevity of the mass of the population will be apparent; and this will be owing as well to the improved habits of life of the people as to the reform in sanitary matters—to the improvements in the dwellings of the labouring classes—the greater attention that is now being paid to drainage—the abolition of intramural interment —the establishment of baths and wash-houses—to the shorter hours of labour that are now becoming daily more general, owing

in a great measure to the exertions of the "Early Closing Association," and of the Secretary of the society, Mr. Lilwall—to the establishment of national play-grounds, and the encouragement to indulge in manly exercises, of which the shorter hours of labour now more readily admit—and to the greater care that is now devoted generally to the promotion of the well-being of that large section of the community which the labouring classes form, a duty which those above them in the social scale have heretofore too much neglected, to the consequent undue increase of the deaths recorded in the Registrar-General's annual returns, and the filling of our prisons and reformatories.

It will probably be found that, in proportion as the energies of the country are devoted to sanitary questions, so will our expenditure upon such institutions as these be diminished; and thus, while we are promoting the good of the people, we shall be at the same time economising the funds of the nation.

If we are not to begin to profit by the valuable information we derive from the Registrar-General's department, which has been in operation since the year 1837, and if we are not to endeavour to adopt methods to check any unnecessary amount of mortality that we thus have the means of ascertaining—what, allow me to ask, is the advantage of the expensive machinery required for the due carrying out of the elaborate system now in operation?

The very object of the introduction of the system of recording the statistics of births, deaths, and marriages, of the state of factories and mines, and of education, was that we might learn where our system of social government was at fault, and that it might be the means of directing our attention to the course necessary to be taken, with the view to remedy the evils that might be observed to exist.

With a view to utilise the materials we possess, some general national statistical system would, however, appear to be necessary. Dr. Farr, in his Report on the International Statistical Congress, held at Paris in 1855, proposed a scheme for the establishment of a Statistical Board, for the purpose of digesting the national statistics, which should consist, he was of opinion, of statistics of finance, population and health, sickness, poor-law, friendly societies, and charity; of learning, art, and science; of statistics of the church and the law; of trade, manufactures, and agriculture; of the army and navy; and of statistics of India and other foreign countries.*

* *Vide* 16th Annual Report of the Registrar-General.

If a comprehensive scheme of this nature were carried out, we should, in the course of a few years, begin to derive some return from the capital which has been sunk in the purchase of the raw material for our subsequent operations, which has heretofore been, to a great extent, unproductive; for, as truly as the magnetic needle guides the mariner through the pathless ocean, so would the digests of the great masses of facts we possess direct the Government through the wide sea of legislation, and point out the course that must most surely be taken, if we wish to improve the resources of the country—if we wish to diminish sickness and to increase health and longevity—to lessen poverty and to promote learning—to take away the incentives that lead to the commission of crime—to ameliorate the condition of a great people—and to further their social and religious progress.

In concluding this paper, I will beg to express a hope that I may not be considered by the members of the Institute to have intruded upon them anything approaching to an *ex-cathedrâ* opinion. I have endeavoured to bring forward recognised authorities upon the different subjects on which I have touched, and I have taken the greatest pains to verify facts.

I may add, that one great motive I have had in coming forward on the present occasion, has been an earnest feeling that it is a duty incumbent upon every one to lend any assistance that lies in his power towards calling attention to matters affecting the health and longevity of the people; and there is no reason, I consider, because I am not myself equal to the task of doing justice to the question, that I should abstain from entering upon a subject which is only now, I may say, *beginning* to arrest public attention, and that I should forbear to open up the store of rich materials that we have at our command from which to derive instruction in such matters.

"Ergo fungar vice cotis, acutum
Reddere quæ ferrum valet, exsors ipsa secandi."

On the Composition for Leave to an Assured to reside Abroad. By Robert Campbell, M.A., *Advocate, Edinburgh, Fellow of Trinity Hall, Cambridge.*

It is not an uncommon occurrence with certain classes of professional men who have settled themselves in this country, and perhaps assured their lives, without any intention of leaving it, to be called to a position which takes them to a residence perma-

nently, or at least for a lengthened period, abroad. The principle on which an Office should increase their payments for assurance forms both a speculative and practical problem of some interest.

If the original contract of assurance be rigidly interpreted—that is, supposing it simply liable to forfeiture in the event of going to live out of the limits specified in the policy—the assured must then stipulate for the liberty by paying the difference between the value of a policy of £100 at death, according to the statistics of mortality at home and that according to those of the given place abroad—the premium payable in each case, during the remainder of life, being that of the original assurance. But it must be remembered that the liability to forfeiture on going abroad without a new contract, though nominally one of the original conditions of the policy, is not one for which, in the ordinary case, the assured receives or the Office gives any value; and therefore the assured, stipulating in the way above indicated, will really give more than the equitable consideration for his liberty.

Suppose the contingency of the party's going to live abroad had been contemplated from the first, let us see how the matter would have stood.

Suppose the party at the time of assuring—say in the year 1860—to be aged m years, and suppose he then stipulate for the additional payment required in the contingency of his being alive and wishing to go abroad at the end of n years:

Let A_m be the present value of a policy of £100 on a life aged m, according to the statistics of this country;

A'_m the same according to the statistics in the foreign place—the premiums payable during life being in both cases those fixed at first assuring.

If the party be alive in the year $1860+n$, and intend then immediately to go abroad, the value of the policy in that year, supposing him allowed to receive it without increase of premium, will be

$$A'_{m+} .$$

The value of the policy, supposing him to remain at home, would be

$$A_{m+n}.$$

The difference, $A'_{m+n}-A_{m+n}$, which would be the amount payable in the year $1860+n$ to obtain the liberty required on a strict interpretation of the conditions.

The value of that payment in the year 1860, at the time of

assuring, supposing it certain that, if alive, he would go abroad at the end of n years, would be

$$p_{m,n} r^n . (A'_{m+n} - A_{m+n})$$

($p_{m,n}$ being the probability of a life aged m surviving n years, r^n the present value of £1 due at the end of n years).*

Now, suppose, in the year $1860+n$, the circumstances emerge that the party is alive and wants to go abroad. By the strict condition of the contract he will forfeit if he goes; but he has received no consideration for this condition. The only equitable conclusion is, that the original contract be amended, and it will stand thus: on the one hand he has a right to go, which we must suppose to have been constituted at first assuring; but, on the other hand, the Company have the right to the sum which he should have paid for the permission had it been part of the original contract—in other words, to the sum which he should have paid them for the permission, improved at compound interest during the n years when it has been owing.

The payment, therefore, that should be made, is—

$$p_{m,n}(A'_{m+n} - A_{m+n}).$$

The principle, therefore, of an equitable adjustment of such payments I take to be this, that when a party, having assured as for a home policy, wishes to take out a wider one, the additional payment for the permission should first be calculated according to the relative statistics of home and foreign mortality, *and then multiplied by the probability of a life of his age when first assured living for the number of years which he has remained in this country since his assurance.*

It should be noticed, as a very important point in the case of statistics of mortality abroad, that not only the ages, but the length of residence in the foreign country, should be looked at.

The suggestion may appear startling, practically, but the theory would apply equally to the case of assured persons dying having broken the conditions without stipulation, except that it is open for consideration whether the original conditions of the policy ought not, in this case, on grounds of general expediency, to be strictly interpreted. Suppose n to be the number of years intervening before breaking the condition, v the number of years after it, the ordinary premiums continuing to be paid up to death, the amount to be deducted from the policy would be

* Notation in Jones' *Annuities*, vol. i., p. 110.

$$\frac{p_{m,n}(A'_{m+n}-A_{m+n})}{r^v}.$$

Perhaps the most equitable conclusion would be this, that when conditions have been violated *without fraud*, the policy should be paid under the above deduction.

On the Discovery of the Law of Human Mortality, and on the antecedent partial Discoveries of Dr. Price and Mr. Gompertz. By T. R. Edmonds, B.A., *formerly of Trinity College, Cambridge.*

In the year 1832 there was published, in my name, an extensive collection of *Life Tables*, founded upon the discovery of the law which, in my belief, governs the mortality, according to age, of all nations and classes of men, from the earliest infancy to extreme old age. In these tables the numbers living or surviving at successive ages have been deduced from a simple formula expressing the proportion of survivors at any age in terms of the mortality. They are the first Tables of this kind ever published, and they have been used daily in the practice of life assurance, and in the valuation of life contingencies, for a period of thirty-two years, including four years before publication.

The following is the law of human mortality:—The whole duration of human life is divided into three well-marked stages, which belong to all animal life, and are—stages of growth, maturity, and decay. The mortality, in all three stages, increases or decreases, uniformly with the age, in geometrical progression, but in a different progression for each of the three stages. The constant ratios of progression belonging severally to the three stages have been ascertained, their values being the same for all populations at the same stages of life.

The three stages of human life may be conveniently designated as those of *infancy, florescence,* and *senescence.* The period of infancy most commonly ends at the age of 9 years; the period of senescence most commonly begins at the age of 55 years. The values of the annual constants of progression in the stages of infancy, florescence, and senescence respectively, are $\frac{1}{1\cdot479108}$, $1\cdot0299117$, and $1\cdot0796923$—the logarithms of these numbers being $-\cdot1700$, $+\cdot0128$, and $+\cdot0333$ respectively. If the force

of mortality continued uniform for one year, at the beginning of any of the three stages of life, be represented by (a), and if the constant annual ratio of geometrical progression be represented by (p), we shall have, for any time or age (x) measured from the beginning of the stage, (ap^x) to represent the force of mortality at that time; the same being true for all values of (x), whether (x) is a whole number or a fraction. When the mortality has been ascertained by observation for any age, and when the exact length of the period of florescence is known, the mortality existing at every other age may be calculated from the formula (ap^x); the quantity (p) having one or other of the three values given above, according to the stage of life observed.

The limits of the periods of infancy and florescence vary slightly from the ages of 9 and 55 years respectively, as well in different populations as in the same population at different epochs. In every population the force of mortality, or the ratio of the dying to the living, is always least at the age or short period immediately preceding the commencement of puberty—say from the age of 9 to the age of 11 years, or from the age of 8 to the age of 12 years. In my principal theoretical tables, I have assumed the existence of a short period, from the age 8 to the age 12 years, wherein the mortality is constant and at a minimum. This assumption is in accordance with appearances presented by large numbers attaining severally their lowest mortality at different ages—near 9 years of age—although each individual may suddenly pass out of the first into the second stage without passing through the supposed intermediate stage.

The first intimation of the nature of the law of human mortality was conveyed to the public in an essay of Dr. Price, read at the Royal Society in April, 1769. This essay is printed in *Price's Observations on Reversionary Payments*, a work which was for many years the chief text-book used by students of the valuation of life contingencies. In this essay Dr. Price made the following remark (page 400, vol. ii., edition 7) on the mortuary register of the parish of Holy Cross, near Shrewsbury:—"This register exhibits, with remarkable regularity and consistency, the progress of human mortality from birth to old age—representing human life, in conformity to other observations, as particularly weak in the first month, and from that age as growing gradually stronger, till at 10 it acquires its greatest strength, which it afterwards loses; but more slowly till 50, and after 50 more rapidly, till at 70 or 75 it is brought back to all the weakness of the first month."

In the above statement may be recognized the chief features of the true law of human mortality. The three stages of human life are designated, their limits are noted, and words are used descriptive of the effect of the different constants belonging to the three separate stages. Dr. Price, to perfect his discovery, would have had to make the following additional remark, viz.:—"That the rate of increase or decrease of the mortality was constant throughout each of the periods designated; that life growing *gradually stronger* was represented by the constant annual ratio of 1·479 to 1; that life losing strength *more slowly* was represented by the constant annual ratio of 1 to 1·0299; and that life losing strength *more rapidly* was represented by the constant annual ratio of 1 to 1·0797.

In the discovery of the law of human mortality, I was not assisted by any knowledge or recollection of the antecedent partial discoveries of Dr. Price or any other writer. My discovery is founded on direct observation of the principal and best-established facts or statements of mortality on record. These facts, collected by me and expressed in their simplest form, denoted observed ratios of dying to living at successive equal intervals of age. At ages above 10 years, the mortality at successive quinquennial intervals of age was thus ascertained from an extensive variety of tables and observations. These successive quinquennial rates seldom indicated any regularity of increase; but, when combined so as to form rates of mortality for successive decennial intervals of age, the uniform rates of progressive increase became manifest, and the two numbers already indicated, as regulating the increase in the periods of florescence and senescence, were discovered. The constant ratio regulating the progressive decrease of the mortality during the period of infancy, was found by observations on the mortality exhibited at this period in a large variety of tables for successive annual and biennial intervals of age. The constant of the period of infancy was found to be the same in all tables and observations of mortality; so also the constant of the period of senescence was found to be the same in all observations. The constant of the period of florescence was found, however, occasionally to vary from its average magnitude, when small sections of a population were observed; but in large aggregates, consisting of mixed population, and in small sections, consisting of homogeneous population, the particular constant already indicated was manifested.

Immediately after the discovery of the law of human mortality, I constructed, for practical use, a table of mortality in which the numbers living or surviving at successive annual intervals of age

were deduced by successive subtractions of the approximate annual decrements obtained from the formula $\Delta y = y \times ap^x \Delta x$. By this table was exhibited a series of annual decrements uniformly progressive with age, excepting at very advanced ages, when the uniformity was interrupted, although the defect was of no importance in practice. By reflection on this defect, manifested at advanced ages, I became convinced that uniformity of decrement according to age could not be obtained by means of an annual decrement of the form $\Delta y = y \times ap^x \Delta x$; and that such uniformity could only be secured by continually diminishing the intervals of age observed, from Δx to dx, or by calculating the decrement in infinitely small intervals of age by the formula $dy = -y \times ap^x dx$.

Until I had arrived at the conclusion that the decrement of the living at any age was of the form $dy = -y \times ap^x dx$, the idea had never occurred to me that there existed anything in common between the formula (g^{p^x}), used by Mr. GOMPERTZ to represent the number living, and my formula (ap^x)—with three permanent values of (p) determined, representing the decrement of life for every year of age. I had previously become acquainted with the formula of Mr. Gompertz by conversation with Mr. Gompertz himself; I had also tried the value of his formula by applying it to interpolation of the numbers living, in a table of mortality of my own selection, and found the result not to be in conformity with Mr. Gompertz's verbal statement. Not believing his theory to be well founded, I soon forgot everything relating to it, except that his formula was (g^{p^x}), and that he used the differential calculus in his investigation, either in descending from the integral to the differential, or in ascending from the differential to the integral. After the true law of human mortality had become known to me, on re-examination of Mr. Gompertz's theory, I became aware of the cause of my original failure to find any truth in his representations. The error was on the side of Mr. Gompertz, who, in his statement to me, made no mention of any limits of age circumscribing the application of his formula. In all probability, at my first examination of his formula, I chose for interpolation two numbers representing the survivors at two ages, on different sides of one of the limits. The result of the comparison would, in this case, evidently be contradictory to Mr. Gompertz's theory.

The statement of his theory made to me does not differ materially from the general statement given by Mr. Gompertz in his paper read at the Royal Society in June, 1825, for he there states it to be a mathematical consequence of his formula being found to

be true by observation, "That the average exhaustions of a man's power to avoid death are such that, at the end of equal infinitely small intervals of time, he loses equal portions (or proportions) of his power to oppose destruction." As no limits of age are here mentioned, it might fairly be inferred from the above statement, that the vital force of man, measured by the ratio of the living to the dying, is in a constant state of decay from birth to the end of life, at one and the same uniform rate. Mr. Gompertz nowhere makes mention of the fact that the vital force increases at a high rate during the period of infancy; nor does he note the existence of any difference between the rate of the annual loss of vital force during the period of florescence, and the greater rate of annual loss of vital force during the period of senescence. All the examples in detail which he adduces in support of his theory are confined to specimens, taken from different tables, of numbers surviving at various ages, between 15 and 55 years, or comprehended in one stage of life only, that of florescence. With respect to ages above 55 years, Mr. Gompertz states the result of his examination of one table of mortality only, without entering into detail as in the other cases.

My knowledge of the law of human mortality is founded on direct observation of the ratios of the dying to the living, at successive intervals of age, in various populations. Mr. Gompertz's knowledge of the law has been obtained indirectly, being founded on compilations made by others, of ratios of dying to living. The laws of mortality in the last century of the populations of Sweden and of Carlisle are contained in the observed ratios of dying to living at various ages, collected and published by Wargentin and Heysham. Mr. Gompertz, in deducing the laws of mortality of these populations, relies upon tables of survivors, according to age, constructed by Price and Milne, who acted as compilers of the ratios of mortality supplied by Wargentin and Heysham. He accepts observations at second hand for his guide when the original ratios of mortality observed were easily accessible. Mr. Gompertz has done nothing without the aid of tables of survivors at successive ages previously constructed by others. Nearly all that he offers to show is, how *interpolations* may be made for intermediate ages when the number of survivors at the beginning, and the number of the survivors at the end, of a large interval of age are given. Mr. Gompertz never shows how a new table of mortality may be constructed independently of other previously-existing tables.

The paper of Mr. Gompertz read at the Royal Society in June, 1825, is entitled, "On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of life contingencies." This paper is divided into two chapters, the first containing 17 pages, and the second containing 58 pages. The subject of both chapters is *interpolation* for intermediate ages between the number of survivors at the beginning, and the number of survivors at the end, of intervals of age, of greater or less extent, given by different tables of mortality. Chapter I., or rather 12 out of its 17 pages, is occupied in showing that *interpolation* may be most correctly effected by the aid of the transcendental (g^{p^x}). The whole of Chapter II. is devoted to showing that interpolations for short intervals of age may be satisfactorily effected by means of a simple geometrical series, of which each term is of the form (g^x). As the two modes of interpolation are, in a certain degree, inconsistent with and opposed to one another, it is not easy to account for the publication of these two chapters side by side. The reader might fairly infer that the author could not have much faith in the virtues of his transcendental (g^{p^x}), when, in the inculcation of its truth and applicability, he had bestowed only one-fifth of the space and attention which he had bestowed on the simple and common geometrical series represented by (g^x).

If the two chapters are viewed together, the reader can hardly avoid coming to the conclusion that the patient labour expended in the production of Chapter II. was the source of the discovery exhibited in Chapter I., and that the ideas connected with the discovery have been presented to the public in an order which is the reverse of the order in which they occurred to the mind of Mr. Gompertz. If anyone acquainted with mathematical formulæ, and with the transcendental (g^{p^x}), had expended, with tolerable success, a vast amount of labour (as Mr. Gompertz apparently had done) in the interpolation of survivors according to age, by means of the geometrical form (g^x), it could hardly have failed to occur to his mind, in the course of his labour, that (g^{p^x}) being a form of much greater elasticity than (g^x), would yield numbers much more nearly coinciding with the facts intended to be represented. After he had, by examination, found his prognostication verified, he would not (at first, at least) regard the quantity (p) as a permanent quantity; for, since (g^{p^x}) is taken only as the substitute of (g^x), and as (g) in (g^x) has an endless number of values, the quantity (p) would also be considered as having an endless number of values. It

would be supposed that (p) would change with every different interpolation, because (g) in (g^x) changes with every different interpolation.

As soon as Mr. Gompertz had discovered that the transcendental (g^{p^x}), when used for interpolation of survivors according to age, gave results nearly coincident with fact, his curiosity would be awakened as to the signification of the constituent parts of (g^{p^x}). The readiest means of examination would be afforded by differentiating the above quantity. The differential obtained would be found to be log. $g \times$ log. $p \times g^{p^x} p^x dx$, which is of the same form as $dy = a \times yp^x dx$, the differential arrived at by my investigation previously to any knowledge of its connexion with (g^{p^x}) as its integral. Thus Mr. Gompertz will have descended from his integral to my differential—there being no reason to suppose that he had ascended, from a differential like mine, to his integral. The former course is simple and obvious; but the latter course, though apparently simple, would not be obvious to a person dealing for the first time with quantities of a new and unknown character. It is often erroneously supposed that Mr. Gomperts has deduced his formula from the hypothesis of equal proportions of vital force being lost in equal intervals of age; but the reverse is the fact, the hypothesis having been arrived at as the mathematical or necessary consequence of the truth of the formula; for Mr. Gompertz states that "the hypothesis was derived from an analysis of the experience alluded to"—the experience alluded to consisting in (g^{p^x}) being found to yield correct interpolated numbers for the survivors according to age in many tables of mortality.

The formula (g^{p^x}) is a general formula, alike applicable to the expression of the true law of human mortality and to the imperfect law announced by Mr. Gompertz. The true law differs from the imperfect law only in respect of the value of (p) being determined in the former case, and being left undetermined in the latter case. According to the true law of human mortality, (p) has a fixed determined value for each of the three periods or stages into which human life is divided—growth, maturity, and decay—for the same and for different populations. According to the imperfect law of mortality declared by Mr. Gomperts, no limit is assigned to the variation of (p), whether in the same or different populations, at any period of life. The original idea entertained by Mr. Gomperts apparently was, that by means of the formula (g^{p^x}), interpolations of survivors between any pair of ages, not very remote from one another, may be effected by assuming different values of (p). It

is only by reference to examples of interpolation cited by Mr. Gompertz that the reader learns, indirectly, that, in the second period of life, or from the age 15 to 55 years, Mr. Gompertz regards (p) as constant in the same population at the same time, though variable for different populations and for the same population at different times. Mr. Gompertz says nothing of the value of (p) at any age under 15 years, and gives the value of (p) in one case only for the period of life above 55 years of age. In examples of the application of his formula to different populations, Mr. Gompertz always treats (p) as a fugitive quantity, and as of no value except for the particular interpolation then required to be made.

The view now taken of Mr. Gompertz's theory of mortality, with reference to the variability of the quantity (p), has evidently been also taken by the late Mr. Galloway, who, I believe, is the only person who has made a practical use of Mr. Gompertz's formula in the construction or adjustment of a new table of mortality. In the year 1841, whilst explaining the construction of tables which he had made of the mortality experienced by the Amicable Assurance Society, Mr. Galloway states that, in the adjustment of these tables, he "had recourse to the highly ingenious method (founded, indeed, on a hypothetical principle) proposed by Mr. Gompertz. . . . . In Table III. the same constants were used through the whole series, from age 45 to age 93 years. In Table IV., from age 24 to age 68, one set of constants was used; and a second set from age 69 to age 93 years." Mr. Galloway does not state the value of the quantity (p) in either of the three cases, nor does he make any allusion to (p) as distinguished from other but less important constants used. He evidently regarded (p) as a fugitive quantity, of no value except for the use to which it had been applied, in smoothing the irregularities which would otherwise have appeared in the tables which he was then constructing. Both Mr. Gompertz and Mr. Galloway have treated (p) as an insignificant quantity, of accidental value, and of no use but to maintain the formula (g^{p^x}). The truth is, however, that (p), with its three determinate values, is independent of all formulæ, has existed as long as man has existed, and forms part of the foundations of the universe.

All my published tables of mortality have been deduced from the formula $y=10^{\frac{k^x a}{\lambda p}(1-p^x)}$. In arriving at this formula, I am not conscious of having received any assistance from the writings of

Mr. Gompertz, beyond the suggestion of the idea that the form which I had discovered for the decrement of human life at every age was one capable of integration by simple and well-known methods. I do not believe that this assistance was of any importance, except with regard to time, for I possessed all the materials for forming the series proposed for summation, and could hardly have failed to discover the law of that series as soon as my attention had been directed to, and concentrated on, the subject. The law of the series may be found as well without as with the aid of the differential calculus. My formula, as given above, is reduced to more simple terms than that given by Mr. Gompertz. The correctness of my formula, or of the process by which it has been obtained, has never been questioned. There is, however, ground for doubting the correctness of the process by which Mr. Gompertz has deduced his formula. To remove this doubt, the publication of his process in a more explicit form is requisite.

My formula, $y=10^{\frac{k^2a}{\lambda p}(1-p^x)}$, gives the number living or surviving at any age (x) in any of the three periods of human life, expressed in terms of the mortality (ap^x) at that age, and known quantities; (k) being the modulus of the common system of logarithms and equal to ·4342945, and (λp) being the common logarithm of (p), the constant ratio of the period. This formula may be written thus:—

$$y=10^{c(1-p^x)}=g^{1-p^x}=\frac{g}{g^{p^x}}.$$

Mr. Gompertz, by changing the sign of (c) and introducing the superfluous quantity (d), makes his corresponding formula $y=dg^{p^x}$. If the change of sign had not been made, Mr. Gompertz would have stated the value of (y) to be $\frac{d}{g^{p^x}}$, instead of $\frac{g}{g^{p^x}}$, my value given above. According to my formula, $y=1$ when $x=0$, whilst Mr. Gompertz's formula gives $y=\frac{d}{g}$ when $x=0$. The integration of Mr. Gompertz's formula requires to be performed between limits of $x=a$ and $x=a+n$, whilst in my formula the limits are of the simplest form between 0 and x. The defect in Mr. Gompertz's formula, caused by the addition of (d), is the same as that which would exist in a table of discount of money at compound interest if any other basis were adopted than the value of the sum of £1 receivable (x) years hence. It is only in cases of interpolation that the quantity (d) is of any utility.

It may be useful here to give an example of the application of my formula $y_x = \frac{g}{g^{p^x}}$; for this purpose we will assume (x) to be equal to 10 years, and to be measured from the beginning of the second and chief of the periods of human life—the period of florescence. In this period $p = 1{\cdot}0299117$, and the value of (a), or the minimum mortality, is, in most populations, on an average, $= {\cdot}0063643$; hence $\frac{k^2 a}{\lambda p}$ or $c = {\cdot}0937799$, and 10^c or $g = 1{\cdot}241023$. Since $p^{10} = 1{\cdot}342765$, we get $g^{p^{10}} = g^{1{\cdot}342765} = 1{\cdot}336363$;

whence $$y_{10} = \frac{g}{g^{p^{10}}} = \frac{1{\cdot}241023}{1{\cdot}336363} = \frac{1}{1{\cdot}076823};$$

otherwise,

$$y_{10} = g^{1-p^{10}} = g^{1-1{\cdot}342765} = g^{-{\cdot}342765} = \frac{1}{g^{{\cdot}342765}} = \frac{1}{1{\cdot}076823},$$

as before.

The formula above given, when expressed in logarithms, becomes $\lambda y_x = \frac{k^2 a}{\lambda p}(1 - p^x)$, whence may be derived the logarithms of the probability of living one year from the age (x), — quantities by the aid of which the construction of mortality tables is most facilitated. The logarithm of such probability is equal to the logarithm of the value of (y) at age $(x+1)$, less the logarithm of the value of (y) for the age (x).

Since—

$$\lambda y_x = \frac{k^2 a}{\lambda p}(1 - p^x), \text{ and } \lambda y_{x+1} = \frac{k^2 a}{\lambda p}(1 - p^{x+1}),$$

we get, by subtraction,

$$\lambda y_{x+1} - \lambda y_x = -\frac{k^2 a}{\lambda p}(p^{x+1} - p^x) = -\frac{k^2 a}{\lambda p} \times (p-1)p^x,$$

$$\text{or } \lambda \frac{y_{x+1}}{y_x} = -mp^x; \quad \text{putting } m = \frac{k^2 a}{\lambda p}(p-1).$$

That is to say, the logarithm (which is negative) of the probability of living one year from the age (x) is equal to (p^x) multiplied by the constant quantity $\frac{k^2 a}{\lambda p}(p-1)$.

If, instead of the common or decimal system of logarithms, having 10 for its base, we use the hyperbolic system, of which the

of the momentary increments may be calculated on principles similar to those on which the sum in one year of momentary increments, derived from constant ratios, is commonly calculated. It may be useful here to observe that the series of survivors in tables of mortality resemble, in a great degree, tables of discount representing the values of £1 receivable (x) years hence; that the annual rate of discount of money is $\frac{1}{1+a}$ when the amount of £1, with compound interest for one year, is $(1+a)$; and that, by a change of sign in the exponent, annual ratios of increasing money or population become annual ratios of decreasing money or population, applicable to the same years or intervals of age, in the same table, according to the direction in which progress is measured.

In the first place, let it be supposed that the rate of interest is constant, being such that £1 increases to £$(1+a)$ in every year. Then, let it be required to find another quantity (a), such that $\left(1+\frac{a}{n}\right)^n=1+a$; the year being divided into an infinite number (n) of equal parts, $\frac{a}{n}$ being a quantity indefinitely small, and $\left(1+\frac{a}{n}\right)$ being a constant ratio repeated or multiplied (n) times in order to produce a result equal to $(1+a)$. In treatises on algebra, it is shown that when $\left(\frac{a}{n}\right)$ is indefinitely small, the quantity $\left(1+\frac{a}{n}\right)=e^{\frac{a}{n}}$, ($e$) being the base of the hyperbolic system of logarithms and equal to 2·7182818. By substituting this value of $\left(1+\frac{a}{n}\right)$ in the first equation, we get $\left(e^{\frac{a}{n}}\right)^n=1+a$, or $e^a=1+a$; that is to say, $a=$hyperbolic logarithm of $(1+a)=\frac{1}{k}\times$common logarithm of $(1+a)$; (k) being the modulus of the common system and equal to ·4342945. Similarly, if (p), the annual ratio at which the interest (a) increases, be taken equal to $(1+b)$, and if (β) have the same relation to (b) that (a) has to (a), we shall have $\beta=\text{hyp. log.}(1+b)=\text{hyp. log.}\,p=\frac{1}{k}\times\text{com. log.}\,p$.

Let it now be assumed that the rate of interest of money, represented by (a) at the beginning of the year, becomes, by

uniform increase throughout the year, (ap) at the end of the year; and let it be required to ascertain the amount of capital and interest at the end of the year in respect of £1 capital possessed at the beginning of the year. It will be seen that the rate of interest of money, which is $\frac{a}{n}$ at the beginning of the first moment of the year, becomes $\frac{a}{n}\left(1+\frac{\beta}{n}\right)$ at the end of the first of the (n) equal intervals into which the year has been divided, and $\frac{a}{n}\left(1+\frac{\beta}{n}\right)^2$ at the end of the second of these equal intervals; for $p=\left(1+\frac{\beta}{n}\right)^n$, and $\left(1+\frac{\beta}{n}\right)$ is the constant ratio borne by the rate of interest at the end of one moment to the rate of interest which existed at the end of the preceding moment. We shall, consequently, have, for the rates of interest existing at the end of the 1st, 2nd, 3rd, to nth moments—

$$\frac{a}{n}\left(1+\frac{\beta}{n}\right),\ \frac{a}{n}\left(1+\frac{\beta}{n}\right)^2,\ \frac{a}{n}\left(1+\frac{\beta}{n}\right)^3,\ \ldots\ldots\ \frac{a}{n}\left(1+\frac{\beta}{n}\right)^n.$$

And, since the ratio of capital and interest together, at the end of any moment, to the capital at the beginning of such moment, is that of (1 + rate of interest) to 1, we shall have for such ratios, in the 1st, 2nd, 3rd, to nth of the (n) infinitely small equal intervals of the year—

$$\left[1+\frac{a}{n}\left(1+\frac{\beta}{n}\right)\right],\ \left[1+\frac{a}{n}\left(1+\frac{\beta}{n}\right)^2\right],\ \left[1+\frac{a}{n}\left(1+\frac{\beta}{n}\right)^3\right]\ \ldots\ldots\ \left[1+\frac{a}{n}\left(1+\frac{\beta}{n}\right)^n\right],$$

which may be written thus, on putting $\frac{a}{n}=a'$, and $\left(1+\frac{\beta}{n}\right)=v$:—

$$(1+a'v),\ (1+a'v^2),\ (1+a'v^3)\ \ldots\ldots\ (1+a'v^n);$$

which last quantities may be written as follows, since $a'v$, $a'v^2$, $a'v^3$, &c., are indefinitely small—

$$e^{a'v},\ e^{a'v^2},\ e^{a'v^3}\ \ldots\ldots\ e^{a'v}.$$

The product of all the above factors, (n) in number, or the value of (y) at the end of the year, will be—

$$y=e^{a'v}\times e^{a'v^2}\times e^{a'v^3}\times \text{ \&c. \&c. } \times e^{a'v^n},$$

$$=e^{a'(v+v^2+v^3+\text{ \&c. }+v^n)}=e^{a'\cdot\frac{v^n-1}{v-1}\times v}.$$

On taking the hyperbolic logarithms of both sides, we get—

$$\text{hyp. log. } y = a' \frac{v^n - 1}{v - 1} \times v = \frac{a}{n} \times \frac{\left(1 + \frac{\beta}{n}\right)^n - 1}{1 + \frac{\beta}{n} - 1} \times \left(1 + \frac{\beta}{n}\right)$$

$$= \frac{a}{\beta}\left(e^{\beta} - 1\right), \text{ omitting } \frac{\beta}{n} \text{ as indefinitely small in } \left(1 + \frac{\beta}{n}\right),$$

$$= \frac{a}{\beta}\left(p - 1\right),$$

which is of the same form as the value previously obtained (p. 180) of the constant used in multiplying (p^x) in order to yield the hyperbolic logarithm of the probability of living one year from the age (x) years.

The value just obtained of the logarithm of the amount of £1 at the end of the first year, when the rate of interest increases by equal proportions in equal infinitely small intervals of time, will, when multiplied successively by $p^1, p^2, p^3, p^4 \ldots . p^{x-1}$, give the values of the logarithms of (y) for the 2nd, 3rd, 4th . . . and xth year. The numbers corresponding to these logarithms will severally represent, for successive years, the capital and interest at the end of each year in respect of £1 capital at the beginning of such year. The multiplication together of all the annual ratios, beginning with the first and ending with the (x)th year, will give the amount of £1 in (x) years when the rate of interest is (a) at the beginning and (ap^x) at the end of the term. If (a) had been taken to represent decrement by mortality, instead of increment by interest of money, a formula in the same terms as that just given would have been obtained, with the difference only of a change of sign from positive to negative in the exponent of $(1 + a'v)$.

The formula now obtained by the most direct method of investigation is in exact agreement with the formula which I have elsewhere obtained by the aid of the differential calculus, neither of them containing any unknown constant like (b), introduced by Mr. Gompertz in the process by which his formula has been obtained. Hence it may be concluded that such a quantity does not belong to the correct formula, and that Mr. De Morgan is not wrong in designating it as superfluous and useless. If Mr. Gompertz has used this unknown quantity in order to obtain a correct result, he must have previously committed an error in the process of integration, of which (b) represents the correction.

THE

ASSURANCE MAGAZINE,

AND

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

Newton's Table of Leases. *By* Professor De Morgan.

By this I do not mean what is usually called Newton's Table, but something which has a better right. First, however, I will speak of the table which, having Newton's certificate of accuracy, is (or was) usually called by his name.

Mr. Edleston, Fellow of Trinity College, in making researches for his biography* of Newton, found out the author of the old table. His name was Mabbot, and he was manciple *(mancipium)* or caterer of King's College. Mabbot published his table in 1686, with the well-known attestation of Newton opposite to the title-page. There seems to have been a special reason for procuring this certificate. The church lessors were beginning to open their eyes to the leniency of their terms, and Mabbot's tables were certainly intended both to urge them to raise their fines, &c., and to point out the proper way of doing it. Subsequent editions of these tables had attached to them the well-known letter which pointed out the advantage under which the tenant was living. As it was natural that the lessees should oppose the new scheme, and they would probably question its accuracy, it seems that Newton was applied to for the testimonial which, as is well known, he gave.

The dispute about the church leases continued till about 1731, when it was at the fiercest, and then burnt out. See a list of

* Attached to his *Correspondence of Sir Isaac Newton and Professor Cotes*; London, 1850; 8vo. This biography is in the form of *annals*, and, besides containing a large quantity of new and curious matter, is of exceedingly convenient reference.

On the Construction of Life Tables, illustrated by a new [illegible]
of the Healthy Districts of England. By W. F[illegible]
M.D., F.R.S.

(Concluded from page 141.)

IV. CONSTRUCTION OF THE COLUMNS d_x, l_x, L_x, P_x, Q_x [illegible]
NOTICES OF SOME OF THEIR PRACTICAL APPLICAT[illegible]

THE series l_x has been constructed, and from that ser[illegible] are deduced to complete the Life Table, consisting n[illegible] columns.

(1.) $d_x = l_x - l_{x+1}$ = number of deaths in the year o[illegible] lowing, out of l_x alive at the age x. By taking x succe[illegible] 0, 1, 2, 3 . . . . to the last age in the Table, the number[illegible] every year of age are obtained. The numbers dying of [illegible] and under the age l_{x+n} are immediately derived from the c[illegible] as (2) $l_x - l_{x+n} = d_x + d_{x+1} \ldots d_{x+n-1}$. When $x+n > \omega = t$[illegible] age in the Table, $l_x = d_x + d_{x+1} \ldots + d_\omega$.

(3.) $L_x = l_x + l_{x+1} \ldots\ldots + l_\omega$. The series is formed [illegible] successive addition of the series l_x, from l_ω upwards.

(3*a*.) $L_x - L_{x+n} = L_{x|n} = l_x + l_{x+1} \ldots + l_{x+n-1}$.

(4.) $\left.\begin{array}{l} P_x = l_{x+1} + \frac{1}{2}d_x \\ P_x = l_x - \frac{1}{2}d_x \end{array}\right\}$ and (5) $P_x = \dfrac{l_x + l_{x+1}}{2}$.

$P_{x+1} = l_{x+1} - \frac{1}{2}d_{x+1} = l_{x+2} + \frac{1}{2}d_{x+1}$.

The series in column P_x is constructed from the two c[illegible] l_x and d_x, or from the single column l_x, as $2P_x = l_x + l_{x+1}$[illegible] $\therefore P_x + \dfrac{l_x + l_{x+1}}{2}$, $\therefore l_x = 2P_x - l_{x+1}$; so, conversely, the se[illegible] can be constructed from the series P_x. The P_x is assu[illegible] represent the population, as expressed by the Life Table, li[illegible] the age x and under the age $x+1$; thus P_{20} = the popula[illegible] the age 20 and under 21 years.

By substituting the successive values of P_x in the equatio[illegible] $P_x + P_{x+1} \ldots P_{x+n}$, we have $\frac{1}{2}l_x + l_{x+1} \ldots + l_{x+n} + \frac{1}{2}l_{x+n+1}$.

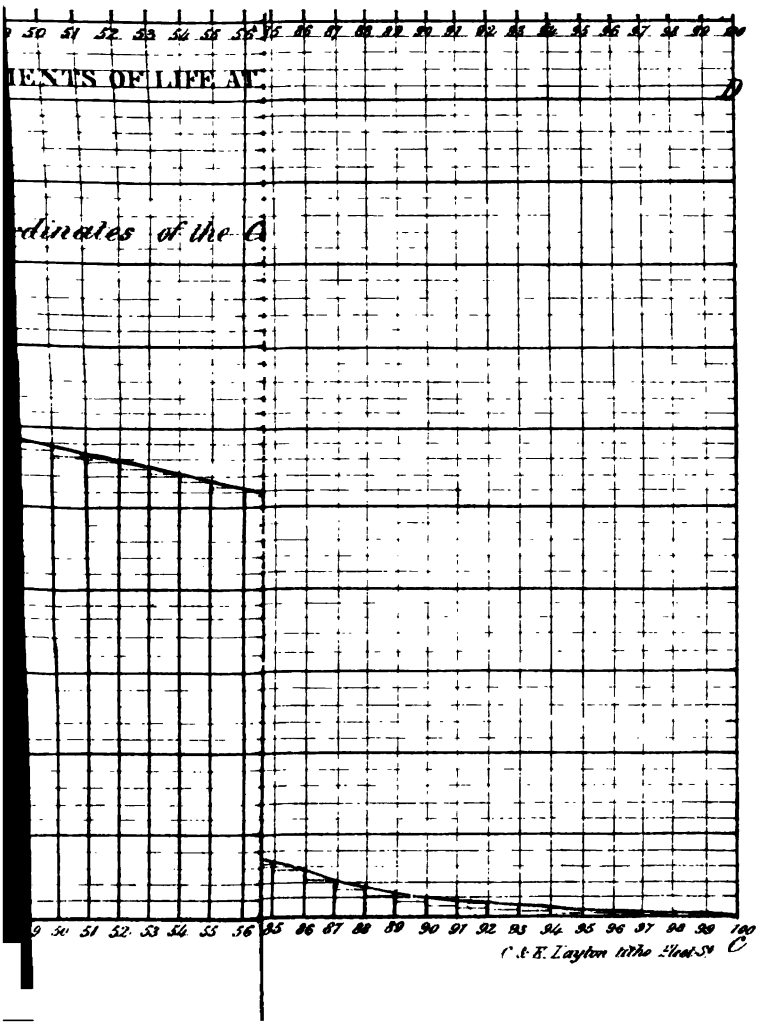
(6.) $Q_x = P_x + P_{x+1} + P_{x+2} \ldots P_{x+n-1} + P_{x+n} \ldots + P_\omega$ [illegible]

$Q_{x+n} = P_{x+n} + P_{x+n+1} + P_{x+n+2} \ldots + P_\omega$.

(7.) $\therefore Q_x - Q_{x+n} = Q_{x|n} = P_x + P_{x+1} + P_{x+2} \ldots P_{x+n-1}$.
column Q_x is constructed by adding up the column P_x, and [illegible] ferring the successive sums to the column Q_x.

By substituting, for the series P_x, its values in l_x, we have

(8). $Q_x = \frac{1}{2}l_x + l_{x+1} + l_{x+2} \ldots\ldots + l_\omega$.


IENTS OF LIFE AT
D
dinates of the A
50 51 52 53 54 55 56 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100
C. & E. Layton lith. Lincoln's Inn
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And by again substituting for the series l_x its corresponding values in d_x, we have—

(9.) $Q_x = \frac{1}{2}d_x + 1\frac{1}{2}d_{x+1} + 2\frac{1}{2}d_{x+2} \ldots\ldots + (\omega + \frac{1}{2})d_\omega$.

(10.) Thus Q_x is equal to the numbers dying in each year of age after the age x, multiplied by the time (expressed in years and fractions of a year) that they have respectively lived over that age; and if $x=0$, then $Q_0 = \frac{1}{2}d_0 + 1\frac{1}{2}d_1 + 2\frac{1}{2}d_2 \ldots\ldots (n+\frac{1}{2})d_{x+n}$, when $(x+n)$ becomes $> \omega$.

(11.) This column Q_x represents, therefore, two distinct orders of facts; it represents the sum of the number of years that will be lived after the age x by the l_x persons then living, and $\therefore \frac{Q_x}{l_x}$ = the mean after-lifetime, of which $\frac{Q_{x|n}}{l_x}$ will be enjoyed before the age $x+n$ is attained, and $\frac{Q_{x+n}}{l_x}$ after the age $x+n$ is attained. At birth the mean after-lifetime is $\frac{Q_0}{l_0}$, the unit here being one year of individual life.

(12.) Q_x also represents the sum of the numbers of men or women living at all ages over the age x, out of Q_0 living at all ages, as Q_x is in all cases the sum of the numbers living in each year of age, represented by the series P_x. The unit is here an individual man.

(13.) Thus, on referring to the Plate appended to this paper, the lifetime of 100,000 children born simultaneously may be represented by 100,000 parallel lines, drawn from AB horizontally in the direction of CD until they cut the curved line BC. And Q_0 is the sum of these lines expressed in the linear units of the scale on the line AC; so $\frac{Q_0}{l_0} = \frac{Q_0}{100{,}000} = \frac{4{,}899{,}665}{100{,}000} = 48{\cdot}99665$; the mean length of those lines = the number of years of mean lifetime.

It will be observed that, in this Table, instead of 100,000 lines, these lines are thrown into 106 groups, each comprising the variable number of lines terminating in each of 106 intervals numbered on the line AC, and representing years of age; and in these short intervals it is assumed that the mean length of the lines terminating in the eleventh interval (10 to 11) is represented by $10\frac{1}{2}$, and so on.

The relative numbers of persons living simultaneously at each interval of age will also be represented in the same Plate, fig. 1, by 106 successive vertical lines, raised from nearly the centre of each

interval between the ordinates on the line AC, and measured in units of which the line AB contains 100,000. The same lines bound the figure representing the two orders of facts, and the numerical units expressing the aggregate length of the vertical lines equal in amount the units expressing the aggregate length of the horizontal lines expressed in the horizontal units.

(14.) I will now explain briefly the nature of the column Y_x, which I have added to the Life Table.* The Life Table (column P_x) exhibits a representative population, such as would be constituted by separating every year 100,000 births as they occurred, and keeping them apart in a separate community, subject to a definite law of mortality. Any population living in the tabular proportions at each year of age may, for the sake of distinction, be called a normally-constituted population.

The ages of the population represented by the Life Table amount, in the aggregate, to Y_0 years; it is the aggregate *number of years which they have already lived,* and, singularly enough, it is also, if the law of mortality remain constant, *the number of years which they will live.* Thus, Q_0 persons in such a population have lived on an average $\frac{Y_0}{Q_0}$ years; *that is their* MEAN AGE, and it is also their mean *after-lifetime.* Y_x is the number of years that Q_x persons have lived *over the age x,* and the mean age of such persons is $x+\frac{Y_x}{Q_x}$; their after-lifetime is $\frac{Y_x}{Q_x}$.

The series Y_x is formed by successively adding up a series of the form $\frac{1}{2}(Q_x+Q_{x+1})$, commencing at $x+1=\omega=$the oldest age in the Table.

$$(15.)\quad \therefore\ Y_0=\tfrac{1}{2}Q_0+Q_1+Q_2\ldots+Q_\omega,$$
$$Y_x=\tfrac{1}{2}Q_x+Q_{x+1}+Q_{x+2}\ldots+Q_\omega.$$

* *See* paper in Appendix to *Registrar-General's Sixth Annual Report*, pp. 544-552.

Extract from the Registrar-General's Sixth Annual Report (1845), p. 528.

"*Note.*—Halley's Table (1693) contained the column P. John Smart made 1,000 'born' the basis of his Table (1738), and introduced the columns *d* and *l*. Simpson adopted Smart's form of Table, which was followed by Kersseboom (1738), Deparcieux (1746), Price (1773), and Milne (1815). The columns S.*y*, *y*, and Δ*y*, in Duvillard's *Loi de Mortalité (en France) dans l'état naturel*,† correspond with the columns L, *l*, *d*, in the new Table. The S.*y* added by Duvillard is our L, and Barrett's column B. Duvillard's short Table (p. 123) has the four columns *d*, *l*, P, Q, for quinquennial or decennial ages, and the 'expectation of life.' Mathieu's Table II. is an expansion of the column Q of Duvillard's short Table, and is that column for each year of age. In a recent report on the Bengal Military Fund, Mr. Davies has a Table (1) containing columns corresponding with the *d*, *l*, L, P, Q, of the English Table, the 'Mortality per cent.', and the 'Expectation of Life' at each age."‡

I have in this paper employed *d*, *l*, L, instead of C, D, N, which have been formerly used by me and others, and should still be used where the factor v^n is introduced.

† *Influence de la Petite Vérole*, p. 161. ‡ *See* the Note (A), p. 558.

By substituting for Q_0, for Q_1, for Q_2, and so on, their values in P_x, it will be found that—

(16.) $Y_0 = \frac{1}{2}P_0 + 1\frac{1}{2}P_1 + 2\frac{1}{2}P_2 + 3\frac{1}{2}P_3 \ldots\ldots + (n+\frac{1}{2})P_n \ldots\ldots + (\omega+\frac{1}{2})P_\omega$.

(17.) But the mean age of the persons (P_0) of the age of 0 and under 1 is nearly $\frac{1}{2}$; and so the series $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, $5\frac{1}{2}$, $6\frac{1}{2}$ .... $(n+\frac{1}{2})$, expresses nearly the mean age of all the persons in the first (P_0), second (P_1), third (P_2), and $(n+1)$th (P_n) years of age, and so for all other ages; consequently the sum of the series (16) Y_0 is the sum of the ages of all the persons living contemporaneously, as they are represented in the Life Table.

In like manner it is shown that—

(18.) $Y_x = \frac{1}{2}P_x + (1+\frac{1}{2})P_{x+1} + (2+\frac{1}{2})P_{x+2} \ldots + (\omega+\frac{1}{2}-x)P_\omega$ is the sum of the number of years that the Q_x persons in the Table have lived over the age x. They have all lived x years; and, consequently, $x + \frac{Y_x}{Q_x}$ gives their average age precisely as $\frac{Y_0}{Q_0}$ gives the average age of the whole community.

(19.) It has been shown that Q_x expresses the number of years that l_x persons will live; in the same manner it may be shown that Q_{x+1} expresses the number of years that l_{x+1} persons will live; $\therefore (l_x + l_{x+1})$ persons will live $(Q_x + Q_{x+1})$ years, $\therefore \frac{1}{2}(l_x + l_{x+1}) = P_x$ persons will live $\frac{1}{2}(Q_x + Q_{x+1})$ years. And the same may be demonstrated for each successive value of x.

But the sum of the series P_x is Q_x = the number of persons living of all ages; and the sum of the series $\frac{1}{2}(Q_x + P_{x+1})$ is Y_x = the number of years that Q_x persons will live; $\therefore \frac{Y_x}{Q_x}$ = the *mean after-lifetime* of all the persons living simultaneously of the age x and upwards. Thus, by the Table D, 4,899,665 persons are living contemporaneously; their mean age is $\frac{Y_0}{Q_0} = \frac{166209701}{4899665} = 33{\cdot}92$ years, and they will life on an average 33·92 years.

(20.) The Life Table serves to determine the value of life annuities, the value of policies, and the premiums of insurance.

This is effected by introducing a new unit, such as £1, 1 franc, 1 dollar, or any other monetary unit. Thus, if £1 is payable at each death, the series d_x will show the number of pounds falling due in each year of age; so if £1 is payable by each person on attaining the age x, and each subsequent year of age, the series l_x shows the number of *pounds* payable every year by the l_x persons; and N_x will be the number of pounds payable in the whole course

of life after the age x: thus $\frac{N_x £1}{l_x}$ = the AVERAGE AMOUNT of an annuity of £1 payable on each life at and after the age x. The money-unit may be introduced into the other columns, and $\frac{Y_x}{Q_x}.£1$ would show the AVERAGE AMOUNT payable under an annuity of £1 on each of Q_x lives. The *present value* of these future payments can always be determined by assuming a given rate of interest. The estimates thus obtained are also always read subject to the qualification that, by hypothesis, the *Life Table* is based on a law of mortality actually to rule for a definite time in the population to which it is applied. The probability of the hypothesis is not here in question.

Under the same circumstances, masses of mankind appear to experience, at the same ages, the same rates of mortality; consequently, if, for several years, d_x persons have died annually on an average out of l_x persons living at the beginning of the year, other things being equal, the probability that the same number will die out of l_x persons in a year to come is greater than any other that can be named, and the fraction expressing that probability is $\frac{d_x}{l_x}$. We know that d_x expressing the numbers dying in a year, l_{x+1} must express the numbers surviving as $l_{x+1}+d_x=l_x$. The chances may be represented by l_x balls: l_{x+1} *white* balls in an urn will represent the chances of living, d_x *black* balls in the same urn will represent the chances of dying. Now, let each of l_x persons pay the sum z for a ticket, and each person that draws a *white* ball be entitled to £1. Before the drawing commences the value of each ticket is $\frac{l_{x+1}}{l_x}$; for l_x (the total chances) : l_{x+1} (the chances in favour of winning on one ticket) : : 1 : $\frac{l_{x+1}}{l_x}=z$.

Put $l_x=30{,}007$, and $l_{x+1}=29{,}647$; then $\frac{l_{x+1}.£1}{l_x}=\frac{29{,}647.£1}{30{,}007}$ $=£\cdot98802$. The amount of money to be paid on l_{x+1} white balls is £29,647, and $£\cdot9802\times30{,}007=z.l_x=£29{,}647$.

In like manner it may be shown that if £1 is paid to each person who draws a *black* ball, the value of each ticket is $\frac{d£1}{l_x}=y£1$, for $y.l_x.£1=d_x£1$, and £1 is to be paid on each of d_x tickets.

Should £1 be paid alike to those who draw white balls and to those who draw black balls, the value of a ticket will be equal to

the sum of the two fractions expressing the several probabilities, namely—

$$\frac{l_{x+1}.£1}{l_x} + \frac{d_x £1}{l_x} = z + y = \frac{l_{x+1} + d_x}{l_x} £1 = \frac{l_x}{l_x} £1 = £1.$$

As one or other of the two kinds of balls *must by hypothesis be drawn*, and £1 is paid for each ball, the receipt of the £1 is certain: certainty is thus in all cases expressed by *unity*.

If every ball as it was drawn were replaced in the urn, although in 30,007 trials *white balls* were not actually drawn 29,647 times, black balls 360 times, still $\frac{29,647}{30,007}$ would express the probability of drawing a white ball, and the value of £1 contingent on that event, more accurately than any other fraction that could be named.

Again, if an urn contained, by hypothesis, an indefinite number of balls, out of which 29,647 white balls and 360 black balls were drawn and then replaced, the probability of again drawing a white ball on trial, and the value of £1 contingent on that event, would be expressed more accurately by $\frac{29,648}{30,009}$* than by any other fraction that could be named—past experience being, by hypothesis, the only means we have here of judging of the future.

Thus a Life Table applicable to the case furnishes the fractions to determine the value of any sums of money dependent on the life or death of a given person, or a certain number of given persons in a given time.

The probability of living two years, expressed by the fraction $\frac{l_{x+2}}{l_x} = \frac{l_x - (d_x + d_{x+1})}{l_x}$, is less than the probability of living one year.

Making n any number of years and fractional parts of years, the fraction $\frac{l_{x+n}}{l_x}$ will invariably express the probability of living n years after the age x. As n approaches zero, the fraction will approximate to 1, the symbol of certainty—thus a person is more likely to live a day than a year, a minute than a day. As n increases, l_{x+n} diminishes in value; and when $x+n$ expresses a year after the age ω in the Life Table, $l_{\omega+1}$ is, by hypothesis, zero, $\therefore \frac{l_{\omega+1}}{l_x} = \frac{0}{l_x} = 0$. The chance of living so long is expressed in this

* The addition of 1 to the numerator, and of 2 to the denominator, may be neglected, when, as in this case, the numbers are large.

case by zero; the chance of dying in the time by 1, the symbol of certainty.

(21.) l_{x+n} expresses the number of chances in favour of surviving n years, and $l_x - l_{x+n}$ the number of chances of dying in the same time—the sum of the two together (l_x) expressing the total number of chances. Thus the fraction $\left(\frac{l_{x+n}}{l_x}\right)$ expressing the probability of living a given time ranges from 1 to 0, and $\frac{l_x - l_{x+n}}{l_x} = 1 - \frac{l_{x+n}}{l_x}$, or the chance of dying in a given time, also ranges from 1 to 0 as n varies. When the two fractions are equal $\frac{l_{x+n}}{l_x} = \frac{l_x - l_{x+n}}{l_x}$, then $l_{x+n} = l_x - l_{x+n}$ and $2l_{x+n} = l_x$, $\therefore l_{x+n} = \frac{l_x}{2}$.

To verify the equations, an age $x+n$ must be chosen at which l_{x+n} is exactly equal to $\frac{1}{2}l_x$. Thus, by the Life Table of healthy districts, 100,000 children born alive are reduced to 50,851 in 58 years, and to 49,895 in 59 years; so the chances are rather in favour of their living 58 years, as they are 50,851 to 49,149; upon the other hand, the chances of their living 59 years (49,895) are less than the chances 50,105 of their dying before attaining that age. Upon trial it will be found that the chances of living to and of dying before $58\frac{851}{956}$ years $= 58 + \frac{50{,}851 - 50{,}000}{d_{58}} = 58 + \frac{851}{956}$ years, or about $58\frac{8}{9}$ years, are nearly equal; hence this is called the *probable lifetime*, or *vie probable* by French writers, for $\frac{l_{58\frac{8}{9}}}{l_0} = \frac{1}{2}$. At the age 20 the probable lifetime is $47\frac{1588}{1638}$, nearly 48 years. The probable lifetime at every age is immediately seen by inspection.

(22.) V. THE THREEFOLD LIFE TABLE—PERSONS, MALES, FEMALES.

The Life Table is threefold: a table having the six columns is made for males, another table is separately made for females; the several columns of the two tables incorporated together form the Table of Persons, which has 100,000, and may have any other number, for its basis. The basis of the Male Table in the illustration is 51,125, while the basis of the Female Table is 48,875. In that proportion males and females were born in the districts. Under this arrangement the number of contemporaneous males and females living at each age in column l_x is shown—thus, 38,388 males and 37,212 females attain the age of 20; 17,145 males

attain the age of 70, and 17,133 females attain the same age. At all ages under 71 the number of males exceeds the females; at the age of 71 and upwards the females exceed the males in number; and upon referring to the columns d_x, it will be seen that the males die off in greater number than females after the age of 42. The age after the second year at which the greatest number of deaths occur is 75 in males, 76 in females.

These numbers all refer to the Life Table for Healthy Districts.

Some of the other properties of the Life Tables, admitting of innumerable applications in the solution of social phenomena, will appear in the following formulæ, which will be found useful in practice.

VI. USEFUL FORMULÆ.

The following formulæ will facilitate the use of the Life Table. The figures must be taken from the Tables of Persons, of Males or Females, applicable to the case. The formulæ are general, and are applicable to any other Life Table.

(23.) $\frac{d_x}{P_x} = m_x =$ the rate of mortality in the year of age following the precise age x.

(24.) $\frac{d_x}{l_x} = \frac{l_x - l_{x+1}}{l_x} = 1 - \frac{l_{x+1}}{l_x} =$ the probability that a person A, of the age x, in average health, will die in the following year.

(25.) $\frac{l_{x+1}}{l_x} = p_x = \frac{l_x - d_x}{l_x} = 1 - \frac{d_x}{l_x} =$ the probability that A, a person of the age x, will live a year; $\therefore 1 - p_x =$ the probability that A, age x, *will die in the year following*, as certainty of life $= 1$.

(26.) $\frac{l_x - l_{x+n}}{l_x} =$ the probability that A, age x, will die in the next n years.

(27.) $\frac{l_{x+n}}{l_x} =$ the probability that A, of age x, will live n years.

(28.) Put $\frac{l_x}{2} = l_{x+n}$, and when l_{x+n} is taken at such an age as to fulfil the conditions of the equation, then n is the *probable lifetime* $=$ *vie probable* $=$ the time that it is an even chance a person of the age x will live.

(29.) $\frac{Q_x}{l_x} = A_x =$ the mean *after-lifetime*, or, as it is often called, the *expectation of life*—an incorrect expression, which is rather applicable to the probable lifetime.

Note.—Upon Demoivre's hypothesis, the *probable lifetime*—that is, the time that a person may fairly expect to live, his *expectation*—was the same as the mean after-lifetime.

(30.) $G_x = x + A_x$ = the mean age at death of persons who have already lived exactly x years.

(31.) $S = c\frac{Q_{x|n}}{l_x}$ = the number of members of any Society between the ages x and $x+n$, which will be permanently sustained by c . . . annual admissions at the age x.

(32.) $c = \frac{Sl_x}{Q_{x|n}}$ = annual recruits of the Society (S).

(33.) $\frac{Sl_{x+n}}{Q_{x|n}}$ = annual members leaving the Society (S) on attaining the age $x+n$.

(34.) $\frac{Sl_{x|n}}{Q_{x|n}}$ = annual deaths in such a Society (S).

(35.) $S\frac{Q_{x+n}}{Q_{x|n}}$ = the aggregate number of persons living who have left such a Society as pensioners or otherwise.

In the following formulæ it is assumed that the population is normally constituted.

(36.) $\frac{Y_x}{Q_x} = A'_x$ = the mean after-lifetime of all persons of the *age x and upwards.*

(37.) $\frac{Y_x - Y_{x+n}}{Q_x - Q_{x+n}} = \frac{Y_{x|n}}{Q_{x|n}}$ = the mean after-lifetime of all persons of the age of x and under the age of $x+n$.

(38.) $c.\frac{Y_{x|n}}{Q_{x|n}}$ = the number of persons of which a Society will *ultimately consist,* recruited by c annual additions of members in the tabular proportions between the age x and $x+n$.

(39.) $c\frac{Y_{x|n} - Y_{x+m|n}}{Q_{x|m}}$ = the number of persons to which a Society joined by c persons of the tabular ages x and under $x+m$ would amount in n years. When $x+n > \omega$, this formula will be reduced to the same form as equation (38); and when $x+m$, as well as $x+n > \omega$, the equation becomes the same as (36).

VII. LIFE TABLE OF THE SIXTY-THREE HEALTHIEST ENGLISH DISTRICTS.

Upon inquiry it was found that in many districts of England the mortality of the population did not exceed the rate of 17 annual deaths to 1,000 living.

For the sake of convenience these were called "healthy districts," consisting of sixty-four, or nearly a tenth part of the total registration districts of England and Wales, and inhabited by nearly a million of people. Sixty-three of these districts have been taken as the basis of the new Life Table, constructed according to the methods previously described.

It will be seen that these districts, generally conterminous with Poor Law Unions, are distributed over the various parts of the country. They comprise:—*Hendon* (with Harrow*) (17), *Lewisham* (17), and *Bromley* (17), in the neighbourhood of London; *Hambledon* (16), *Dorking* (17), *Reigate* (16), and *Godstone* (17), on the southern slope of the Surrey hills; *East Ashford* (17), in East Kent; *Blean* (including Herne Bay) (17), between Canterbury and the sea. Ten districts of Sussex—*Battle* (16), near Hastings; *Eastbourne*, around Beachy Head (15); *Hailsham* (17), *Uckfield* (17), *East Grinstead* (17), *Cuckfield* (16), *Steyning*, near Brighton, (16); *Petworth* (17), *Worthing* (17), and *Midhurst* (17). Seven districts of Hampshire—The *Isle of Wight*, separated from the mainland by the sea (17); *Lymington* (17), *Christchurch* (16), *Ringwood* (17), *New Forest* (17), *Catherington* (17), and *Alresford* (17). *Wokingham* (17), and *Easthampstead* (16), in Berkshire, south of the Thames; *Ongar* (17), in Essex, east of Epping Forest; *Mutford* (17), including Lowestoft, on the Suffolk coast; *Henstead* (17), south of Norwich; *Kingsbridge* (17), on the south coast of Devon; *Okehampton* (16), *Crediton* (17), *Barnstaple* (17), *Torrington* (17), *Bideford* (17), *Holsworthy* (16), stretching from the centre over Dartmouth and Exmoor, along the coast of the Bristol Channel; *Stratton* (17), *Camelford* (17), and *Launceston* (17), in the adjacent parts of Cornwall, and, further south, *St. Columb* (17); *Williton* (17), in Somerset, also on the Bristol Channel; *Winchcomb* (17), to the east of Cheltenham and the Cotswold Hills around the sources of the Thames; *King's Norton* (17), in Worcestershire, adjoining Birmingham; *Melton Mowbray* (17), in Leicestershire; *Southwell* (17), about Sherwood Forest, in the centre of Nottinghamshire; *Garstang* (16), in Lancashire, looking northward over Lancaster Bay; *Easingwold* (17), in the North Riding of Yorkshire; *Guisborough* (16), on the eastern coast north of Whitby. Then follow five border districts of Northumberland on the southern face of the Cheviot Hills—*Belford* (17), *Glendale* (15), *Rothbury*

* The annual deaths to 1,000 living of all ages, inserted in parentheses, are deduced from returns of the living at the Censuses of 1841 and 1851, and the deaths registered in the ten years 1841 to 1850. (*See* Registrar-General's *Sixteenth Report*, pp. 141 to 153.)

(15), *Bellingham* (17), *Haltwhistle* (16) (is omitted in the Table), *Longtown* (17) and *Brampton* (17) on the border, and *Bootle* (16) on the coast of Cumberland; the *East Ward* (17) of Westmoreland; *Haverfordwest* (17), on the western point of South Wales; *Builth* (16), *Corwen* (17), *Pwllheli* (17), on Carnarvon Bay, and *Anglesey* (17) complete the list. These districts, and others nearly equally healthy, have been thus described:—

"Such is the variety of the soil of England, that, tested by the rates of mortality, the children reared out of a given number born, the longevity of the inhabitants, the freedom from common epidemics, or the immunity from cholera, healthy districts are found in nearly every county. Large tracts of country are, however, so much healthier than the rest, that they may be justly called Salubrious Fields; and it is remarkable that here the finest races of animals are bred. The north districts of Northumberland, around the beautiful Cheviot Hills, covered with grasses, ferns, and wild thyme, extending from the region of the heaths to the rich cultivated land at their bases, touching each other or intersected by narrow valleys—the districts extending from the Tees, over the North and East Ridings of York, to Leicestershire, Herefordshire, and parts of Shropshire—some of the districts of Gloucestershire about the Cotswold Hills—parts of Wales—North Devon, including Dartmoor and Exmoor—the Surrey and Sussex hills, with the Southdowns—have given names to the best breeds of sheep, fowls, cattle, and horses in the kingdom. . . . . .

"The dry and most inland are not always the healthiest regions of the country. The salubrious fields are sometimes watered by running streams, and diversified by lakes. The dew is abundant. They are often veiled, not by infectious fogs, but by mists drawn from the sky as it breathes over them. The mountains rise above, the ocean rolls at the distance below them, as on the coast of Sussex, North Devon, the western region of Wales, extending under Snowdon and Cader Idris in a vast amphitheatre round Cardigan Bay—the lake land and moors of the North, rising between the Irish Sea and the German Ocean. The land is sometimes heathy, but may be covered by the sweetest herbage, and bees feeding on the flowers. The cereal grains, the hop, the timber, are often of the finest quality. The animals are healthy, the native breeds are vigorous, and those fine varieties are produced at intervals which men of the genius of Bakewell, Ellman, Tomkins, Colling, and O'Kelly, make the permanent stock of the country. Industry and the army receive their best recruits from the population, while they get their worst from the people of the low parts of sickly towns. Agriculture has reclaimed many unhealthy districts on the plains, so that a considerable extent of the cultivated land is now in a state of comparative salubrity; and vast systems of drainage have subdued the noxious fens, although carried out less efficiently than is desirable, and interfered with by milldams on the rivers, descending like the Nene from the inland high lands."*

The sanitary condition of the people in these districts is, however, still in many respects defective.

* *Report to the Registrar-General on Cholera*, pp. xcv., xcvi.

CONCLUSION.

Halley first pointed out the financial applications of the Life Table, and first calculated the values of life annuities. That branch of science, in the various forms of life insurance, has since received great developments. The new Table shows that the duration of life, among large classes of the population, by no means in unexceptionable sanitary conditions, exceeds the term of the ordinary tables, and proves that life annuities cannot be sold advantageously by Offices, or by the Government, to large classes of lives, for less than the values deducible from the new Table.

A new branch of science has been developed since Halley's day: it is the science of public health—and here a new application of the Life Table is found.

It is probable, upon physiological grounds, that man goes through all the phases of his natural development in a hundred years; and that the period of active life seldom extends beyond eighty years. But this is a very indefinite measure, as the rates of mortality in all the intermediate ages are left undetermined after it has been ascertained in what proportions men attain the extreme limits.

Generations of men, under all circumstances, die at all ages; but the proportions vary indefinitely under different conditions, from a slight tribute to death each year, down to the point of extermination by pestilence. If we ascertain at what rate a generation of men dies away under the least unfavourable existing circumstances, we obtain a standard by which the loss of life, under other circumstances, is measured; and this I have endeavoured to determine in the Life Table of English Healthy Districts. And recollecting that the science of public health was almost inaugurated in England by a former president of this Society,* who encouraged and crowned the sanitary discoveries of Captain Cook, I feel assured that it will receive with favour this imperfect attempt to supply sanitary inquirers with a scientific instrument.

In a subsequent paper I hope to be able to lay before the Society the mortality by different kinds of diseases at each age, as they have been deduced from the same series of observations.

* Sir John Pringle.

HEALTHY DISTRICTS.

TABLE A.—*Population*, 1851—*Deaths in the Five Years* 1849 *to* 1853—*Average Annual Mortality per Cent., and Logarithms of the Mortality.*

AGES.	POPULATION.			DEATHS.			AVERAGE ANNUAL MORTALITY TO 100 LIVING (m).			LOGARITHMS OF THE MORTALITY (λm).		
	Persons.	Males.	Females.	Persons.	Males.	Females.	Persons.	Males.	Females.	Persons.	Males.	Females
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.
All ages ..	996,773	493,525	503,248	87,345	43,736	43,609	1·753	1·772	1·733	$\bar{2}$·2436718	$\bar{2}$·2485599	$\bar{2}$·2388240
Under 5 ..	130,635	65,700	64,935	26,361	14,282	12,079	4·036	4·348	3·720	$\bar{3}$·6059323	$\bar{3}$·6382536	$\bar{3}$·5705821
5–	122,406	61,733	60,673	4,209	2,080	2,129	·688	·674	·702	$\bar{3}$·8374062	$\bar{3}$·8285759	$\bar{3}$·8462102
10–	110,412	56,651	53,761	2,377	1,087	1,290	·431	·384	·480	$\bar{3}$·6340429	$\bar{3}$·5840519	$\bar{3}$·6811523
15–	181,339	90,066	91,273	6,603	3,113	3,490	·728	·691	·765	$\bar{3}$·8622801	$\bar{3}$·8396482	$\bar{3}$·8835130
25–	136,892	65,422	71,470	5,869	2,675	3,194	·857	·818	·894	$\bar{3}$·9332160	$\bar{3}$·9126300	$\bar{3}$·9512411
35–	108,056	52,734	55,322	5,208	2,447	2,761	·964	·928	·998	$\bar{3}$·9840521	$\bar{3}$·9675733	$\bar{3}$·9991965
45–	85,244	42,383	42,861	5,252	2,698	2,554	1·232	1·273	1·192	$\bar{2}$·0906909	$\bar{2}$·1048802	$\bar{2}$·0761886
55–	62,857	31,105	31,752	7,001	3,568	3,433	2·228	2·594	2·162	$\bar{2}$·3478365	$\bar{2}$·3606246	$\bar{2}$·3349327
65–	39,453	18,860	20,593	10,313	5,173	5,140	5·228	5·486	4·992	$\bar{2}$·7183350	$\bar{2}$·7392308	$\bar{2}$·6982734
75–	16,737	7,718	9,019	10,297	4,946	5,351	12·304	12·817	11·866	$\bar{1}$·0900631	$\bar{1}$·1077793	$\bar{1}$·0743066
85–	2,614	1,097	1,517	3,581	1,555	2,026	27·399	28·350	26·711	$\bar{1}$·4377287	$\bar{1}$·4525536	$\bar{1}$·4266538
95 & upwds.	128	56	72	274	112	162	42·813	40·000	45·000	$\bar{1}$·6315706	$\bar{1}$·6020600	$\bar{1}$·6532125

Note.—The ages at death of 146 persons—viz., 123 males and 23 females—were not stated; in calculating the mortality they have been distributed proportionally over the several ages in the Table. The Table may be read thus:—136,892 persons (of whom 65,422 were males, 71,470 were females at the age of 25 and under 35) were enumerated in 1851; at the same ages, 5,869 (2,675 males and 3,194 females) died in the five years 1849 to 1853; consequently the annual rates of mortality per cent. were ·857, ·818, and ·894.

Number of Deaths, at Five Periods of Age, in the Healthy Districts, in 1848 *to* 1855.

YEARS.	AGES.														
	Persons.					Males.					Females.				
	0.	1.	2.	3.	4.	0.	1.	2.	3.	4.	0.	1.	2.	3.	4.
1848	2,935	832	458	371	312	1,678	442	244	204	162	1,257	390	214	167	150
1849	2,932	858	541	427	292	1,637	452	263	207	154	1,295	406	278	220	138
1850	2,969	859	466	331	301	1,676	453	231	164	144	1,293	406	235	167	157
1851	3,185	932	543	341	288	1,769	502	274	179	148	1,416	430	269	162	140
1852	3,405	860	567	389	297	1,913	446	273	206	140	1,492	414	294	183	157
1853	3,370	946	554	376	287	1,888	514	293	179	137	1,482	432	261	197	150
1854	3,404	1,047	601	386	311	1,903	539	317	197	165	1,501	508	284	189	146
1855	3,350	907	533	445	297	1,948	483	257	230	156	1,402	424	276	215	141

Number of Births in Sixty-three Healthy Districts of England, 1848 *to* 1855.

Years.	Persons.	Males.	Females.
1848	28,679	14,756	13,923
1849	29,128	14,751	14,377
1850	29,699	15,176	14,523
1851	30,163	15,465	14,698
1852	30,370	15,557	14,813
1853	29,214	15,010	14,204

Age.	Males.
	2)29,507=births in 1848 and 1849.
0	14,754=births on January 1, 1849.
1	13,117=living on January 1, 1850.
2	12,664=living on January 1, 1851.
3	12,390=living on January 1, 1852.
4	12,184=living on January 1, 1853.
5	12,047=living on January 1, 1854.

Age.	Males.
0	1,637=deaths in 1849.
1	453=deaths in 1850.
2	274=deaths in 1851.
3	206=deaths in 1852.
4	137=deaths in 1853.

TABLE B.—*The several Values of* λp_x *on which the Life Table of Healthy Districts is based; also the corresponding Values of* p_x *and* $(1-p_x)$.

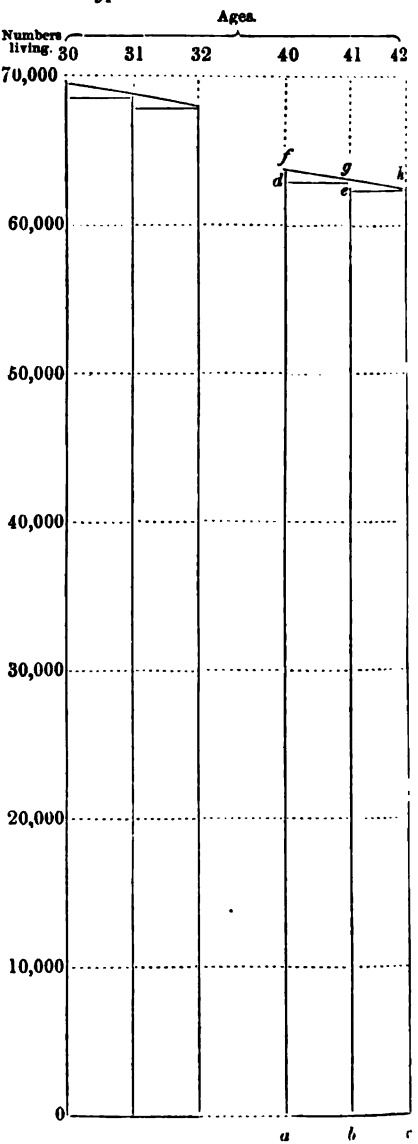
Age x.	λp_x =logarithms of the probability of living one year after the age x.		p_x =probability of *living* a year.		$(1-p_x)$ =probability of *dying* in a year.	
	Males.	Females.	Males.	Females.	Males.	Females.
0	$\bar{1}$·9480215	$\bar{1}$·9577796	·88720	·90736	·11280	·09264
1	$\bar{1}$·9844929	$\bar{1}$·9859276	·96492	·96812	·03508	·03188
2	$\bar{1}$·9904341	$\bar{1}$·9904679	·97821	·97829	·02179	·02171
3	$\bar{1}$·9932422	$\bar{1}$·9932928	·98456	·98467	·01544	·01533
7	$\bar{1}$·9970729	$\bar{1}$·9969512	·99328	·99300	·00672	·00700
12	$\bar{1}$·9984539	$\bar{1}$·9980197	·99645	·99545	·00355	·00455
20	$\bar{1}$·9969724	$\bar{1}$·9966528	·99305	·99232	·00695	·00768
30	$\bar{1}$·9964260	$\bar{1}$·9960967	·99180	·99105	·00820	·00895
40	$\bar{1}$·9959051	$\bar{1}$·9956263	·99062	·98998	·00938	·01002
50	$\bar{1}$·9943048	$\bar{1}$·9946669	·98697	·98780	·01303	·01220
60	$\bar{1}$·9895894	$\bar{1}$·9902049	·97631	·97770	·02369	·02230
70	$\bar{1}$·9751357	$\bar{1}$·9773538	·94436	·94919	·05564	·05081
80	$\bar{1}$·9420680	$\bar{1}$·9463182	·87512	·88373	·12488	·11627
90	$\bar{1}$·8747315	$\bar{1}$·8809176	·74943	·76018	·25057	·23982

Note.—Age x is in this Table the precise age. Age 12 is applied frequently to all persons of the age of 12 and under the age of 13; but in this Table it applies only to persons of the precise age of 12 years, neither more nor less. The λp_7 was, in both cases, derived from the formula $\left(\frac{2-m}{2+m}\right)$. The λp_{12}, deduced from this formula, is for males $\bar{1}$·9983497, and for females $\bar{1}$·9979153, which may be regarded either as the constant or the mean values of λp_{10}, λp_{11}, λp_{12}, λp_{13}, and λp_{14}; but as these are the terminations of an ascending and a descending series, it is probable, and quite in conformity with other observations, that one, two, or more of these

values will exceed the mean value. The logarithms of p_{12} adopted are given above; and the two arithmetical means of the five logarithms, λp_{10}, λp_{11}, λp_{12}, λp_{13}, and λp_{14}, resulting from the interpolation, are $\bar{1}\cdot9983688$ for males, and $\bar{1}\cdot9979435$ for females.

The values of λp_{20}, λp_{30} . . . . . are derived from the formula $y_x = 10^{\frac{\lambda^2 m}{\lambda r}(1-r^x)}$.

Note on the two Hypotheses.



Let b be the decrement of the ordinate y in a unit of time, then the decrement Δy of the ordinate in the time x, represented by the abscissa, will be $\Delta y = -bx$, on Demoivre's hypothesis; and as it is always proportional to the time, it will be, in an infinitely short time, $dy = -bdx$.

Passing to the integral $y = c - bx$; and if $y = a$ at the origin, when $x = 0$, $c = a$, $\therefore y = a - bx$; and if $b = 1$, then $y = a - x$. This evidently represents very closely short portions of the Life Table curve; and the smaller x is taken, the nearer is the approximation to the corresponding value of y.

Again: let Δy be the decrement of the ordinate y in the indefinite time Δx, represented by the abscissa; and let the mortality (m), represented by the ratio of the area $abfg$ to the area dfg, be $\frac{d_0}{P_0} = m_0$; let also m_0 increase at the rate r in a unit of time, so that $\frac{geh}{bcgh} = \frac{d_1}{P_1} = m_1 = m_0 r$, and generally, within given limits, $m_0 r^x = m_x$; then $\Delta y = -y m_x \Delta x$ nearly, Δx being any small portion of time.

The error increases as the time Δx is extended, from the circumstance that on the one hand m_x varies by hypothesis momentarily, and that y, from which the varying proportional part is taken, constantly grows shorter. But by passing to the

limit, and making the time dx infinitely short, m_x and y during that infinitely short time may be considered constant, and $dy=-ym_x dx$ will be the true decrement. Substituting $m_0 r^x$ for m_x, the equation becomes $dy=-ym_0 r^x dx$, from which the value of y can be derived, as before shown; for $\frac{dy}{y}=-m_0 r^x dx$, and, integrating both sides, $\lambda_\epsilon y=\lambda_\epsilon c-\frac{m_0 r^x}{\lambda_\epsilon r}$; here λ_ϵ stands for the logarithm having ϵ for its base.

At the origin of the curve, when $x=0$, let $y=1$, and then $\lambda_\epsilon c=\frac{m_0}{\lambda_\epsilon r}$. Now, substituting this value for $\lambda_\epsilon c$, we have $\lambda_\epsilon y=\frac{m_0}{\lambda_\epsilon r}-\frac{m_0 r^x}{\lambda_\epsilon r}$, $\therefore \lambda_\epsilon y=\frac{m_0}{\lambda_\epsilon r}(1-r^x)$; and, passing to the number, $y=\epsilon^{\frac{m_0}{\lambda_\epsilon r}(1-r^x)}$. Putting k for the modulus of the common logarithm (λ) having 10 for its base, we have $\lambda_\epsilon y=\frac{\lambda y}{k}$, and $\lambda_\epsilon r=\frac{\lambda r}{k}$, $\therefore \frac{\lambda y}{k}=\frac{km}{\lambda r}(1-r^x)$; or, passing to the number, $y=10^{\frac{km}{\lambda r}(1-r^x)}$

Upon the one hypothesis, out of a generation of men an *equal quantity of life** is destroyed in equal times, out of diminishing quantities in existence, the *proportion* that perishes of the residual life constantly *increasing*.

Upon the other hypothesis, a *decreasing proportion* of the residual life is destroyed from birth down to the age of puberty; in the after ages, a *proportion increasing* at different rates is destroyed in equal times. The *quantity* of life *destroyed* in equal times may be the same, or different upon this hypothesis; and in very short intervals of age the differences between *the quantities of life destroyed* may be so inconsiderable that they may be neglected.

The two hypotheses may be illustrated. Assume that at every beat of the heart an equal quantity of vital force on an average is consumed in excess of that produced; or if this does not happen at distant ages, assume that it happens during two consecutive years, two consecutive days, two consecutive pulses of a generation of men, and is represented by the deaths in the two intervals. This will give an idea of the first hypothesis.

The second hypothesis will be represented by assuming that, in addition to the existing force, a certain amount of vital force is produced, while a certain amount is also destroyed at every beat of the heart—the quantity destroyed exceeding the quantity produced in a diminishing ratio, and then in an increasing ratio—the proportional part destroyed being for this purpose always represented by the proportional number of hearts beating to the number of hearts ceasing to beat at every instant of age among a generation of men. The respirations, the sensations, the secretions, nutrition, and all the vital acts, may be conceived, like the heart, to influence the continuance of the vital force, implying here simply the force which sustains life.

* The quality or the intensity of life at different ages is purposely left out of consideration.

TABLE B 1.—*Life Table of Healthy English Districts.—Logarithms of the Numbers of Males and Females living at each Year of Age.*

λl_x.				λl_x.			
Age. x.	Males.	Age x.	Females.	Age x.	Males.	Age x.	Females.
0	4·7086364	0	4·6890835	55	4·4351998	55	4·4177773
1	4·6566579	1	4·6468631	56	4·4279544	56	4·4116015
2	4·6411508	2	4·6327907	57	4·4203212	57	4·4052190
3	4·6315849	3	4·6232586	58	4·4122719	58	4·3981522
4	4·6248271	4	4·6165514	59	4·4037768	59	4·3901691
5	4·6193109	5	4·6110606	60	4·3943905	60	4·3812819
6	4·6148376	6	4·6065737	61	4·3839799	61	4·3714868
7	4·6112225	7	4·6028950	62	4·3725154	62	4·3607637
8	4·6082954	8	4·5998462	63	4·3599518	63	4·3490765
9	4·6059001	9	4·5972658	64	4·3462281	64	4·3363727
10	4·6038946	10	4·5950094	65	4·3312678	65	4·3225837
11	4·6021511	11	4·5929497	66	4·3149786	66	4·3076249
12	4·6005560	12	4·5909763	67	4·2972528	67	4·2913951
13	4·5990100	13	4·5889960	68	4·2779668	68	4·2737774
14	4·5974279	14	4·5869326	69	4·2569814	69	4·2546384
15	4·5957387	15	4·5847269	70	4·2341418	70	4·2338287
16	4·5938855	16	4·5823368	71	4·2092775	71	4·2111825
17	4·5918259	17	4·5797373	72	4·1822024	72	4·1865180
18	4·5895314	18	4·5769202	73	4·1527146	73	4·1596372
19	4·5869878	19	4·5738947	74	4·1205968	74	4·1303259
20	4·5841951	20	4·5706868	75	4·0856157	75	4·0983537
21	4·5811675	21	4·5673396	76	4·0475228	76	4·0634741
22	4·5780527	22	4·5639166	77	4·0060534	77	4·0254242
23	4·5748607	23	4·5604237	78	3·9609277	78	3·9839252
24	4·5716008	24	4·5568665	79	3·9118498	79	3·9386819
25	4·5682808	25	4·5532498	80	3·8585083	80	3·8893831
26	4·5649078	26	4·5495779	81	3·8005763	81	3·8357013
27	4·5614874	27	4·5458546	82	3·7377111	82	3·7772929
28	4·5580244	28	4·5420830	83	3·6695542	83	3·7137979
29	4·5545223	29	4·5382656	84	3·5957318	84	3·6448405
30	4·5509835	30	4·5344046	85	3·5158541	85	3·5700284
31	4·5474095	31	4·5305013	86	3·4295159	86	3·4889532
32	4·5438005	32	4·5265566	87	3·3362962	87	3·4011904
33	4·5401557	33	4·5225708	88	3·2357583	88	3·3062992
34	4·5364730	34	4·5185435	89	3·1274500	89	3·2038228
35	4·5327494	35	4·5144739	90	3·0109034	90	3·0932880
36	4·5289808	36	4·5103606	91	2·8856349	91	2·9742056
37	4·5251620	37	4·5062016	92	2·7511453	92	2·8460701
38	4·5212864	38	4·5019942	93	2·6069196	93	2·7083599
39	4·5173467	39	4·4977353	94	2·4524273	94	2·5605372
40	4·5133342	40	4·4934212	95	2·2871223	95	2·4020479
41	4·5092393	41	4·4890475	96	2·1104426	96	2·2323219
42	4·5050512	42	4·4846093	97	1·9218108	97	2·0507729
43	4·5007579	43	4·4801012	98	1·7206337	98	1·6567982
44	4·4963465	44	4·4755172	99	1·5063024	99	1·6497793
45	4·4918029	45	4·4708506	100	1·2781926	100	1·4290811
46	4·4871119	46	4·4660943	101	1·0356640	101	1·1940526
47	4·4822570	47	4·4612404	102	0·7780608	102	0·9440265
48	4·4772210	48	4·4562807	103	0·5047118	103	0·6783194
49	4·4719852	49	4·4512061	104	0·2149296	104	0·3962318
50	4·4665301	50	4·4460074	105	9·9080117	105	0·0970476
51	4·4608349	51	4·4406743	106	9·5832396	106	9·7800351
52	4·4548778	52	4·4351962	107	9·2398792	107	9·4444460
53	4·4486358	53	4·4295620	108	8·8771808	108	9·0895160
54	4·4420848	54	4·4237598	109	8·4943792	109	8·7144646

TABLE C.—*Healthy Districts.*

Age.	Living at each Age (l_x).			Dying in each Year of Age (d_x).			Age.
x.	Persons.	Males.	Females.	Persons.	Males.	Females.	x.
0	100,000	51,125	48,875	10,295	5,767	4,528	0
1	89,705	45,358	44,347	3,005	1,591	1,414	1
2	86,700	43,767	42,933	1,885	953	932	2
3	84,815	42,814	42,001	1,305	661	644	3
4	83,510	42,153	41,357	1,051	532	519	4
5	82,459	41,621	40,838	847	427	420	5
6	81,612	41,194	40,418	682	341	341	6
7	80,930	40,853	40,077	555	275	280	7
8	80,375	40,578	39,797	459	223	236	8
9	79,916	40,355	39,561	391	186	205	9
10	79,525	40,169	39,356	347	161	186	10
11	79,178	40,008	39,170	324	146	178	11
12	78,854	39,862	38,992	319	142	177	12
13	78,535	39,720	38,815	328	144	184	13
14	78,207	39,576	38,631	350	154	196	14
15	77,857	39,422	38,435	379	168	211	15
16	77,478	39,254	38,224	414	186	228	16
17	77,064	36,068	37,996	451	205	246	17
18	76,613	38,863	37,750	489	227	262	18
19	76,124	38,636	37,488	524	248	276	19
20	75,600	38,388	37,212	552	267	285	20
21	75,048	38,121	36,927	562	272	290	21
22	74,486	37,849	36,637	571	277	294	22
23	73,915	37,572	36,343	577	281	296	23
24	73,338	37,291	36,047	583	284	299	24
25	72,755	37,007	35,748	588	287	301	25
26	72,167	36,720	35,447	591	288	303	26
27	71,576	36,432	35,144	593	289	304	27
28	70,983	36,143	34,840	595	290	305	28
29	70,388	35,853	34,535	596	291	305	29
30	69,792	35,562	34,230	598	292	306	30
31	69,194	35,270	33,924	599	292	307	31
32	68,595	34,978	33,617	599	292	307	32
33	67,996	34,686	33,310	601	293	308	33
34	67,395	34,393	33,002	601	293	308	34
35	66,794	34,100	32,694	603	295	308	35
36	66,191	33,805	32,386	604	296	308	36
37	65,587	33,509	32,078	608	298	310	37
38	64,979	33,211	31,768	610	300	310	38
39	64,369	32,911	31,458	613	302	311	39
40	63,756	32,609	31,147	618	306	312	40
41	63,138	32,303	30,835	623	310	313	41
42	62,515	31,993	30,522	630	315	315	42
43	61,885	31,678	30,207	638	320	318	43
44	61,247	31,358	29,889	645	326	319	44
45	60,602	31,032	29,570	656	334	322	45
46	59,946	30,698	29,248	666	341	325	46
47	59,280	30,357	28,923	679	350	329	47
48	58,601	30,007	28,594	692	360	332	48
49	57,909	29,647	28,262	706	370	336	49
50	57,203	29,277	27,926	722	381	341	50
51	56,481	28,896	27,585	740	394	346	51
52	55,741	28,502	27,239	758	407	351	52
53	54,983	28,095	26,888	777	420	357	53

TABLE C (*continued*).

Age.	Living at each Age (l_x).			Dying in each Year of Age (d_x).			Age.
x.	Persons.	Males.	Females.	Persons.	Males.	Females.	x.
54	54,206	27,675	26,531	798	435	363	54
55	53,408	27,240	26,168	820	451	369	55
56	52,588	26,789	25,799	843	467	376	56
57	51,745	26,322	25,423	894	483	411	57
58	50,851	25,839	25,012	956	501	455	58
59	49,895	25,338	24,557	1,040	542	498	59
60	48,855	24,796	24,059	1,123	587	536	60
61	47,732	24,209	23,523	1,205	631	574	61
62	46,527	23,578	22,949	1,281	672	609	62
63	45,246	22,906	22,340	1,356	712	644	63
64	43,890	22,194	21,696	1,430	752	678	64
65	42,460	21,442	21,018	1,501	789	712	65
66	40,959	20,653	20,306	1,571	826	745	66
67	39,388	19,827	19,561	1,638	861	777	67
68	37,750	18,966	18,784	1,705	895	810	68
69	36,045	18,071	17,974	1,767	926	841	69
70	34,278	17,145	17,133	1,825	954	871	70
71	32,453	16,191	16,262	1,876	978	898	71
72	30,577	15,213	15,364	1,921	999	922	72
73	28,656	14,214	14,442	1,955	1,013	942	73
74	26,701	13,201	13,500	1,980	1,022	958	74
75	24,721	12,179	12,542	1,991	1,023	968	75
76	22,730	11,156	11,574	1,987	1,016	971	76
77	20,743	10,140	10,603	1,966	1,000	966	77
78	18,777	9,140	9,637	1,931	977	954	78
79	16,846	8,163	8,683	1,875	943	932	79
80	14,971	7,220	7,751	1,803	902	901	80
81	13,168	6,318	6,850	1,713	851	862	81
82	11,455	5,467	5,988	1,608	794	814	82
83	9,847	4,673	5,174	1,491	731	760	83
84	8,356	3,942	4,414	1,360	662	698	84
85	6,996	3,280	3,716	1,224	591	633	85
86	5,772	2,689	3,083	1,084	520	564	86
87	4,688	2,169	2,519	943	448	495	87
88	3,745	1,721	2,024	805	380	425	88
89	2,940	1,341	1,599	675	316	359	89
90	2,265	1,025	1,240	555	257	298	90
91	1,710	768	942	444	204	240	91
92	1,266	564	702	350	159	191	92
93	916	405	511	269	122	147	93
94	647	283	364	201	89	112	94
95	446	194	252	146	65	81	95
96	300	129	171	104	45	59	96
97	196	84	112	71	31	40	97
98	125	53	72	48	21	27	98
99	77	32	45	31	13	18	99
100	46	19	27	19	8	11	100
101	27	11	16	12	5	7	101
102	15	6	9	7	3	4	102
103	8	3	5	4	1	3	103
104	4	2	2	2	1	1	104
105	2	1	1	1	1	..	105
106	1		1	1	.	1	106

TABLE D.—*Healthy Districts.—Persons.*

Age.	Dying in each Year of Age, 0–1, 1–2, to 105–106.	Born and Living at each Age.	Sum of the Numbers Born and Living at each Age (x) from x to the last Age in the Table.	Population, or the living in each Year of Age 0 to 1, 1 to 2, &c.	(1) Sum of the Living, and of the Living of every Age (x) and upwards to the last Age in the Table; also (2) the Years which the Persons (l_x) will live.	(1) The Years which the Persons at the Age (x) and upwards will live; also (2) the Years which they have lived over x.	Age.
		Σd_x.	Σl_x.	$\frac{1}{2}(l_x + l_{x+1}) = l_{x+1} + \frac{1}{2}d_x$.	ΣP_x	$\Sigma\frac{1}{2}(Q_x + Q_{x+1}) = Y_{x+1} + (Q_{x+1} + \frac{1}{2}P_x)$.	
x.	d_x.	l_x.	L_x.	P_x.	Q_x.	Y_x.	x.
0	10,295	100,000	4,951,908	92,611	4,899,665	166,209,701	0
1	3,005	89,705	4,851,908	88,202	4,807,054	161,356,341	1
2	1,885	86,700	4,762,203	85,758	4,718,852	156,593,388	2
3	1,305	84,815	4,675,503	84,162	4,633,094	151,917,415	3
4	1,051	83,510	4,590,688	82,985	4,548,932	147,326,402	4
5	847	82,459	4,507,178	82,036	4,465,947	142,818,963	5
6	682	81,612	4,424,719	81,270	4,383,911	138,394,034	6
7	555	80,930	4,343,107	80,653	4,302,641	134,050,757	7
8	459	80,375	4,262,177	80,145	4,221,988	129,788,443	8
9	391	79,916	4,181,802	79,721	4,141,843	125,606,527	9
10	347	79,525	4,101,886	79,352	4,062,122	121,504,545	10
11	324	79,178	4,022,361	79,016	3,982,770	117,482,099	11
12	319	78,854	3,943,183	78,694	3,903,754	113,538,837	12
13	328	78,535	3,864,329	78,371	3,825,060	109,674,430	13
14	350	78,207	3,785,794	78,032	3,746,689	105,888,556	14
15	379	77,857	3,707,587	77,668	3,668,657	102,180,882	15
16	414	77,478	3,629,730	77,271	3,590,989	98,551,059	16
17	451	77,064	3,552,252	76,838	3,513,718	94,998,706	17
18	489	76,613	3,475,188	76,369	3,436,880	91,523,407	18
19	524	76,124	3,398,575	75,862	3,360,511	88,124,711	19
20	552	75,600	3,322,451	75,323	3,284,649	84,802,131	20
21	562	75,048	3,246,851	74,767	3,209,326	81,555,144	21
22	571	74,486	3,171,803	74,201	3,134,559	78,383,202	22
23	577	73,915	3,097,317	73,626	3,060,358	75,285,743	23
24	583	73,338	3,023,402	73,047	2,986,732	72,262,198	24
25	588	72,755	2,950,064	72,461	2,913,685	69,311,989	25
26	591	72,167	2,877,309	71,872	2,841,224	66,434,535	26
27	593	71,576	2,805,142	71,279	2,769,352	63,629,247	27
28	595	70,983	2,733,566	70,685	2,698,073	60,895,535	28
29	596	70,388	2,662,583	70,091	2,627,388	58,232,804	29
30	598	69,792	2,592,195	69,493	2,557,297	55,640,462	30
31	599	69,194	2,522,403	68,894	2,487,804	53,117,911	31
32	599	68,595	2,453,209	68,296	2,418,910	50,664,554	32
33	601	67,996	2,384,614	67,695	2,350,614	48,279,792	33
34	601	67,395	2,316,618	67,095	2,282,919	45,963,025	34
35	603	66,794	2,249,223	66,492	2,215,824	43,713,654	35
36	604	66,191	2,182,429	65,889	2,149,332	41,531,076	36
37	608	65,587	2,116,238	65,283	2,083,443	39,414,688	37
38	610	64,979	2,050,651	64,674	2,018,160	37,363,887	38
39	613	64,369	1,985,672	64,062	1,953,486	35,378,064	39
40	618	63,756	1,921,303	63,447	1,889,424	33,456,609	40
41	623	63,138	1,857,547	62,827	1,825,977	31,598,909	41
42	630	62,515	1,794,409	62,200	1,763,150	29,804,345	42
43	638	61,885	1,731,894	61,566	1,700,950	28,072,295	43
44	645	61,247	1,670,009	60,925	1,639,384	26,402,128	44
45	656	60,602	1,608,762	60,274	1,578,459	24,793,206	45
46	666	59,946	1,548,160	59,612	1,518,185	23,244,885	46
47	679	59,280	1,488,214	58,941	1,458,573	21,756,505	47
48	692	58,601	1,428,934	58,255	1,399,632	20,327,403	48
49	706	57,909	1,370,333	57,556	1,341,377	18,956,899	49
50	722	57,203	1,312,424	56,842	1,283,821	17,644,300	50
51	740	56,481	1,255,221	56,111	1,226,979	16,388,899	51
52	758	55,741	1,198,740	55,362	1,170,868	15,189,976	52
53	777	54,983	1,142,999	54,594	1,115,506	14,046,789	53

TABLE D (*continued*).

Age.	Dying in each Year of Age, 0–1, 1–2, to 105–106.	Born and Living at each Age.	Sum of the Numbers Born and Living at each Age (x) from x to the last Age in the Table.	Population, or the Living in each Year of Age 0 to 1, 1 to 2, &c.	(1) Sum of the Living, and of the Living of every Age (x) and upwards to the last Age in the Table; also (2) the Years which the Persons (L_x) will live.	(1) The Years which the Persons at the Age (x) and upwards will live; also (2) the Years which they have lived over x.	Age.
		Σd_x.	Σl_x.	$\frac{1}{2}(l_x + l_{x+1}) = l_{x+1} + \frac{1}{2}d_x$.	ΣP_x.	$\Sigma\frac{1}{2}(Q_x + Q_{x+1}) = Y_{x+1} + (Q_{x+1} + \frac{1}{2}P_x)$.	
x.	d_x.	l_x.	L_x.	P_x.	Q_x.	Y_x.	x.
54	798	54,206	1,088,016	53,808	1,060,912	12,958,580	54
55	820	53,408	1,033,810	52,997	1,007,104	11,924,572	55
56	843	52,588	980,402	52,167	954,107	10,943,966	56
57	894	51,745	927,814	51,298	901,940	10,015,943	57
58	956	50,851	876,069	50,373	850,642	9,139,652	58
59	1,040	49,895	825,218	49,375	800,269	8,314,197	59
60	1,123	48,855	775,323	48,293	750,894	7,538,615	60
61	1,205	47,732	726,468	47,130	702,601	6,811,867	61
62	1,281	46,527	678,736	45,887	655,471	6,132,831	62
63	1,356	45,246	632,209	44,568	609,584	5,500,304	63
64	1,430	43,890	586,963	43,175	565,016	4,913,004	64
65	1,501	42,460	543,073	41,709	521,841	4,369,575	65
66	1,571	40,959	500,613	40,173	480,132	3,868,589	66
67	1,638	39,388	459,654	38,570	439,959	3,408,544	67
68	1,705	37,750	420,266	36,897	401,389	2,987,869	68
69	1,767	36,045	382,516	35,161	364,492	2,604,929	69
70	1,825	34,278	346,471	33,366	329,331	2,258,017	70
71	1,876	32,453	312,193	31,515	295,965	1,945,369	71
72	1,921	30,577	279,740	29,617	264,450	1,665,162	72
73	1,955	28,656	249,163	27,678	234,833	1,415,520	73
74	1,980	26,701	220,507	25,711	207,155	1,194,527	74
75	1,991	24,721	193,806	23,726	181,444	1,000,227	75
76	1,987	22,730	169,085	21,736	157,718	830,646	76
77	1,966	20,743	146,355	19,760	135,982	683,796	77
78	1,931	18,777	125,612	17,811	116,222	557,694	78
79	1,875	16,846	106,835	15,909	98,411	450,377	79
80	1,803	14,971	89,989	14,070	82,502	359,921	80
81	1,713	13,168	75,018	12,311	68,432	284,454	81
82	1,608	11,455	61,850	10,651	56,121	222,178	82
83	1,491	9,847	50,395	9,102	45,470	171,382	83
84	1,360	8,356	40,548	7,676	36,368	130,463	84
85	1,224	6,996	32,192	6,383	28,692	97,933	85
86	1,084	5,772	25,196	5,230	22,309	72,432	86
87	943	4,688	19,424	4,217	17,079	52,739	87
88	805	3,745	14,736	3,342	12,862	37,768	88
89	675	2,940	10,991	2,603	9,520	26,577	89
90	555	2,265	8,051	1,988	6,917	18,358	90
91	444	1,710	5,786	1,488	4,929	12,436	91
92	350	1,266	4,076	1,090	3,441	8,251	92
93	269	916	2,810	782	2,351	5,355	93
94	201	647	1,894	547	1,569	3,395	94
95	146	446	1,247	372	1,022	2,099	95
96	104	300	801	249	650	1,263	96
97	71	196	501	160	401	737	97
98	48	125	305	101	241	416	98
99	31	77	180	61	140	226	99
100	19	46	103	37	79	116	100
101	12	27	57	21	42	56	101
102	7	15	30	11	21	24	102
103	4	8	15	7	10	9	103
104	2	4	7	2	3	3	104
105	1	2	3	1	1	.	105
106	1	1	1	.	..	.	106

TABLE E.—*Healthy Districts.—Males.*

Age.	Dying in each Year of Age 0–1, 1–2, to 104–105.	Born and Living at each Age.	Sum of the Numbers Born and Living at each Age (x) from x to the last Age in the Table.	Population, or the Living in each Year of Age 0 to 1, 1 to 2, &c.	(1) Sum of the Living, and of the Living of every Age (x) and upwards to the last Age in the Table; also (2) the Years which the Males (l_x) will live.	(1) The Years which the Males at the Age (x) and upwards will live; also (2) the Years which they have lived over x.	Age.
		Σd_x.	Σl_x.	$\frac{1}{2}(l_x + l_{x+1}) = l_{x+1} + \frac{1}{2}d_x$.	ΣP_x.	$\Sigma\frac{1}{2}(Q_x + Q_{x+1}) = Y_{x+1} + (Q_{x+1} + \frac{1}{2}P_x)$.	
x.	d_x.	l_x.	L_x.	P_x.	Q_x.	Y_x.	x.
0	5,767	51,125	2,509,635	46,915*	2,482,745	84,008,921	0
1	1,591	45,358	2,458,510	44,562	2,435,830	81,549,633	1
2	953	43,767	2,413,152	43,291	2,391,268	79,136,084	2
3	661	42,814	2,369,385	42,483	2,347,977	76,766,462	3
4	532	42,153	2,326,571	41,887	2,305,494	74,439,726	4
5	427	41,621	2,284,418	41,408	2,263,607	72,155,176	5
6	341	41,194	2,242,797	41,023	2,222,199	69,912,273	6
7	275	40,853	2,201,603	40,716	2,181,176	67,710,585	7
8	223	40,578	2,160,750	40,466	2,140,460	65,549,767	8
9	186	40,355	2,120,172	40,262	2,099,994	63,429,540	9
10	161	40,169	2,079,817	40,089	2,059,732	61,349,677	10
11	146	40,008	2,039,648	39,935	2,019,643	59,309,990	11
12	142	39,862	1,999,640	39,791	1,979,708	57,310,314	12
13	144	39,720	1,959,778	39,648	1,939,917	55,350,502	13
14	154	39,576	1,920,058	39,499	1,900,269	53,430,409	14
15	168	39,422	1,880,482	39,338	1,860,770	51,549,889	15
16	186	39,254	1,841,060	39,161	1,821,432	49,708,788	16
17	205	39,068	1,801,806	38,965	1,782,271	47,906,937	17
18	227	38,863	1,762,738	38,750	1,743,306	46,144,148	18
19	248	38,636	1,723,875	38,512	1,704,556	44,420,217	19
20	267	38,388	1,685,239	38,254	1,666,044	42,734,917	20
21	272	38,121	1,646,851	37,985	1,627,790	41,088,000	21
22	277	37,849	1,608,730	37,711	1,589,805	39,479,203	22
23	281	37,572	1,570,881	37,431	1,552,094	37,908,253	23
24	284	37,291	1,533,309	37,149	1,514,663	36,374,875	24
25	287	37,007	1,496,018	36,864	1,477,514	34,878,786	25
26	288	36,720	1,459,011	36,576	1,440,650	33,419,704	26
27	289	36,432	1,422,291	36,287	1,404,074	31,997,342	27
28	290	36,143	1,385,859	35,998	1,367,787	30,611,412	28
29	291	35,853	1,349,716	35,708	1,331,789	29,261,624	29
30	292	35,562	1,313,863	35,416	1,296,081	27,947,689	30
31	292	35,270	1,278,301	35,124	1,260,665	26,669,316	31
32	292	34,978	1,243,031	34,832	1,225,541	25,426,213	32
33	293	34,686	1,208,053	34,539	1,190,709	24,218,088	33
34	293	34,393	1,173,367	34,247	1,156,170	23,044,648	34
35	295	34,100	1,138,974	33,952	1,121,923	21,905,602	35
36	296	33,805	1,104,874	33,657	1,087,971	20,800,655	36
37	298	33,509	1,071,069	33,360	1,054,314	19,729,512	37
38	300	33,211	1,037,560	33,061	1,020,954	18,691,878	38
39	302	32,911	1,004,349	32,760	987,893	17,687,455	39
40	306	32,609	971,438	32,456	955,133	16,715,942	40
41	310	32,303	938,829	32,148	922,677	15,777,037	41
42	315	31,993	906,526	31,836	890,529	14,870,434	42
43	320	31,678	874,533	31,518	858,693	13,995,823	43
44	326	31,358	842,855	31,195	827,175	13,152,889	44
45	334	31,032	811,497	30,865	795,980	12,341,311	45
46	341	30,698	780,465	30,527	765,115	11,560,764	46
47	350	30,357	749,767	30,182	734,588	10,810,912	47
48	360	30,007	719,410	29,827	704,406	10,091,415	48
49	370	29,647	689,403	29,462	674,579	9,401,923	49
50	381	29,277	659,756	29,087	645,117	8,742,075	50
51	394	28,896	630,479	28,699	616,030	8,111,501	51

* P_0 is $\frac{1}{2}(l_0+l_1)\times(\cdot 9725)$. The factor ·9725 has been introduced, as the number living in the first year is less than the arithmetical mean of those born and surviving a year.

TABLE E (*continued*).

Age.	Dying in each Year of Age 0–1, 1–2, to 104–105.	Born and Living at each Age.	Sum of the Numbers Born and Living at each Age (x) from x to the last Age in the Table.	Population, or the Living in each Year of Age 0 to 1, 1 to 2, &c.	(1) Sum of the Living, and of the Living of every Age (x) and upwards to the last Age in the Table; also (2) the Years which the Males (l_x) will live.	(1) The Years which the Males at the Age x and upwards will live; also (2) the Years which they have lived over x.	Age.
		Σd_x.	Σl_x.	$\frac{1}{2}(l_x + l_{x+1}) = l_{x+1} + \frac{1}{2}d_x$.	ΣP_x.	$\Sigma\frac{1}{2}(Q_x + Q_{x+1}) = Y_{x+1} + (Q_{x+1} + \frac{1}{2}P_x)$.	
x.	d_x.	l_x.	L_x.	P_x.	Q_x.	Y_x.	x.
52	407	28,502	601,583	28,298	587,331	7,509,821	52
53	420	28,095	573,081	27,885	559,033	6,936,639	53
54	435	27,675	544,986	27,458	531,148	6,391,548	54
55	451	27,240	517,311	27,014	503,690	5,874,129	55
56	467	26,789	490,071	26,556	476,676	5,383,946	56
57	483	26,322	463,282	26,080	450,120	4,920,548	57
58	501	25,839	436,960	25,589	424,040	4,483,468	58
59	542	25,338	411,121	25,067	398,451	4,072,223	59
60	587	24,796	385,783	24,502	373,384	3,686,305	60
61	631	24,209	360,987	23,894	348,882	3,325,172	61
62	672	23,578	336,778	23,242	324,988	2,988,237	62
63	712	22,906	313,200	22,550	301,746	2,674,870	63
64	752	22,194	290,294	21,818	279,196	2,384,399	64
65	789	21,442	268,100	21,047	257,378	2,116,112	65
66	826	20,653	246,658	20,240	236,331	1,869,258	66
67	861	19,827	226,005	19,397	216,091	1,643,047	67
68	895	18,966	206,178	18,518	196,694	1,436,654	68
69	926	18,071	187,212	17,608	178,176	1,249,219	69
70	954	17,145	169,141	16,668	160,568	1,079,847	70
71	978	16,191	151,996	15,702	143,900	927,613	71
72	999	15,213	135,805	14,714	128,198	791,564	72
73	1,013	14,214	120,592	13,707	113,484	670,723	73
74	1,022	13,201	106,378	12,690	99,777	564,093	74
75	1,023	12,179	93,177	11,668	87,087	470,661	75
76	1,016	11,156	80,998	10,648	75,419	389,408	76
77	1,000	10,140	69,842	9,640	64,771	319,313	77
78	977	9,140	59,702	8,651	55,131	259,362	78
79	943	8,163	50,562	7,692	46,480	208,556	79
80	902	7,220	42,399	6,769	38,788	165,922	80
81	851	6,318	35,179	5,892	32,019	130,519	81
82	794	5,467	28,861	5,070	26,127	101,446	82
83	731	4,673	23,394	4,308	21,057	77,854	83
84	662	3,942	18,721	3,611	16,749	58,951	84
85	591	3,280	14,779	2,984	13,138	44,007	85
86	520	2,689	11,499	2,429	10,154	32,361	86
87	448	2,169	8,810	1,945	7,725	23,422	87
88	380	1,721	6,641	1,531	5,780	16,669	88
89	316	1,341	4,920	1,183	4,249	11,655	89
90	257	1,025	3,579	897	3,066	7,997	90
91	204	768	2,554	666	2,169	5,380	91
92	159	564	1,786	484	1,503	3,544	92
93	122	405	1,222	344	1,019	2,283	93
94	89	283	817	239	675	1,436	94
95	65	194	534	161	436	880	95
96	45	129	340	107	275	525	96
97	31	84	211	68	168	303	97
98	21	53	127	43	100	169	98
99	13	32	74	25	57	91	99
100	8	19	42	15	32	46	100
101	5	11	23	9	17	22	101
102	3	6	12	4	8	9	102
103	1	3	6	3	4	3	103
104	1	2	3	1	1	1	104
105	1	1	1	..	..	..	105
			..		..	..	106

TABLE F.—*Healthy Districts.—Females.*

Age.	Dying in each Year of Age 0–1, 1–2, to 105–106.	Born and Living at each Age.	Sum of the Numbers Born and Living at each Age (x) from x to the last Age in the Table.	Population, or the Living in each Year of Age 0 to 1, 1 to 2, &c.	(1) Sum of the Living, and of the Living of every Age (x) and upwards to the last Age in the Table; also (2) the Years which the Females (l_x) will live.	(1) The Years which the Females at the Age (x) and upwards will live; also (2) the Years which they have lived over x.	Age.
		Σd_x.	Σl_x.	$\frac{1}{2}(l_x + l_{x+1}) = l_{x+1} + \frac{1}{2}d_x$.	ΣP_x.	$\Sigma\frac{1}{2}(Q_x + Q_{x+1}) = Y_{x+1} + (Q_{x+1} + \frac{1}{2}P_x)$.	
x.	d_x.	l_x.	L_x.	P_x.	Q_x.	Y_x.	x.
0	4,528	48,875	2,442,273	45,696*	2,416,920	82,200,780	0
1	1,414	44,347	2,393,398	43,640	2,371,224	79,806,708	1
2	932	42,933	2,349,051	42,467	2,327,584	77,457,304	2
3	644	42,001	2,306,118	41,679	2,285,117	75,150,953	3
4	519	41,357	2,264,117	41,098	2,243,438	72,886,676	4
5	420	40,838	2,222,760	40,628	2,202,340	70,663,787	5
6	341	40,418	2,181,922	40,247	2,161,712	68,481,761	6
7	280	40,077	2,141,504	39,937	2,121,465	66,340,172	7
8	236	39,797	2,101,427	39,679	2,081,528	64,238,676	8
9	205	39,561	2,061,630	39,459	2,041,849	62,176,987	9
10	186	39,356	2,022,069	39,263	2,002,390	60,154,868	10
11	178	39,170	1,982,713	39,081	1,963,127	58,172,109	11
12	177	38,992	1,943,543	38,903	1,924,046	56,228,523	12
13	184	38,815	1,904,551	38,723	1,885,143	54,323,928	13
14	196	38,631	1,865,736	38,533	1,846,420	52,458,147	14
15	211	38,435	1,827,105	38,330	1,807,887	50,630,993	15
16	228	38,224	1,788,670	38,110	1,769,557	48,842,271	16
17	246	37,996	1,750,446	37,873	1,731,447	47,091,769	17
18	262	37,750	1,712,450	37,619	1,693,574	45,379,259	18
19	276	37,488	1,674,700	37,350	1,655,955	43,704,494	19
20	285	37,212	1,637,212	37,069	1,618,605	42,067,214	20
21	290	36,927	1,600,000	36,782	1,581,536	40,467,144	21
22	294	36,637	1,563,073	36,490	1,544,754	38,903,999	22
23	296	36,343	1,526,436	36,195	1,508,264	37,377,490	23
24	299	36,047	1,490,093	35,898	1,472,069	35,887,323	24
25	301	35,748	1,454,046	35,597	1,436,171	34,433,203	25
26	303	35,447	1,418,298	35,296	1,400,574	33,014,831	26
27	304	35,144	1,382,851	34,992	1,365,278	31,631,905	27
28	305	34,840	1,347,707	34,687	1,330,286	30,284,123	28
29	305	34,535	1,312,867	34,383	1,295,599	28,971,180	29
30	306	34,230	1,278,332	34,077	1,261,216	27,692,773	30
31	307	33,924	1,244,102	33,770	1,227,139	26,448,595	31
32	307	33,617	1,210,178	33,464	1,193,369	25,238,341	32
33	308	33,310	1,176,561	33,156	1,159,905	24,061,704	33
34	308	33,002	1,143,251	32,848	1,126,749	22,918,377	34
35	308	32,694	1,110,249	32,540	1,093,901	21,808,052	35
36	308	32,386	1,077,555	32,232	1,061,361	20,730,421	36
37	310	32,078	1,045,169	31,923	1,029,129	19,685,176	37
38	310	31,768	1,013,091	31,613	997,206	18,672,009	38
39	311	31,458	981,323	31,302	965,593	17,690,609	39
40	312	31,147	949,865	30,991	934,291	16,740,667	40
41	313	30,835	918,718	30,679	903,300	15,821,872	41
42	315	30,522	887,883	30,364	872,621	14,933,911	42
43	318	30,207	857,361	30,048	842,257	14,076,472	43
44	319	29,889	827,154	29,730	812,209	13,249,239	44
45	322	29,570	797,265	29,409	782,479	12,451,895	45
46	325	29,248	767,695	29,085	753,070	11,684,121	46
47	329	28,923	738,447	28,759	723,985	10,945,593	47
48	332	28,594	709,524	28,428	695,226	10,235,988	48
49	336	28,262	680,930	28,094	666,798	9,554,976	49
50	341	27,926	652,668	27,755	638,704	8,902,225	50
51	346	27,585	624,742	27,412	610,949	8,277,398	51
52	351	27,239	597,157	27,064	583,537	7,680,155	52
53	357	26,888	569,918	26,709	556,473	7,110,150	53

* P_0 is $\frac{1}{2}(l_0 + l_1) \times (\cdot 98037)$. The factor ·98037 has been introduced, as the number living in the first year is less than the arithmetical mean of those born and surviving a year.

TABLE F (*continued*).

Age.	Dying in each Year of Age, 0–1, 1–2, to 105–106.	Born and Living at each Age.	Sum of the Numbers Born and Living at each Age (x) from x to the last Age in the Table.	Population, or the Living in each Year of Age 0 to 1, 1 to 2, &c.	(1) Sum of the Living, and of the Living of every Age (x) and upwards to the last Age in the Table; also (2) the Years which the Females (l_x) will live.	(1) The Years which the Females at the Age (x) and upwards will live; also (2) the Years which they have lived over x.	Age.
		Σd_x.	Σl_x.	$\frac{1}{2}(l_x + l_{x+1}) = l_{x+1} + \frac{1}{2}d_x$.	ΣP_x.	$\Sigma\frac{1}{2}(Q_x + Q_{x+1}) = Y_{x+1} + (Q_{x+1} + \frac{1}{2}P_x)$.	
x.	d_x.	l_x.	L_x.	P_x.	Q_x.	Y_x.	x.
54	363	26,531	543,030	26,350	529,764	6,567,032	54
55	369	26,168	516,499	25,983	503,414	6,050,443	55
56	376	25,799	490,331	25,611	477,431	5,560,020	56
57	411	25,423	464,532	25,218	451,820	5,095,395	57
58	455	25,012	439,109	24,784	426,602	4,656,184	58
59	498	24,557	414,097	24,308	401,818	4,241,974	59
60	536	24,059	389,540	23,791	377,510	3,852,310	60
61	574	23,523	365,481	23,236	353,719	3,486,695	61
62	609	22,949	341,958	22,645	330,483	3,144,594	62
63	644	22,340	319,009	22,018	307,838	2,825,434	63
64	678	21,696	296,669	21,357	285,820	2,528,605	64
65	712	21,018	274,973	20,662	264,463	2,253,463	65
66	745	20,306	253,955	19,933	243,801	1,999,331	66
67	777	19,561	233,649	19,173	223,868	1,765,497	67
68	810	18,784	214,088	18,379	204,695	1,551,215	68
69	841	17,974	195,304	17,553	186,316	1,355,710	69
70	871	17,133	177,330	16,698	168,763	1,178,170	70
71	898	16,262	160,197	15,813	152,065	1,017,756	71
72	922	15,364	143,935	14,903	136,252	873,598	72
73	942	14,442	128,571	13,971	121,349	744,797	73
74	958	13,500	114,129	13,021	107,378	630,434	74
75	968	12,542	100,629	12,058	94,357	529,566	75
76	971	11,574	88,087	11,088	82,299	441,238	76
77	966	10,603	76,513	10,120	71,211	364,483	77
78	954	9,637	65,910	9,160	61,091	298,332	78
79	932	8,683	56,273	8,217	51,931	241,821	79
80	901	7,751	47,590	7,301	43,714	193,999	80
81	862	6,850	39,839	6,419	36,413	153,935	81
82	814	5,988	32,989	5,581	29,994	120,732	82
83	760	5,174	27,001	4,794	24,413	93,528	83
84	698	4,414	21,827	4,065	19,619	71,512	84
85	633	3,716	17,413	3,399	15,554	53,926	85
86	564	3,083	13,697	2,801	12,155	40,071	86
87	495	2,519	10,614	2,272	9,354	29,317	87
88	425	2,024	8,095	1,811	7,082	21,099	88
89	359	1,599	6,071	1,420	5,271	14,922	89
90	298	1,240	4,472	1,091	3,851	10,361	90
91	240	942	3,232	822	2,760	7,056	91
92	191	702	2,290	606	1,938	4,707	92
93	147	511	1,588	438	1,332	3,072	93
94	112	364	1,077	308	894	1,959	94
95	81	252	713	211	586	1,219	95
96	59	171	461	142	375	738	96
97	40	112	290	92	233	434	97
98	27	72	178	58	141	247	98
99	18	45	106	36	83	135	99
100	11	27	61	22	47	70	100
101	7	16	34	12	25	34	101
102	4	9	18	7	13	15	102
103	3	5	9	4	6	6	103
104	1	2	4	1	2	2	104
105	1*	1*	2	1	1	..	105
106	1*	1*	1	..	..	..	106

* The values of l_{104}, l_{105}, and l_{106}, decimally carried out, are 2·490, 1·250, and 0·603; and their differences are 1·240, 0·647, and 0·325. The apparent anomaly, that no death happens between the ages 105 and 106, arises from the omission of decimals.

TABLE G.—*Healthy Districts Life Table.—The Mean After-Lifetime (or the Expectation of Life) at the Age x, and at the Age x and upwards; also the Mean Ages of the Living and the Mean Ages at Death (constructed from Tables D, E, F).*

PERSONS.					
Age (or past Lifetime).	Mean After-Lifetime of Persons of the Age x.	Mean After-Lifetime of Persons of the Age x and *upwards*.	Mean Age of Persons living of the Age x and *upwards*.	Mean Age at Death. Of Persons actually living at the Age x.	Mean Age at Death. Of Persons actually living at the Age x and *upwards*.
x.	$A_x = \frac{Q_x}{D_x}$.	$A'_x = \frac{Y_x}{Q_x}$.	$x + A'_x$.	$x + A_x$.	$x + 2A'_x$.
0	49·00	33·92	33·92	49·00	67·84
5	54·16	31·98	36·98	59·16	68·96
10	51·08	29·91	39·91	61·08	69·82
15	47·12	27·85	42·85	62·12	70·70
20	43·45	25·82	45·82	63·45	71·64
25	40·05	23·79	48·79	65·05	72·58
30	36·64	21·76	51·76	66·64	73·52
35	33·17	19·73	54·73	68·17	74·46
40	29·64	17·71	57·71	69·64	75·42
45	26·05	15·71	60·71	71·05	76·42
50	22·44	13·74	63·74	72·44	77·48
55	18·86	11·84	66·84	73·86	78·68
60	15·37	10·04	70·04	75·37	80·08
65	12·29	8·37	73·37	77·29	81·74
70	9·61	6·86	76·86	79·61	83·72
75	7·34	5·51	80·51	82·34	86·02
80	5·51	4·36	84·36	85·51	88·72
85	4·10	3·41	88·41	89·10	91·82
90	3·05	2·65	92·65	93·05	95·30
95	2·29	2·05	97·05	97·29	99·10
100	1·72	1·47	101·47	101·72	102·94

Age (or past Lifetime).	MALES. Mean After-Lifetime of Males of the Age x.	MALES. Mean Age at Death of Males actually living at the Age x.	FEMALES. Mean After-Lifetime of Females of the Age x.	FEMALES. Mean Age at Death of Females actually living at the Age x.
x.	$A_x = \frac{Q_x}{D_x}$.	$x + A_x$.	$A_x = \frac{Q_x}{D_x}$.	$x + A_x$.
0	48·56	48·56	49·45	49·45
5	54·39	59·39	53·93	58·93
10	51·28	61·28	50·88	60·88
15	47·20	62·20	47·04	62·04
20	43·40	63·40	43·50	63·50
25	39·93	64·93	40·18	65·18
30	36·45	66·45	36·85	66·85
35	32·90	67·90	33·46	68·46
40	29·29	69·29	30·00	70·00
45	25·65	70·65	26·46	71·46
50	22·03	72·03	22·87	72·87
55	18·49	73·49	19·24	74·24
60	15·06	75·06	15·69	75·69
65	12·00	77 00	12·58	77·58
70	9·37	79·37	9·85	79·85
75	7·15	82·15	7·52	82·52
80	5·37	85·37	5·64	85·64
85	4·01	89·01	4·19	89·19
90	2·99	92·99	3·11	93·11
95	2·25	97·25	2·32	97·32
100	1·69	101·69	1·75	101·75

The Table may be read thus:—Persons in the Healthy Districts of England of the precise age 20 will live, on an average, 43·45 years; while persons of the age 20 and *upwards*, living in a normally-constituted population of the same character, will live on an average 25·82 years. The mean age of persons of the age 20 and *upwards* is 45·82 years; the mean age at death of persons living at the precise age 20 will be 63·45, while the mean age at death of persons actually living at the age x and *upwards* will be 71·64 years.

On Gompertz's Law of Mortality. *By* PROFESSOR DE MORGAN.

I HAVE a suspicion that Mr. Edmonds intends the fifteen pages on the "Law of Human Mortality," which appear in the last Number of this *Journal*, to stand in place of an answer to my remarks "On an unfair suppression of due acknowledgment to the writings of Mr. Benjamin Gompertz," printed in the July Number; for though no allusion is made to the charge, still less statement of it or answer to the evidence produced, there is one mention of me which looks so like a distortion of my paper, that I think the suspicion is justified. Mr. Edmonds says (p. 181), "Mr. De Morgan, in his office of self-constituted judge between Mr. Gompertz and me, overlooks this important error . . ." Now, though my paper does not deal with the truths or errors of either, but only with the question whether Mr. Edmonds's mention of Mr. Gompertz was suppressive; and though I never said, and certainly never thought, that there was or could be any question pending between Mr. Gompertz and Mr. Edmonds; and though I was not the judge, either self-constituted or otherwise, but only the promoter of an accusation of unfair suppression for others to judge of;—there is in the quotation that remote likeness to an account of my proceeding which often exists between that which cannot be answered and that which it is convenient to substitute for it. Some acute adviser seems to have whispered, "When you cannot answer what needs answer, answer something else, and keep what you ought to answer out of sight; this will do for all who are to see only one side, and nothing will do for those who are to see both." I shall make two short remarks, and then leave the whole to those who have read the whole.

It will be asked, in turning over page after page, "What has all this talk about the better and worse of this and that method to do with the year 1832 and the question whether the account then given by Mr. Edmonds was, or was not, an unfair suppression of what had been done by Mr. Gompertz?" What did Mr. Edmonds *then* bring forward? What *had* Mr. Gompertz brought forward? What is there in what Mr. Edmonds then brought forward which will redeem his description of Mr. Gompertz's method from the imputation of unfair suppression? If Mr. Edmonds had given all the description he has now given, weak as it is, there would have been foolish and unfounded self-assertion, but at least there would not have been suppression. With the contents of Mr. Edmonds's

paper I have nothing to do; and this because no account he may *now* give, be it true or be it false, can affect the question whether what he *then* gave was or was not unfair suppression. This want of allusion to my accusation relieves me from all necessity of further rejoinder.

Secondly: the process of bringing Dr. Price into the paper is one which has been repeated many and many a time. When A is charged with dealing unfairly with the writings of B, he tries to prove—sometimes he does prove—that he has dealt just as unfairly with C. Dr. Price calls attention to the manner in which the Holy-Cross data and others exhibit the periods of infancy, manhood, and old age. Mr. Edmonds calls this a *discovery*, and then adds, that, "to perfect his discovery," he should have remarked "that the rate was *constant* throughout each of the periods." There was no discovery in what Dr. Price *did* remark, and as to what he *should* have remarked, all we can say is, not that it *does* prove a discovery, but that it *would have* done so if the remark *had been* made. O! these auxiliary verbs! What queer auxiliaries they sometimes are! The introduction of Dr. Price and Mr. Gompertz, as a pair of imperfect predecessors to Mr. Edmonds, can only be explained thus:—Dr. Price was no predecessor at all; Mr. Gompertz was a full and entire predecessor. Take the mean, and each of them was half a predecessor. The policy of such an introduction is of a very questionable stamp, though, no doubt, there is assurance enough in it for a whole life.

[We have considered this question with a good deal of care, and we cannot but think that there is great truth and justness in the remarks which Professor De Morgan has thought fit to make upon it. When Mr. Edmonds admits "that the honour belongs to Mr. Gompertz of first discovering that some connexion existed between tables of mortality and the algebraic expression a^{b^x}," it seems to us that there is an end of the matter. What else of any moment in connexion with it has been discovered? The giving another form to an expression or another phase to an hypothesis already suggested, cannot be looked upon as a discovery. Mr. Edmonds has applied or made use of Mr. Gompertz's suggestion or "discovery" with great ingenuity, neatness, and effect. We do not see how it can be said with truth that he has done more than this. Of the less commendable features of his work we say nothing.—Ed. *A. M.*]

On the Stability of Results based upon Average Calculations, considered with reference to the Number of Transactions embraced. By ROBERT CAMPBELL, M.A., *Advocate, Edinburgh, and Fellow of Trinity Hall, Cambridge.*

[Read before the Institute, 31st December, 1860, and ordered by the Council to be printed.]

AN article published in this *Journal* last January (vol. viii., p. 316), proposed itself the problem of furnishing a test whether the degree of uniformity, or the reverse, observed in a table of statistics, required to be assigned to any other cause than the relative magnitude of the numbers of combinations which each figure in the table represented. The formulæ obtained in that article gave a general solution to the problem proposed, by giving an expression for the ratios of the numbers of such combinations which would be embraced under any figure in the table; and, consequently, expressing the relative frequency with which each number should be expected to occur from the mere consideration of counting the combinations represented by it.

These formulæ will aid in the solution of another and a much more useful problem—namely, to give us the power of prediction, in some very important practical cases, as to the *stability* to be expected in future results calculated from the observation of an average. The simplest case is that where the data on which the calculation is founded are taken from so large sources as to be considered complete.

The problem proposed in the following pages is this: *To show the degree of stability to be expected in the numbers representing the deaths in a given period to be experienced in the course of a life assurance business of given amount*—the problem, for the present, being treated on the supposition that the data on which the calculations of the Office are based are sufficiently large to furnish the true average.

The relation, it may be remarked, between the extensiveness of the data and the accuracy of the average deduced from them, is implicitly given by the formulæ in page 321 of the above article, which may be the subject of future expansion. Enough, for the present, to observe, that when a (the number representing the number of years for which observations are taken) is very large, the formulæ approximate to expressions independent of a altogether—which, looking at the considerations on which these

formulæ are obtained, means to say that the true average has been approximately arrived at. Again: the true average being arrived at, the formulæ on page 324 represent the relative frequency with which numbers differing from the average by any given amount may be expected to occur; but as these formulæ may be obtained more simply and from less complicated considerations than those involved in the above article, I shall proceed to obtain them by a more direct method.

Suppose that we have a sufficient number of observations for obtaining with certainty the *average* percentage of persons of a given class to whom a certain event happens in a given time.

Let a society consist of n persons, who are supposed so far unconnected with each other that the event contemplated happening to one does not make it more or less likely to happen to another, and that of these n persons we know of no reason why it should happen to one rather than another, but that they are all of the given class on which the above observations are made. And suppose that, according to the average above found, the number out of n persons to which the event should happen is b.

Let us designate the above persons by $A_1, A_2 \ldots A_n$; then, by the above hypothesis, the probability of the event happening to A_1 is

$$\frac{b}{n},$$

and the probability of its *not* happening to A_2 is

$$1-\frac{b}{n}.$$

The probability, therefore, of its happening to A_1 and not to A_2 is

$$\frac{b}{n}\left(1-\frac{b}{n}\right);$$

and that of its happening to A_1 and neither to A_2, A_3, nor A_n, is

$$\frac{b}{n}\left(1-\frac{b}{n}\right)^{n-1}.$$

But, of course, the same will be the probability of its happening to A_2 and not to A_1, A_3, nor A_n; and, therefore, the probability of its happening to one person only out of the whole society is

$$n.\frac{b}{n}\left(1-\frac{b}{n}\right)^{n-1}=b.\left(1-\frac{b}{n}\right)^{n-1}.$$

The probability of its happening to both A_1 and A_2 is

$$\left(\frac{b}{n}\right)^2,$$

and that of its happening to both A_1 and A_2, and not to any of the rest,

$$\left(\frac{b}{n}\right)^2\left(1-\frac{b}{n}\right)^{n-2}.$$

The probability, therefore, of its happening to *two persons only* out of the society is

$$\frac{n(n-1)}{2}.\left(\frac{b}{n}\right)^2\left(1-\frac{b}{n}\right)^{n-2};$$

that of its happening to *three persons only*,

$$\frac{n(n-1)(n-2)}{1.2.3}.\left(\frac{b}{n}\right)^3\left(1-\frac{b}{n}\right)^{n-3}.$$

The law is evident, and we shall only further write down the probability of its happening to exactly b persons out of the Society, and no more nor less, which is

$$\frac{n(n-1)(n-2)\ldots(n-b+1)}{1.2.3\ldots\ldots\quad b}\left(\frac{b}{n}\right)^b\left(1-\frac{b}{n}\right)^{n-b}.*$$

This will be the *most probable number*, for it has been obtained from the last by multiplying by

$$\frac{n-b+1}{b}.\frac{b}{n}.\left(1-\frac{b}{n}\right)^{-1}=\frac{n-b+1}{b}.\frac{b}{n-b}=\frac{n-b+1}{n-b}, \text{ which is} > 1;$$

and the next would be obtained from it by multiplying by

$$\frac{n-b}{b+1}.\frac{b}{n}.\left(1-\frac{b}{n}\right)^{-1}=\frac{n-b}{b+1}.\frac{b}{n-b}=\frac{b}{b+1}, \text{ which is} < 1.$$

If we now compare the probability of the number b being the exact number out of the society to whom the event happens, with that of $b-1$, $b-2$, and so on being the number, and then with that of $b+1$, $b+2$, and so on being the number, the ratios will be—

* So far, this investigation is substantially identical with one which is well known—the language of the above being, for the sake of clearness, adapted to the case here contemplated. The simplifications which follow, admitting the definite working out of the problem in the most useful practical form, are believed to be new.

The mathematical demonstration of the stability of average results, a principle on the practical truth of which the whole fabric of assurance business rests, has, I believe, hitherto rested on very general grounds borrowed from investigations of La Place, which were chiefly worked out in a totally different direction, and apply only to the problem now under consideration in the case where the numbers are very large. In other words, it has been proved that the results of extensive transactions of this kind approach to stability, but no definite relation has hitherto been given between the extent of transactions and the degree of stability to be expected.

Probability of b being the number to that of $(b-1)$

$$=\frac{n-b+1}{b}\cdot\frac{b}{n-b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad a_1$$

Probability of b being the number to that of $(b-2)$

$$=\frac{(n-b+1)(n-b+2)}{b \qquad (b-1)}\cdot\left(\frac{b}{n-b}\right)^2 \quad . \quad . \quad . \quad . \quad a_2$$

Probability of b being the number to that of $(b-3)$

$$=\frac{(n-b+1)(n-b+2)(n-b+3)}{b \qquad (b-1) \qquad (b-2)}\cdot\left(\frac{b}{n-b}\right)^3 \quad . \quad a_3$$

and so on; and

Probability of b being the number to that of $(b+1)$

$$=\frac{b+1}{n-b}\cdot\frac{n-b}{b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad a_1$$

Probability of b being the number to that of $(b+2)$

$$=\frac{(b+1)(b+2)}{(n-b)(n-b-1)}\cdot\left(\frac{n-b}{b}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad a_2$$

Probability of b being the number to that of $(b+3)$

$$=\frac{(b+1)\,(b+2)\,(b+3)}{(n-b)\,(n-b-1)(n-b-2)}\cdot\left(\frac{n-b}{b}\right)^3 \quad . \quad . \quad a_3$$

and so on.

It is evident that, after getting a certain length, these numbers will increase with enormous rapidity. That rapidity of increase is the test of the stability of calculations proceeding upon average results, and we shall best show how it may be applied to practical cases, by dealing with actual numbers and taking advantage of all the approximate simplifications which they will supply to us.

It is plain that if n is very large compared to b, we may neglect such factors as $\frac{n-b+1}{n-b}$, $\frac{n-b+2}{n-b}$, $\frac{n-b}{n-b-1}$; in which case the above ratios will be very much simplified, and will *depend on the value of b alone.* They will become

$$a_1=\frac{b}{b}=1, \qquad a_1=\frac{b+1}{b},$$

$$a_2=\frac{b}{b-1}, \qquad a_2=\frac{(b+1)(b+2)}{b^2},$$

$$a_3=\frac{b^2}{(b-1)(b-2)}, \qquad a_3=\frac{(b+1)(b+2)(b+3)}{b^3},$$

and so on.

The practical extent to which we may extend this simplification will soon appear from a numerical example. Suppose $n=1{,}000$, and let $b=7$, which will very nearly represent the mortality among

persons of 20 years of age according to the Carlisle Tables; we have then $n-b=993$, and, substituting these values in the expression first given for a_1, a_a, &c., we have—

$$a_1=\frac{994}{7}\cdot\frac{7}{993}=\frac{994}{993}\times 1,$$

$$a_2=\frac{994.995}{7\ .\ 6}\times\left(\frac{7}{993}\right)^2=\frac{994}{993}\cdot\frac{995}{993}\times\frac{7}{6},$$

$$a_3=\frac{994.995.996}{7\ .\ 6\ .\ 5}\cdot\left(\frac{7}{993}\right)^3=\frac{994}{993}\cdot\frac{995}{993}\cdot\frac{996}{993}\times\frac{7}{6}\cdot\frac{7}{5},$$

and so on; and

$$a_1=\frac{8}{993}\cdot\frac{993}{7}=\frac{8}{7},$$

$$a_2=\frac{8\ .\ 9}{993.992}\cdot\left(\frac{993}{7}\right)^2=\frac{8}{7}\cdot\frac{9}{7}\cdot\frac{993}{992},$$

$$a_3=\frac{8\ .\ 9\ .\ 10}{993.992.991}\cdot\left(\frac{993}{7}\right)^3=\frac{8}{7}\cdot\frac{9}{9}\cdot\frac{10}{7}\times\frac{993}{992}\cdot\frac{993}{991},$$

and so on.

Now, it will not make much difference in the rapidity with which these ratios, a_1, a_2, a_3, &c., and a_1, a_2, a_3, increase, *especially when we get beyond the first two or three*, if we neglect such a fraction as $\frac{996}{993}$ in comparison with $\frac{7}{5}$, or $\frac{993}{991}$ in comparison with $\frac{10}{7}$. We need only remember, that if we had not neglected these fractions the stability of results would have been found to be rather greater.

Again: let $n=1{,}000$, as before, and let $b=96$, which will nearly represent the mortality at the age of 75 from the same tables; then $n-b=904$, and we have—

$$a_1=\frac{905}{96}\cdot\frac{96}{904}=\frac{905}{904}\times 1,$$

$$a_2=\frac{905.906}{96.95}\left(\frac{96}{904}\right)^2=\frac{905}{904}\cdot\frac{906}{904}\times\frac{96}{95},$$

$$a_3=\frac{905.906.907}{96.95.94}\left(\frac{96}{904}\right)^3=\frac{905}{904}\cdot\frac{906}{904}\cdot\frac{907}{904}\times\frac{96}{95}\cdot\frac{96}{94},$$

and

$$a_1=\frac{97}{904}\cdot\frac{904}{96}=\frac{97}{97},$$

$$a_2=\frac{97.98}{904.903}\cdot\left(\frac{904}{96}\right)^2=\frac{97}{96}\cdot\frac{98}{96}\times\frac{904}{903},$$

$$a_3=\frac{97.98.99}{904.903.902}\cdot\left(\frac{904}{96}\right)^3=\frac{97}{96}\cdot\frac{98}{96}\cdot\frac{99}{96}\times\frac{904}{903}\cdot\frac{904}{902}.$$

Now, as before, and having an eye to the use we are about to make of these figures, it will not materially affect our results if we neglect the fraction involving the large figures in comparison of the others corresponding to them. We might, perhaps, retain all through a_1, a_2, a_3, the factor $\frac{905}{904}$; but even this may, for the present, be neglected—only remembering that in what follows we shall slightly underestimate the stability of results to be expected from considerations based on an average.

Now, suppose the lives assured in an Office comprise various ages between 20 and 75 (if there are a few over the latter age, which we fixed on merely for illustration, it will not materially affect our results); and suppose that, according to the tables, taking the due proportion of lives to each age, the whole number of lives that should fall next year is 96. It scarcely requires proof that what I call the stability of this expectation—that is, the rapidity of increase of the numbers representing the ratio of the probability of this result to the respective results of 95, 94, 93, &c., 97, 98, 99, &c., lives falling—will be intermediate to the stability of expected results in two hypothetical cases—namely, these:—

Take first the case of an Office which should have nothing but lives aged 75 assured, and whose expected number of deaths next year was 96.

Then take the case of an Office which should have nothing but lives aged 20 assured, and whose expected number of deaths next year was also 96.

The first hypothetical Office must have just 1,000 lives assured, and the values of a_1, a_2, &c., in its case, are just those found above on p. 220.

The second hypothetical Office must have a number of lives assured equal to $96 \times \frac{1,000}{7} = 13,914$. We should then have to make $n = 13,914$, $b = 96$, and $n - b = 13,818$.

But from what has been shown above, in estimating the stability of either, we might have, for all practical purposes, neglected the value of n. The stability in either case, therefore, is nearly the same—depends on $b = 96$ alone, and not on n—and may be estimated from the values of a_1, a_2, &c., on p. 219; and the stability thus estimated, being a sufficient approximation to the stability of results to be expected in either of the hypothetical cases, is a sufficient approximation to that in the real case which lies between them.

The formulæ on p. 219, then, easily supply the means of constructing tables by which it may be shown at a glance, in the case of an Office expecting (according to average calculations) that a certain number (say 50) of lives will fall in the course of the year, what are the relative probabilities of the number of lives actually falling being 50, 49, 48, &c., 51, 52, 53, &c.

This being done, it will easily appear how, finally, we can form a table showing, if the number on an average calculation be given, what is the probability of the true number being found to differ from the calculated one by more than a given percentage of its amount, which I conceive to be the best form in which the relation between the stability of an assurance business and the number of its transactions can be exhibited.

After what has been said, the tables which follow will sufficiently explain themselves. In what follows we shall adopt the following notation :—

$$\phi_1(b) = \frac{b}{b} = 1,$$

$$\phi_2(b) = \frac{b-1}{b},$$

$$\phi_3(b) = \frac{(b-1)(b-2)}{b^2},$$

$$\cdots\cdots\cdots$$

$$\phi_n(b) = \frac{(b-1)(b-2)\ldots\ldots(b-n+1)}{b^{n-1}},$$

and

$$\psi_1(b) = \frac{b}{b+1},$$

$$\psi_2(b) = \frac{b^2}{(b+1)(b+2)},$$

$$\psi_3(b) = \frac{b^3}{(b+1)(b+2)(b+3)},$$

$$\cdots\cdots\cdots\cdots$$

$$\psi_n(b) = \frac{b^n}{(b+1)(b+2)\ldots\ldots(b+n)}.$$

the reciprocals of the expressions on p. 219.

TABLE I.—*Preparatory Division Table, by Logarithms, giving the value of* $\log. \frac{b+x}{b}$ *or* $\frac{b}{b-x}$, *for all values of* b *from* 1 *to* 20 *and* x *from* 1 *to* 20.

Divisors (x)	Dividend (b) 1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
1	·0000000	·3010300	·4771213	·6020600	·6989700	·7781513	·8450980	·9030900	·9542425	1·0000000
2		·0000000	·1760913	·3010300	·3979400	·4771213	·5440680	·6020600	·6532125	·6989700
3			·0000000	·1249387	·2218487	·3010300	·3679767	·4259687	·4771212	·5228787
4				·0000000	·0969100	·1760913	·2430380	·3010300	·3521825	·3979400
5					·0000000	·0791813	·1461280	·2041200	·2552726	·3010300
6						·0000000	·0669467	·1249387	·1760912	·2218487
7							·0000000	·0579920	·1091445	·1549020
8								·0000000	·0511525	·0969100
9									·0000000	·0457575
	$b=11.$	$b=12.$	$b=13.$	$b=14.$	$b=15.$	$b=16.$	$b=17.$	$b=18.$	$b=19.$	$b=20.$
$x=$ 1	1·0413927	1·0791812	1·1139434	1·1461280	1·1760913	1·2041200	1·2304489	1·2552725	1·2787536	1·3010300
2	·7403627	·7781512	·8129134	·8450980	·8750613	·9030900	·9294189	·9542425	·9777236	1·0000000
3	·5642714	·6020599	·6368221	·6690067	·6989700	·7269987	·7533276	·7781512	·8016323	·8239087
4	·4393327	·4771212	·5118834	·5440680	·5740313	·6020600	·6283889	·6532125	·6766936	·6989700
5	·3424227	·3802112	·4149734	·4471580	·4771213	·5051500	·5314789	·5563025	·5797836	·6020600
6	·2632414	·3010300	·3357921	·3679767	·3979400	·4259687	·4522976	·4771212	·5006023	·5228787
7	·1962947	·2340832	·2688454	·3010300	·3309933	·3590220	·3853509	·4101745	·4336556	·4559320
8	·1283027	·1760912	·2108534	·2430380	·2730013	·3010300	·3273589	·3521825	·3756636	·3979400
9	·0871502	·1249387	·1597009	·1908855	·2218488	·2498775	·2762064	·3010300	·3245111	·3467875
10	·0413927	·0791812	·1139434	·1461280	·1760913	·2041200	·2304489	·2552725	·2787536	·3010300
11	·0000000	·0377885	·0725507	·1047353	·1346986	·1627273	·1890562	·2138798	·2373609	·2596373
12		·0000000	·0347622	·0669468	·0969101	·1249388	·1512677	·1760913	·1995724	·2218488
13			·0000000	·0321846	·0621479	·0901766	·1165055	·1413291	·1648102	·1870866
14				·0000000	·0299633	·0579920	·0843209	·1091445	·1326256	·1549020
15					·0000000	·0280287	·0543576	·0791812	·1026623	·1249387
16						·0000000	·0263289	·0511525	·0746336	·0969100
17							·0000000	·0248236	·0483047	·0705811
18								·0000000	·0234811	·0457575
19									·0000000	·0222764
20										·0000000

It is evident that, commencing at the bottom of a column corresponding to any given value of b, and adding vertically upwards, the numbers produced will be the logs. of the quantities $\frac{1}{\phi_1(b)}, \frac{1}{\phi_2(b)}, \frac{1}{\phi_3(b)}$, and so on; and adding horizontally, we shall, in the same way, produce the logs. of $\frac{1}{\psi_1(b)}, \frac{1}{\psi_2(b)}, \frac{1}{\psi_3(b)}$, and so on.

TABLE I. (*continued*).

		$b=21.$	$b=22.$	$b=23.$	$b=24.$	$b=25.$	$b=26.$	$b=27.$	$b=28.$	$b=29.$	$b=30.$	$b=31.$
$n=$	1	1·3222193	1·3424227	1·3617278	1·3802112	1·3979400	1·4149733	1·4313638	1·4471580	1·4623980	1·4771213	1·4913617
	2	1·0211893	1·0413927									
	3	·8450980	·8653014	·8846065								
	4	·7201593	·7403627	·7596678	·7781513							
	5	·6232493	·6434527	·6627578	·6812412	·6989700						
	6	·5440680	·5642714	·5835765	·6020600	·6197887	·6368221					
	7	·4771213	·4973247	·5166298	·5351132	·5528420	·5698754	·5862658				
	8	·4191293	·4393327	·4486378	·4771213	·4948500	·5118834	·5282738	·5440680			
	9	·3679768	·3881802	·4074853	·4259687	·4436975	·4607309	.4771213	·4929155	·5081555		
	10	·3222193	·3424227	·3617278	·3802112	·3979400	·4149734	·4313638	·4471580	·4623980	·4771213	
	11	·2808266	·3010300	·3203351	·3388185	·3565473	·3735806	·3899711	·4057653	·4210053	·4357286	·4499690
	12	·2430381	·2632414	·2825466	·3010300	·3187588	·3357921	·3521826	·3679767	·3832168	·3979410	·4121805
	13	·2082759	·2284793	·2477844	·2662678	·2839966	·3010300	·3174204	·3332146	·3484546	·3631779	·3774183
	14	·1760913	·1962947	·2155998	·2340832	·2518120	·2688454	·2852358	·3010300	·3162700	·3309933	·3452337
	15	·1461280	·1663314	·1856365	·2041199	·2218487	·2388820	·2552725	·2710667	·2863067	·3010300	·3152704
	16	·1180993	·1283027	·1576078	·1760912	·1938200	·2108534	·2272438	·2430380	·2582780	·2730013	·2872417
	17	·0917704	·1129738	·1312789	·1497623	·1674911	·1845244	·2009149	·2167091	·2319491	·2466724	·2609128
	18	·0669468	·0871502	·1064553	·1249387	·1426675	·1597009	·1760913	·1908855	·2071255	·2218488	·2360892
	19	·0434657	·0636691	·0829742	·1014576	·1191864	·1362197	·1526102	·1684044	·1836444	·1983677	·2126081
	20	·0211893	·0413927	·0606978	·0791812	·0969100	·1139434	·1303338	·1461280	·1613680	·1760913	·1903317

		$b=32.$	$b=33.$	$b=34.$	$b=35.$	$b=36.$	$b=37.$	$b=38.$	$b=39.$	$b=40.$	$b=41.$	$b=42.$
		1·5051500	1·5185139	1·5314789	1·5440680	1·5563025	1·5682017	1·5797836	1·5910646	1·6020600		
$n=$	12	·4259687										
	13	·3912066	·4045705									
	14	·3590220	·3723859	·3853509								
	15	·3290587	·3424226	·3553876	·3679767							
	16	·3010300	·3143939	·3273589	·3399480	·3521825						
	17	·2747011	·2880650	·3010300	·3136191	·3258536	·3377528	·3493347				
	18	·2498775	·2632414	·2762064	·2887955	·3010300	·3129292	·3245111	·3357921			
	19	·2263964	·2397593	·2527253	·2653144	·2775489	·2894481	·3010300	·3123110	·3233064	·3340303	
	20	·2041200	·2174839	·2304489	·2430380	·2552725	·2671717	·2787536	·2900346	·3010300	·3117539	·3222192

TABLE II.—*Logarithms found by adding those in the last Table as indicated.*

	$b=1$.	$b=2$.	$b=3$.	$b=4$.	$b=5$.	$b=6$.	$b=7$.	$b=8$.	$b=9$.	$b=10$.	$b=11$.	$b=12$.	$b=13$.	$b=14$.
$1-\log.\ \phi_{14}$													4·6869838	3·9582556
$1-\log.\ \phi_{13}$												4·2698375	3·5730404	3·1131576
$1-\log.\ \phi_{12}$											3·8441639	3·1906563	2·7601270	2·4441509
$1-\log.\ \phi_{11}$										3·4402369	2·8027712	2·4125051	2·1233049	1·9000829
$1-\log.\ \phi_{10}$									3·0284194	2·4402369	2·0624085	1·8104452	1·6114215	1·4529249
$1-\log.\ \phi_{9}$								2·6191994	2·0741769	1·7412669	1·4981371	1·3333240	1·1964481	1·0849482
$1-\log.\ \phi_{8}$							2·2132554	1·7161094	1·4209664	1·2183882	1·0588044	·9531128	·8606560	·7859182
$1-\log.\ \phi_{7}$						1·8115752	1·3681574	1·1140494	·9438432	·8204482	·7163817	·6520828	·5918106	·5408802
$1-\log.\ \phi_{6}$					1·4156687	1·0334239	·8240894	·6880807	·5916607	·5194182	·4531403	·4179996	·3809572	·3499947
$1-\log.\ \phi_{5}$				1·0280287	·7166987	·5563026	·4561127	·3870507	·3363882	·2975695	·2568456	·2419084	·2212563	·2038667
$1-\log.\ \phi_{4}$			·6532126	·4259687	·3187587	·2552726	·2130747	·1829307	·1602970	·1426675	·1285429	·1169697	·1073129	·0991314
$1-\log.\ \phi_{3}$		·3010300	·1760913	·1249387	·0969100	·0791813	·0669467	·0579920	·0511525	·0457575	·0413927	·0377885	·0347622	·0321846
$1-\log.\ \phi_{2}$	·0000000	·0000000	·0000000	·0000000	·0000000	·0000000	·0000000	·0000000	·0000000	·0000000	·0000000	·0000000	·0000000	·0000000
$1-\log.\ \phi_{1}$														
$1-\log.\ \psi_{1}$	·3010300	·1760913	·1249387	·0969100	·0791813	·0669467	·0579920	·0511525	·0457575	·0413927	·0377885	·0347622	·0321846	·0299633
$1-\log.\ \psi_{2}$	·7781513	·4771213	·3467874	·2730013	·2253093	·1918854	·1671365	·1480625	·1329077	·1205739	·1103492	·1017090	·0943325	·0879553
$1-\log.\ \psi_{3}$	1·3802113	·8750613	·6478174	·5160393	·4294293	·3679766	·3220385	·2763652	·2578464	·2345173	·2150845	·1986191	·1845091	·1722762
$1-\log.\ \psi_{4}$	2·0791813	1·3521826	1·0157941	·8170693	·6847018	·5898253	·5183332	·4524564	·4175473	·3806453	·3497831	·3235579	·3010146	·2814207
$1-\log.\ \psi_{5}$	2·8573326	1·8962506	1·4417628	1·1692518	·9857318	·8530667	·7524164	·6633098	·6084328	·5567366	·5125104	·4748256	·4423437	·4140463
$1-\log.\ \psi_{6}$	3·7024306	2·4983106	1·9188840	1·5671918	1·3281545	1·1540967	1·0212618	·9063478	·8302816	·7608566	·7015666	·6509169	·6071539	·5689483
$1-\log.\ \psi_{7}$	4·6055206	3·1515231	2·4417627	2·0065245	1·7083657	1·4898888	1·3222918	1·2793491	1·0801591	·9913055	·9154464	·8504893	·7942405	·7450396
$1-\log.\ \psi_{8}$		3·8504931	3·0060341	2·4836457	2·1233391	1·8578655	1·6532851	1·4803791	1·3563655	1·2465780	1·1528073	1·0723381	1·0025164	·9413343
$1-\log.\ \psi_{9}$		4·5908558	3·6080940	2·9955291	2·5704971	2·2558055	2·0123071	1·8077380	1·6573955	1·5253316	1·4124446	1·3153762	1·2309957	1·1569341
$1-\log.\ \psi_{10}$			4·2449161	3·5395971	3·0476184	2·6817742	2·3976580	2·1599205	1·9819066	1·8263616	1·6932712	1·5786176	1·4787801	1·3910173
$1-\log.\ \psi_{11}$				4·1136384	3·5527684	3·1340718	2·8088325	2·5355841	2·3286941	2·1485809	1·9943012	1·8611642	1 7450479	1·6428293
$1-\log.\ \psi_{12}$					4·0842473	3·6111930	3·2424881	2·9335241	2·6966709	2·4910036	2·3146363	2·1621942	2·0290445	1·9116747
$1-\log.\ \psi_{13}$						4·1117953	3·6984201	3·3526534	3·0848511	2·8527314	2·6534548	2·4809530	2·3300745	2·1969105
$1-\log.\ \psi_{14}$							4·1755414	3·7919861	3·4923364	3·2329426	3·0100021	2·8167451	2·6474949	2·4979405
$1-\log.\ \psi_{15}$								4·2406239	3·9183051	3·6308826	3·3835827	3·1689277	2·9807095	2·8142105
$1-\log.\ \psi_{16}$									4·3620026	4·0458560	3·7735538	3·5369044	3·3291641	3·1452038
$1-\log.\ \psi_{17}$											4·1793191	3·9201212	3·6923420	3·4904375
$1-\log.\ \psi_{18}$												4·3180622	4·0697603	3·8494595
$1-\log.\ \psi_{19}$														4·2218454

In any column corresponding to a particular value of b placed at the top, the numbers from the centre upwards are successively the values of $1-\log.\ \phi(b)$, and those successively from the centre downwards are the values of $1-\log.\ \psi(b)$.

TABLE II. (*continued*).

	$b=15$.	$b=16$.	$b=17$.	$b=18$.	$b=19$.	$b=20$.	$b=25$.	$b=30$.	$b=40$.	$b=50$.	$b=70$.	$b=100$.	$b=200$.	$b=500$.
$1-\log.\ \phi_{100}$														4·616996
$1-\log.\ \phi_{90}$														3·707105
$1-\log.\ \phi_{80}$														2·903012
$1-\log.\ \phi_{70}$														2·202209
$1-\log.\ \phi_{60}$													4·294021	1·602296
$1-\log.\ \phi_{50}$													2·911505	1·100982
$1-\log.\ \phi_{40}$											6·149176	3·9501612	1·817768	·696080
$1-\log.\ \phi_{30}$										4·872150	3·1861803	2·1083913	·994793	·385494
$1-\log.\ \phi_{25}$								4·1556512	4·256354	3·181821	2·1268572	1·4245980	·679801	·264940
$1-\log.\ \phi_{20}$								3·6785299	2·5156797	1·919986	1·3066302	·8847140	·4266829	·1672201
$1-\log.\ \phi_{19}$							4·2275419	3·2428013	2·2358390	1·7123767	1·1691024	·7931990	·3838315	·1503952
$1-\log.\ \phi_{18}$							3·6746999	2·8448603	1·9762017	1·5185567	1·0400077	·7070129	·3428729	·1344722
$1-\log.\ \phi_{17}$						4·5095366	3·1798499	2·4816824	1·7358695	1·3381006	·9191856	·6260910	·3037940	·1194493
$1-\log.\ \phi_{16}$				4·5790489	4·1531142	3·8105666	2·7361524	2·1506891	1·5140207	1·1706095	·8064814	·5503703	·2675818	·1053247
$1-\log.\ \phi_{15}$		4·7411803	4·2066949	3·8008977	3·4764206	3·2085066	2·3382124	1·8496591	1·3099007	1·0157075	·7017461	·4797992	·2337235	·0920964
$1-\log.\ \phi_{14}$	4·3487785	3·8380903	3·4533673	3·1476852	2·8966370	2·6856279	1·9816651	1·5766578	1·1228140	·8730402	·6048361	·4142977	·2022064	·0797627
$1-\log.\ \phi_{13}$	3·4737172	3·1110916	2·8249784	2·5913827	2·3960347	2·2296959	1·6629063	1·3300854	·9521178	·7422719	·5156130	·3538170	·1730180	·0683217
$1-\log.\ \phi_{12}$	2·7747472	2·5090316	2·2934995	2·1142615	1·9623791	1·8317559	1·3789097	1·1082366	·7972158	·6230855	·4339430	·2982997	·1461458	·0577715
$1-\log.\ \phi_{11}$	2·2007159	2·0038816	1·8412019	1·7040870	1·5867155	1·4849684	1·1270977	·9098689	·6575538	·5151801	·3596970	·2476897	·1215776	·0481104
$1-\log.\ \phi_{10}$	1·7235946	1·5779129	1·4558510	1·3519045	1·2622044	1·1839384	·9052490	·7337776	·5326151	·4182701	·2927503	·2019322	·0993012	·0393365
$1-\log.\ \phi_{9}$	1·3256546	1·2188909	1·1284921	1·0508745	·9834508	·9243011	·7114290	·5788756	·4219168	·3320840	·2329821	·1609736	·0793046	·0314480
$1-\log.\ \phi_{8}$	·9946613	·9178609	·8522857	·7956020	·7460899	·7024523	·5439379	·4441770	·3250068	·2563633	·1802758	·1247614	·0615758	·0244431
$1-\log.\ \phi_{7}$	·7216600	·6679834	·6218368	·5817222	·5465175	·5153657	·4012704	·3287835	·2414607	·1908618	·1345183	·0932443	·0461031	·0183200
$1-\log.\ \phi_{6}$	·4998112	·4638634	·4327806	·4056309	·3817073	·3604637	·2820840	·2317734	·1708796	·1353445	·0955997	·0663722	·0328748	·0130769
$1-\log.\ \phi_{5}$	·3237199	·3011361	·2815129	·2643018	·2490817	·2355250	·1851740	·1525921	·1128876	·0895870	·0634151	·0440958	·0218794	·0087121
$1-\log.\ \phi_{4}$	·1890213	·1761973	·1650074	·1551573	·1464194	·1386150	·1094533	·0904441	·0671301	·0533748	·0378610	·0263670	·0131055	·0052288
$1-\log.\ \phi_{3}$	·0921112	·0860207	·0806865	·0759761	·0717858	·0680339	·0539410	·0446866	·0332718	·0265027	·0188380	·0131387	·0065417	·0026102
$1-\log.\ \phi_{2}$	·0299633	·0280287	·0263289	·0248236	·0234811	·0222764	·0177288	·0147233	·0109954	·0087739	·0062489	·0043648	·0021769	·0008695
$1-\log.\ \phi_{1}$	·0000000	·0000000	·0000000	·0000000	·0000000	·0000000	·0000000	·0000000	·0000000	·0000000	·0000000	·0000000	·0000000	·0000000

$1 - \log. \psi_1$	·0280287	·0263289	·0248236	·0234811	·0222764	·0211893	·0170333	·0142404	·0107239	·0086002	·0061603	·0043214	·0021661	·0008677
$1 - \log. \psi_2$	·0823863	·0774814	·0731283	·0692386	·0657421	·0625820	·0504571	·0422691	·0319132	·0256335	·0183948	·0129216	·0064875	·0026014
$1 - \log. \psi_3$	·1615675	·1521150	·1437094	·1361854	·1294112	·1232798	·0996751	·0836617	·0633217	·0509394	·0366197	·0257588	·0129535	·0051994
$1 - \log. \psi_4$	·2642298	·2490250	·2354798	·2232356	·2123854	·2024610	·1641331	·1380193	·1047144	·0843632	·0607534	·0427921	·0215537	·0086599
$1 - \log. \psi_5$	·3891685	·3671243	·3484536	·3296909	·3138430	·2993710	·2433444	·2049660	·1558669	·1257559	·0907167	·0639814	·0322776	·0129813
$1 - \log. \psi_6$	·5352965	·4954270	·4797325	·4546296	·4330294	·4133144	·3367361	·2841472	·2165647	·1749739	·1264323	·0892873	·0451148	·0181618
$1 - \log. \psi_7$	·7016279	·6530348	·6294948	·5972971	·5692491	·5436482	·4439461	·3752276	·2866026	·2318788	·1678250	·1186711	·0600551	·0241998
$1 - \log. \psi_8$	·8872644	·8291260	·7969859	·7569980	·7218593	·6897762	·5645200	·4778899	·3657838	·2963368	·2148216	·1520949	·0770884	·0310935
$1 - \log. \psi_9$	1·0913843	1·0229460	·9815103	·9330893	·8902637	·8511442	·6980589	·5918332	·4539199	·3682188	·2678507	·1895214	·0962047	·0388413
$1 - \log. \psi_{10}$	1·3132330	1·2337994	1·1824252	1·1239748	1·0739081	1·0272355	·8441869	·7167719	·5508299	·4474001	·3253427	·2309141	·1173940	·0474415
$1 - \log. \psi_{11}$	1·5521150	1·4610432	1·3991343	1·3311003	1·2722758	1·2175672	1·0025494	·8524345	·6563401	·5337598	·3887297	·2762371	·1406465	·0568924
$1 - \log. \psi_{12}$	1·8073875	1·7040812	1·6310834	1·5529491	1·4848839	1·4216872	1·1728111	·9985625	·7702834	·6271815	·4574456	·3254551	·1659524	·0671924
$1 - \log. \psi_{13}$	2·0784542	1·9623592	1·8777558	1·7890383	1·7112803	1·6391711	1·3546547	1·1549097	·8924993	·7275520	·5314257	·3785335	·1933020	·0783390
$1 - \log. \psi_{14}$	2·3647609	2·2353605	2·1386686	2·0389158	1·9510396	1·8696200	1·5477793	1·3212411	1·0228331	·8347620	·6106070	·4354384	·2226858	·0903329
$1 - \log. \psi_{15}$	2·6657909	2·5226022	2·4133697	2·3021572	2·2037649	2·1126580	1·7518993	1·4973323	1·1611358	·9487054	·6949279	·4961362	·2540943	·1031701
$1 - \log. \psi_{16}$	2·9810613	2·8236322	2·7014347	2·5783636	2·4690793	2·3679305	1·9777432	1·6829688	1·3072638	1·0692793	·7843284	·5605942	·2875181	·1178498
$1 - \log. \psi_{17}$	3·3101200	3·1380261	3·0024647	2·8671591	2·7466282	2·6351022	2·1920525	1·8779454	1·4610787	1·1963841	·8787497	·6287801	·3229478	·1313703
$1 - \log. \psi_{18}$	3·6525426	3·4653850	3·3160838	3·1681891	3·0360763	2·9138558	2·4275810	2·0820653	1·6224467	1·3299230	·9781344	·7006621	·3603743	·1467301
$1 - \log. \psi_{19}$	4·0079302	3·8053330	3·6419374	3·4811183	3·3371063	3·2038904	2·6730937	2·2951401	1·7912387	1·4698021	1·0824264	·7762091	·3997884	·1629275
$1 - \log. \psi_{20}$		4·1575155	3·9796902	3·8056294	3·6494173	3·5049204	2·9283662	2·5169888	1·9673300	1·6159301	1·1915709	·8553903	·441181	·1799608
$1 - \log. \psi_{21}$			4·3290249	4·1414215	3·9727237	3·8166743								
$1 - \log. \psi_{22}$					4·3067540	4·1388935								
$1 - \log. \psi_{25}$							4·343929	3·7519882	2·953195	2·437287	1·808244	1·304772	·677512	·276761
$1 - \log. \psi_{30}$							5·974835	5·1828759	4·104960	3·402553	2·538658	1·840690	·961994	·395923
$1 - \log. \psi_{40}$										5·730061	4·318592	3·159106	1·671280	·693775
$1 - \log. \psi_{50}$												4·786890	2·561198	1·071943
$1 - \log. \psi_{60}$												6·703453	3·624532	1·529025
$1 - \log. \psi_{70}$													4·854624	2·063595
$1 - \log. \psi_{80}$														2·674295
$1 - \log. \psi_{90}$														3·359811
$1 - \log. \psi_{100}$														4·118878

The numbers above and below ϕ_{20} and ψ_{20} are found with the help of the Table for $\Gamma(n+1)$ in the *Encyclopædia Metropolitana*, on "Probabilities."

TABLE III.—*Giving the Successive Values of ϕ, ψ, for any Value of b.*

	$b=1.$	$b=2.$	$b=3.$	$b=4.$	$b=5.$	$b=6.$	$b=7.$	$b=8.$	$b=9.$	$b=10.$	$b=11.$	$b=12.$	$b=13.$	$b=14.$	$b=15.$	$b=16.$	$b=17.$
ϕ_{16}																	
ϕ_{15}																·00002	·00006
ϕ_{14}															·00004	·00015	·00035
ϕ_{13}													·00002	·00011	·00034	·00077	·00150
ϕ_{12}												·00005	·00027	·00077	·00168	·00310	·00509
ϕ_{11}											·00013	·00065	·00174	·00360	·00630	·00911	·01441
ϕ_{10}										·00036	·00128	·00387	·00753	·01259	·01890	·02643	·03501
ϕ_{9}									·00094	·00363	·00866	·01547	·02447	·03524	·04724	·06041	·07439
ϕ_{8}								·00240	·00843	·01814	·03176	·04642	·06361	·08223	·10124	·12082	·14051
ϕ_{7}							·00612	·00192	·03793	·06048	·08734	·11140	·13783	·16446	·18982	·21390	·23887
ϕ_{6}						·01543	·04284	·07690	·11380	·15120	·19214	·22280	·25597	·28752	·31631	·34367	·36916
ϕ_{5}					·03840	·09259	·14993	·20508	·25606	·30240	·35226	·38194	·41595	·44669	·47455	·49988	·52298
ϕ_{4}				·09375	·19200	·27145	·34985	·41016	·46091	·50400	·55355	·57292	·60082	·62536	·64711	·66650	·68390
ϕ_{3}			·22222	·37500	·48000	·55556	·61225	·65625	·69136	·72000	·74380	·76389	·78106	·79592	·80889	·82031	·83045
ϕ_{2}		·50000	·66667	·75000	·80000	·83333	·85714	·87500	·88889	·90000	·90909	·91667	·92307	·92857	·93333	·93750	·94118
ϕ_{1}	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000
0	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000

ψ_1	·50000	·66667	·75000	·80000	·83333	·85714	·87500	·88889	·90000	·90909	·91667	·92307	·92857	·93333	·93750	·94118	·94444
ψ_2	·16667	·33333	·45000	·53052	·59524	·64286	·68056	·71111	·73636	·75758	·77570	·79121	·80476	·81667	·82721	·83660	·84503
ψ_3	·04167	·13333	·22453	·30476	·37202	·42857	·47639	·52922	·55227	·58275	·60942	·63297	·65387	·67255	·68934	·70451	·71828
ψ_4	·00833	·04444	·09643	·15238	·20668	·25714	·30316	·35281	·38234	·41625	·44691	·47473	·50002	·52309	·54421	·56360	·58146
ψ_5	·00139	·01270	·03616	·06773	·10334	·14026	·17684	·21712	·24636	·27750	·30725	·33510	·36112	·38544	·40816	·42941	·44828
ψ_6	·00020	·00318	·01205	·02709	·04697	·07013	·09522	·12407	·14782	·17344	·19881	·22340	·24708	·26981	·29154	·31958	·33133
ψ_7	·00002	·00071	·00362	·00985	·01957	·03236	·04761	·05256	·08315	·10202	·12150	·14109	·16060	·17987	·19878	·22231	·23470
ψ_8		·00014	·00099	·00328	·00753	·01587	·02222	·03308	·04401	·05668	·01034	·08466	·09942	·11446	·12964	·14821	·15959
ψ_9		·00003	·00025	·00101	·00269	·00555	·00972	·01557	·02201	·02983	·03869	·04838	·05875	·06967	·08102	·09485	·10435
ψ_{10}			·00006	·00029	·00090	·00208	·00400	·00693	·01043	·01492	·02026	·02639	·03321	·04064	·04861	·05838	·06570
ψ_{11}				·00008	·00028	·00073	·00155	·00291	·00469	·00710	·01013	·01377	·01799	·02276	·02805	·03469	·03989
ψ_{12}					·00008	·00024	·00057	·00117	·00201	·00323	·00485	·00688	·00935	·01227	·01558	·01977	·02338
ψ_{13}						·00008	·00020	·00044	·00082	·00140	·00227	·00330	·00500	·00644	·00834	·01090	·01325
ψ_{14}							·00006	·00016	·00032	·00059	·00098	·00153	·00225	·00318	·00432	·00582	·00727
ψ_{15}								·00006	·00012	·00023	·00041	·00068	·00105	·00153	·00216	·00300	·00386
ψ_{16}									·00004	·00009	·00017	·00029	·00047	·00072	·00121	·00150	·00198
ψ_{17}											·00007	·00012	·00020	·00032	·00050	·00073	·00099
ψ_{18}												·00005	·00009	·00014	·00022	·00034	·00048
ψ_{19}														·00006	·00010	·00016	·00023
ψ_{20}																·00007	·00011

It is sufficiently obvious how this last Table is formed, from the one preceding it. It requires, of course, the extension of this Table to give for the particular values required all the successive values of $\phi_1(b)$, $\phi_2(b)$, &c., $\psi_1(b)$, $\psi_2(b)$, &c., till they become so small as to be safely neglected. All the ψ's below the given percentage being added for the numerator, and the whole sum of the ϕ's and ψ's, + unity, to include the chance of the average number itself being taken for the denominator, the numbers in the final Table are obtained.

TABLE III. (*continued*).

	$b=18$.	$b=19$.	$b=20$.	$b=25$.	$b=30$.	$b=40$.	$b=50$.	$b=70$.	$b=100$.	$b=200$.	$b=500$.
ϕ_{100}											·00002
ϕ_{90}											·00019
ϕ_{80}											·00125
ϕ_{70}											·00628
ϕ_{60}										·00005	·02499
ϕ_{50}										·00102	·07925
ϕ_{40}									·00011	·01521	·20133
ϕ_{30}							·00001	·00065	·00779	·10121	·41163
ϕ_{25}						·00006	·00066	·00747	·03762	·20902	·54332
ϕ_{20}					·00021	·00305	·01203	·04936	·13040	·37438	·68042
ϕ_{19}				·00006	·00057	·00581	·01939	·06775	·16099	·41321	·70730
ϕ_{18}				·00021	·00143	·01057	·03030	·09120	·19633	·45460	·73372
ϕ_{17}			·00003	·00066	·00330	·01837	·04590	·12045	·23654	·49683	·75954
ϕ_{16}	·00003	·00007	·00015	·00184	·00707	·03062	·06751	·15614	·28169	·54003	·78465
ϕ_{15}	·00016	·00033	·00062	·00459	·01414	·04899	·09645	·19873	·33128	·58382	·80892
ϕ_{14}	·00071	·00127	·00206	·01043	·02651	·07537	·13396	·24841	·38521	·62776	·83222
ϕ_{13}	·00256	·00402	·00589	·02173	·04676	·11166	·18102	·30506	·44277	·67140	·85443
ϕ_{12}	·00769	·01100	·01473	·04179	·07794	·15951	·23818	·36801	·50315	·71426	·87544
ϕ_{11}	·01977	·02590	·03274	·07463	·12306	·21900	·30537	·43682	·56534	·75583	·89514
ϕ_{10}	·04447	·05468	·06547	·12438	·18460	·29335	·38171	·50962	·62816	·79567	·91341
ϕ_{9}	·08895	·10388	·11904	·19434	·26371	·37852	·46550	·58481	·69028	·83310	·93015
ϕ_{8}	·16010	·17943	·19840	·28580	·35960	·47314	·55416	·66027	·75031	·86781	·94527
ϕ_{7}	·26199	·28411	·30523	·39694	·46905	·57351	·64437	·73364	·80678	·89928	·95869
ϕ_{6}	·39298	·41523	·43605	·52229	·58644	·67472	·73224	·80242	·85828	·92710	·97034
ϕ_{5}	·54412	·56353	·58140	·65287	·70373	·77110	·81360	·86414	·90345	·95087	·98014
ϕ_{4}	·69846	·71381	·72675	·77722	·81200	·85678	·88435	·91651	·94109	·97027	·98804
ϕ_{3}	·83951	·84765	·85500	·88320	·90222	·92625	·94080	·95755	·97020	·98505	·99401
ϕ_{2}	·94444	·94737	·95000	·96000	·96667	·97500	·98000	·98571	·99000	·99500	·99800
ϕ_{1}	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000

0	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000
ψ_1	·94737	·95000	·95238	·96154	·96774	·97561	·98039	·98592	·99010	·99502	·99800
ψ_2	·85263	·85952	·86580	·89031	·90735	·92915	·94268	·95858	·97068	·98517	·99403
ψ_3	·75083	·74232	·75287	·79492	·82478	·86433	·88933	·91914	·94241	·97061	·98810
ψ_4	·59809	·61322	·62739	·68528	·72775	·78575	·82845	·86945	·90617	·95158	·98026
ψ_5	·46807	·48546	·50191	·57102	·62378	·69845	·74859	·81149	·86302	·92887	·97055
ψ_6	·35105	·36895	·38609	·46054	·51988	·60734	·66838	·74742	·81417	·90133	·95904
ψ_7	·25276	·26962	·28599	·35979	·42148	·51689	·58630	·67948	·76090	·87085	·94580
ψ_8	·17499	·17707	·20428	·27257	·33275	·43074	·50543	·60979	·70454	·83736	·93091
ψ_9	·11166	·12875	·14088	·20042	·25596	·35163	·42833	·54032	·64637	·80130	·91445
ψ_{10}	·07517	·08435	·09392	·14316	·19196	·28130	·35694	·47278	·58761	·76314	·89652
ψ_{11}	·04665	·05342	·06059	·09942	·14046	·22063	·29258	·40857	·52937	·72336	·87722
ψ_{12}	·02799	·03274	·03787	·06717	·10033	·16971	·23595	·34878	·47266	·68241	·85666
ψ_{13}	·01625	·01944	·02295	·04419	·06000	·12809	·18726	·29415	·41828	·64076	·83495
ψ_{14}	·00914	·01129	·01350	·02833	·04773	·09488	·14630	·24513	·36691	·59884	·81221
ψ_{15}	·00499	·00626	·00772	·01771	·03182	·06900	·11254	·20187	·31905	·55706	·78855
ψ_{16}	·00264	·00339	·00431	·01053	·02075	·04929	·08526	·16431	·27505	·51580	·76284
ψ_{17}	·00136	·00179	·00232	·00643	·01325	·03459	·06362	·13221	·23508	·47539	·73897
ψ_{18}	·00068	·00092	·00122	·00374	·00828	·02385	·04678	·10516	·19922	·43614	·71330
ψ_{19}	·00033	·00046	·00063	·00212	·00507	·01617	·03390	·08271	·16741	·39830	·68718
ψ_{20}	·00016	·00022	·00031	·00118	·00304	·01078	·02421	·06433	·13951	·36209	·66075
ψ_{25}				·00005	·00018	·00111	·00367	·01555	·04957	·21013	·52770
ψ_{30}						·00008	·00040	·00289	·01443	·10914	·40186
ψ_{40}								·00005	·00069	·02132	·20242
ψ_{50}									·00002	·00275	·08473
ψ_{60}										·00024	·02958
ψ_{70}										·00001	·00864
ψ_{80}											·00212
ψ_{90}											·00044
ψ_{100}											·00008

Finally, TABLE IV., *showing the probability on the hypothesis of the preceding investigation of the occurrences in a given period exceeding the average by more than a named percentage.*

Per Cent.	Average Numbers.									
	20.	25.	30.	40.	50.	70.	100.	200.	500.	1,000.
5	·35544	·37062	·38137	·33820	·35417	·33189	·28711	·23407	·19821	·05604
10	·27939	·29981	·25568	·23413	·21553	·18387	·14711	·07755	·02002	·00087
15	·21250	·23659	·20270	·11705	·14156	·10658	·06314	·01766	·00074	·00001–
20	·15677	·13668	·11963	·09247	·07216	·04483	·02263	·00275	·00001–	
25	·11218	·09998	·08897	·03551	·04239	·02114	·00675	·00029		
30	·07788	·07144	·04625	·02806	·01726	·00672	·00166	·00002		
40	·03433	·02244	·01482	·00658	·00297	·00063	·00006			
50	·01347	·00919	·00396	·00120	·00037	·00004				

The numbers above the columns denote the numbers that should occur according to the observed average; the numbers on the left, the percentage of deviation in excess of the average. The numbers in a column (say the one headed 50) opposite a certain percentage (say 25), give the probability of the occurrences in a particular case, in which, according to the average observations, they should be 50, exceeding this number by 25 per cent.—that is, in this case, of their being 63 or more. A table such as the above appears to be the best form of representing, at a glance, the relation between the stability of the probable results and the magnitude of the field on which they are looked for.

It is easy to see how the table (IV.) thus finally arrived at, gives the solution of the problem with which we commenced.

Table III. gives a more detailed view of the probabilities of each particular number occurring, and will also itself furnish a very good general idea of what I call the stability of these results.

The case chiefly contemplated in the foregoing pages is that of life assurance; but the columns corresponding to small values of b will give a good idea of the stability to be expected in such a business as marine assurance (for instance).

The somewhat large values of b will express the kind of risks incurred in fire and life businesses, which have not secured a large connexion. The columns for higher values of b explain how the risk arising from fluctuations of luck becomes diminished as the number of transactions increase—becoming, indeed, in the case of very large Offices, an element of much subordinate importance to the fluctuations in the money market, which appear less reducible to law.

Take, for example, the case of an Office which calculates upon losing 70 lives in the course of the year. The odds against the losses in the course of the year *exceeding* this number by 20 per cent.—that is, being greater than 84, are about 24 to 1—that is, a result which we may expect to occur only about once in 25 years.

But, looking at the possible losses in a septennial valuation of assets, we shall have to look at the result of taking 490 (say 500) as the calculated average. I must remark, by the way, that this case does not precisely fulfil the conditions of the problem, because the losses in each year are not entirely independent of one another; but the result of the distinction will simply be, practically to increase slightly the stability to be expected.

In the case now contemplated, the chances are about 5 to 1 against the losses exceeding the average by 5 per cent.—and about 50 to 1 against their exceeding the average by 10 per cent. Again: in the case now contemplated, it is considerably more than 10,000 to 1 against the number exceeding the average by 20 per cent.; and the enormous ratio in which these numbers afterwards diminish, is just the mathematical expression for the certainty of which practice has assured us that the results of such contingent transactions on a sufficiently large scale, will, within certain limits, compensate each other with unerring certainty.

I must remark, that the method by which the general part of the problem is treated in the present article, might have been offered with some hesitation, but for the identity of its result with the limiting case of the problem as more completely analysed in the former article referred to. The former method seems unexceptionable, as it affords a complete analysis of the combinations of events represented by the different numbers, and shows completely what an enormous proportion of the possible combinations of occurrences is represented by the arrangements, which nearly equalize the number of occurences in each year.

I believe the foregoing pages will be of some interest in a practical as well as a theoretical point of view. No doubt, those who are deeply conversant with the practical working of Offices, may acquire a kind of empirical knowledge of the relation between the stability and extent of such business; but, even to the experienced, it is thought that greater precision of ideas may be attained by the expression of this relation in figures; and, moreover, it is something to be assured that the stability thus experienced requires for its explanation no recondite principle of moral or physical laws, but is simply demonstrated by an arithmetical computation of the

number of possible events, between which there are no data for assigning preference of expectation.

For the sake of readers less familiar with the principles on which the reasoning with which we started is founded, it may be well to state the result of the present problem in its most elementary form.

Take the case above supposed, where 500 is the average. The proposition is this:—Out of various parts of the country, and from various occupations, &c., a number of people has been taken, of whom, according to the laws of mortality, which (for the present) we suppose known with sufficient accuracy, 500 would die in the course of the year. For every combination of circumstances which would involve the death of a number greater than 10 per cent. of the above, there are 50 which would involve the number being below this limit. For every combination of circumstances involving the death of a number exceeding the 500 by 20 per cent., there are more than 10,000 equally probable combinations of circumstances which will keep the number within that limit. It must be kept in view that the problem assumés—what, of course, is not quite realized—the true ascertainment of the average law. This, no doubt, in the first form of the problem, is implicitly involved along with the oscillations about the average here investigated; but, I think, the assumption already made will be scarcely disputed—namely, that it may be sufficiently ascertained, in order to give the problem of the oscillations here treated a sufficient standing ground of its own. That being admitted, the stability of the results, practically important as it is, is simply the arithmetical consequence, and the law of the oscillations about the average is that expressed by the above tables.

The American Life Underwriters' Convention.

[We drew attention to this important movement in a previous Number,* and now present our readers with the Report of a committee of the Convention on vital statistics. It will be observed from this Report that data have been already collected for the required purpose more extensive than those from which our "Combined Experience" Table has been deduced, and that the records of many Companies are still to be supplied. This we need hardly say is only as it should be, since it seems probable that divisions of the experience must be made on account of differences of

* *See* vol. viii., p. 268.

climate in so wide a range of country, and since also so many of the Companies are but of recent origination and can afford but slender information as to the lives of persons of advanced age. Great skill and judgment will evidently be required in the general arrangement of the data, to say nothing of that demanded in the ordinary processes of the work. The carrying out of the whole, it appears, has been confided to the committee, and we have reason to believe that no better or wiser course could have been adopted. In a future Number we hope to give the Reports sent in by other committees of the Convention.—ED. *A. M.*]

Report of the Committee on Vital Statistics.—At a Convention of American Life Underwriters, held in this city one year ago, the following preamble and resolutions, offered by Mr. Chickering, of Pittsfield, Mass., were unanimously adopted:—

"Whereas it is the opinion of this Convention (in accordance with the Report already adopted) that it is very desirable that each of the Companies in the United States should unite in contributing towards the determination of the general experience of the rate of mortality among assured lives in the United States—

"Resolved—That Messrs. Homans and Eadie, of New York, and Russell, of Hartford, be appointed a committee to carry into effect the Report on Vital Statistics, adopted by the Convention.

"Resolved—That in conferring with the different Life Companies, and soliciting statements from those not represented in this Convention, the committee assure them that such statements shall be regarded as strictly confidential.

"Resolved—That this committee be instructed to treat all statements furnished by the several Companies as strictly confidential, and to make public only the combined experience ascertained as the result of such statements. That as soon as the necessary blanks can be procured, said committee furnish them to the different Companies, requesting as early replies, and up to as recent dates, as the committee shall see fit.

"Resolved—That the expenses of collecting and publishing said statistics be paid by the Companies jointly, and each Company contributing their experience shall be entitled to one or more copies of the same."

The committee thus appointed have diligently considered the important subjects committed to their care, and have spared no pains to induce all American Life Companies to unite in a project of such practical utility and interest.

In order to carry out the objects thus contemplated, the committee addressed a circular letter to each of the following Companies:—Union Mutual, Portland, Maine; National, Montpelier, Vermont; Massachusetts Hospital, Boston, Mass.; New England Mutual, Boston, Mass.; State Mutual, Worcester, Mass.; Berkshire Mutual, Pittsfield, Mass.; Massachusetts Mutual, Springfield, Mass.; American Temperance, Hartford, Connecticut; Charter Oak, Hartford, Connecticut; Ætna, Hartford, Connecticut; Connecticut Mutual, Hartford, Connecticut; American Mutual, New Haven, Connecticut; Mutual Life, New York; New York, New York; United States, New York; Manhattan, New York; Knickerbocker, New York; New York Life and Trust, New York; Guardian, New York; Washington, New York; Equitable, New York; Home, Brooklyn, N. Y.; Royal, of London and Liverpool, England; Inter-

national, London, England; Liverpool and London, England; Eagle and Albion, London, England; British Commercial, London, England; Mutual Benefit, Newark, N. J.; Penn Mutual, Philadelphia, Penn.; Pennsylvania, Philadelphia, Penn.; Girard, Philadelphia, Penn.; American, Philadelphia, Penn.; United States, Philadelphia, Penn.; Philadelphia, Philadelphia, Penn.; Baltimore, Baltimore, Md.; Southern Mutual, Columbia, S. C.; Wisconsin Mutual, Milwaukie, Wisconsin; Canada, Hamilton, Canada West; in all, thirty-nine Companies.

To this circular favourable replies were received from the following Companies, who have agreed to contribute from the records of their policies the data requested to a general fund:—Union Mutual, Portland, Maine; National, Montpelier, Vermont; N. E. Mutual, Boston, Mass.; Berkshire, Pittsfield, Mass.; Am. Temperance, Hartford, Connecticut; Ætna, Hartford, Connecticut; Charter Oak, Hartford, Connecticut; Connecticut Mutual, Hartford, Connecticut; New York Life, New York, N. Y.; Manhattan, New York, N. Y.; Knickerbocker, New York, N. Y.; Mutual Life, New York, N. Y.; Washington, New York, N. Y.; Equitable, New York, N.Y.; Guardian, New York, N. Y.; Mutual Benefit, Newark, N. J.; Penn Mutual, Philadelphia, Penn.; Girard Life and Trust, Philadelphia, Penn.; Am. Life and Trust, Philadelphia, Penn.; Southern Mutual, Columbia, S.C.; Canada Life, Canada; Royal, Liverpool and London, England; in all, twenty-two Companies.

The United States, of New York, contemplate the publication of their experience in a separate form.

Completed returns from the following Companies have been received by the committee:—National, Etna, Connecticut Mutual, Royal (of Liverpool and London), Equitable, Girard Life and Trust, American Temperance, Charter Oak, New York Life, Mutual Life of New York, Washington, American Life and Trust, and Southern Mutual; in all, thirteen Companies.

No replies have been received from other Companies, with the exception of one letter from the Secretary of the American Mutual, of New Haven, declining to furnish the *data*, but offering to furnish *results* deduced by themselves, which proposition the committee would respectfully refer to the Convention for action.

The committee have thought that the object contemplated in their appointment would be the better attained if the contributions from the various Companies were to remain unopened until further action by the Convention, when it was hoped that all contributing Companies, including those *not* represented in the last Convention, would be present, and have a voice in the final disposition of the data. The committee have then confined themselves to the duty of inducing as many Companies as possible to contribute their quotas to a general fund, and now place the various contributions at the disposal of the Convention in the same order as when first received.

The committee have thus collected in a simple, concise, yet comprehensive form, the elementary data according to the official record of thirteen Life Companies, and have, in addition, data promised to them from nine other Companies. It is not yet known how many lives and policies, or rather *years of life*, are thus embraced, but the number is undoubtedly larger than that from which was deduced the celebrated "Actuaries'," or "Combined Experience" Table of Mortality.

It now remains for the Convention to decide upon what shall be done

with the valuable materials thus gathered, at the cost of so much time and labour.

There is at the present moment a fund of no less than 22,000,000 dollars held in trust and invested by American Life Companies, with annual premiums amounting to more than 7,000,000 dollars, covering policies of about 180,000,000 dollars on the lives of nearly 60,000 American citizens. Is it not time, in a mere pecuniary point of view, that some efforts were made to ascertain the laws governing the duration of life in the localities thus covered? It has been well said that a knowledge of the laws of mortality is the very essence and foundation of the system of life insurance; in fact, a table of mortality may be called the keystone of the arch upon which the vast superstructure of life insurance is based, and upon the accuracy of our tables depend, in a great measure, the stability and safety of the great institution (amounting, in this country, to so many millions of a most sacred fund), in which we are all so much interested.

We have now, Mr. President, from the best of all sources—viz., abstracts from the actual records of the Companies themselves, the preliminary data for constructing tables and deducing sound and reliable information of the greatest practical value. In short, Sir, we have now the opportunity of substituting certainty for uncertainty, facts for mere conjecture, and of placing the whole system of insurance, heretofore greatly dependent upon individual judgment, or at least upon foreign observations, upon the broad foundation of scientific investigation. If the contributing Companies would so arrange their books as that, at stated times—once in five or ten years, for instance—a *census* might be taken, so that we then could correct or corroborate our tables from actual experience, little more could be desired.

In placing the contributions in the hands of the Convention, the committee would recommend that the task of deducing practical tables having special reference to the comparative mortality in the different sections into which our country has been divided, at different ages or epochs of life, both among native and foreign assured, the determination of the extra risk on voyages, &c., and the value of selection, &c., be confided to a proper committee.

SHEPPARD HOMANS.
THOMAS W. RUSSELL.
JOHN EADIE.

New York, May, 1860.

A long discussion seems to have followed the reading of the Report. We have space only for some observations by Mr. Homans, the chairman of the committee, with reference to their *modus operandi.* That gentleman says, "The committee have not consulted together very much on the best plan to be adopted in relation to the matter before the Convention; but it seems to me that the best plan would be, perhaps, to appoint a committee to deduce the results, and give them authority to employ a person to arrange these data in a convenient form. I take it for granted that no person employed in a Life Company could give the time necessary for such a work. I know it would be impossible for me to do it, but it could be done by a clerk at a salary of, say 50 dollars a month, who would arrange the matters in such a way that the committee could at once deduce the information they require, and the different tables that they think requisite, or that the Con-

vention thinks requisite. That, I think, would be the cheapest and the most certain way of arriving at the information sought for. Of course, the tables of mortality could only be deduced by persons who have given some little attention to that subject. There are many persons in this country who would be willing to give their assistance freely—and, indeed, this is not a very small labour—for the sake of helping on the cause of Life Insurance. I have no doubt that any committee which the Convention will appoint will freely give their information and labour."

CORRESPONDENCE.

AUTHORSHIP OF THE TREATISE ON PROBABILITY PUBLISHED BY THE SOCIETY FOR THE DIFFUSION OF USEFUL KNOWLEDGE.

To the Editor of the Assurance Magazine.

DEAR SIR,—In your last Number (p. 143) my friend Sir John Lubbock says, in a note relative to the Useful Knowledge treatise on probability by himself and the late Mr. Drinkwater Bethune, that "if anyone shall pretend that this work was written by De Morgan, I can produce the letter of my lamented friend with which he furnished our manuscript to Mr. Coates." This is a sly joke about the state of things which I shall describe; but to those who are not in the secret there will be something mysterious about it. I am glad of the opportunity of once more setting right a mistake which has now lasted at least fifteen years.

When anything anonymous on mathematics turns up in the Useful Knowledge publications, it was written by me: this is one of the fundamental laws of thought. Bacon says that the mind delights in springing up to general maxims, that it may find rest; and the mind of the catalogue-maker finds rest in the maxim that I am the author of all the anonymous mathematics published by the Useful Knowledge Society. About 1845, a binder, in obedience to the general law, stamped an issue of Lubbock and Drinkwater (Bethune) on *Probability* with the title "De Morgan on Probabilities," in gold letters. In the *Arabian Nights*, when a traveller tells the Sultan some particularly veracious story, it is ordered to be written in letters of gold and deposited in the archives of the kingdom. The story above is as well entitled to the bright alphabet as any which the Princess Schehezerade (or Scheherezade, I forget which) ever told for one day more of life; but our manners, though not wholly averse from lying, properly gilt, do not absolutely require us to perpetuate false titles; I therefore printed one correction of this mistake in my "Arithmetical Books" (1847), and others in other places. But what can a sober black-and-white statement, shut up inside a book, do against a brilliant misstatement on the outside? The only chance is to make the statement long enough to attract attention, and I have spun this letter out accordingly.

Yours truly,

November 8, 1860. A. DE MORGAN.

THE LATE MR. HILLMAN'S TABLES OF THE VALUES OF LIFE ASSURANCE POLICIES.

To the Editor of the Assurance Magazine.

SIR,—Having recently had occasion to use the values of policies at 4 per cent. "Experience" rate of mortality, I very naturally sought to avail myself of the labours of the late Mr. Hillman, who, in his *Value of Life Policies* published the values at the rate above mentioned—that being the only published table, according to the "combined experience" at 4 per cent., with which I am acquainted. For the object I had in hand this promised to be a valuable assistance, inasmuch as it would save the necessity of my forming the values which I had there hoped to find were at my service. Instead, however, of obtaining the expected aid, I discovered that a considerable number of the values I had based upon Mr. Hillman's figures were untrue, owing to the extreme inaccuracy of the table referred to, not a page of which is without numerous errors.

It is painful to criticise the work of a deceased member of the profession, but I am impelled to do so as a duty to others who, like myself, may be tempted to avail themselves of these tables, which are stated to be the result of many years' labour, and to have been printed "for circulation among those engaged in the business of life assurance," in consequence of several of Mr. Hillman's more immediate friends being desirous of possessing a copy. The work is commended on the ground that, "if it will not much abridge the labour of the computations, it will form results by which they may be verified." The compiler also expresses his obligation to four gentlemen named, and others, for assistance rendered him during the compilation of the work. It therefore naturally follows that a certain amount of confidence will be reposed in the values thus accredited, which are there so extensively collected, and which, if trustworthy, would be of great value. Unfortunately, such confidence would be sadly misplaced, inasmuch as in the table at 4 per cent. Experience (to which my examination has been limited), at pages 69 to the end, the number of errors is, on an average, 10 to each page; besides nearly nine hundred differences which I have noted to the extent of + or —1 and 2 in the final figures, and which appear to have arisen from incorrect interpolation of the logarithmic results.

That this communication may not be considered undeserving the notice of the profession of actuaries, and to show that I am not influenced by any feeling towards the deceased gentleman by whose name the tables are sanctioned, and with whom I was in no way acquainted, I append a list of the incorrect values above referred to, in order that others may possibly be spared the inconvenience to which I have been subjected in correcting the erroneous results at which I had arrived by confiding in so untrustworthy a pioneer. Allow me, in conclusion, to observe that some of the errors are so obvious as to be readily detected by inspection of the differences, by which means I first alighted upon a portion of them—sufficient, indeed, to throw distrust upon the whole. A recomputation of the entire table by myself has shown how far that distrust was well founded.

As further evidence of the want of attention in the compilation, I may instance Tables IV., V., and VI., being the preparatory tables

for computing the present values of policies at the rates of 3, 3½, and 4 per cent. Experience, the logarithms of the annuities payable immediately ($\lambda\overline{1+a}$) from age 93 to the final age, in each of the three tables, are given with a negative index—an error manifest to the merest tyro, and which, had those logarithms been used in the computation of the Tables of Values Nos. X., XI., and XII., founded on them, would have produced values of one-tenth only of the correct results.

It is very probable that the errors to which I thus draw attention may have been observed by others, who may have felt, as I do, a delicacy in making the fact public. I trust, however, that I shall be acquitted of having transgressed the requirements of good feeling, for I may truly say that I have no other object in view than to save others from falling into a difficulty similar to that which I have encountered.

I am, Sir,

Your obedient servant,

Eagle Life Office,
10th December, 1860.

SAMUEL L. LAUNDY.

A Schedule of Ninety-nine Errors referred to in the foregoing Letter.

Page 69,	Age	14,	column	3	years,	·01517	should	be	·01523
,,	,,	15	,,	5	,,	·02770	,,		·02735
,,	,,	17	,,	2	,,	·01122	,,		·01127
,,	,,	18	,,	2	,,	·01172	,,		·01169
,,	,,	23	,,	3	,,	·02185	,,		·02176
,,	,,	23	,,	5	,,	·01748	,,		·03748
,,	,,	27	,,	5	,,	·04400	,,		·04397
,,	,,	29	,,	1	,,	·00894	,,		·00890
,,	,,	31	,,	3	,,	·02994	,,		·03005
,,	,,	33	,,	2	,,	·02141	,,		·02146
,,	,,	39	,,	5	,,	·07321	,,		·07342
,,	,,	40	,,	2	,,	·02927	,,		·02933
,,	,,	45	,,	1	,,	·01800	,,		·01797
,,	,,	49	,,	5	,,	·10705	,,		·10715
,,	,,	50	,,	3	,,	·06567	,,		·06570
,,	,,	51	,,	2	,,	·04501	,,		·04507
,,	,,	52	,,	3	,,	·07133	,,		·07032
,,	,,	55	,,	2	,,	·05067	,,		·05168
,,	,,	56	,,	3	,,	·08146	,,		·08046
Page 70,	,,	16	,,	9	,,	·05596	,,		·05494
,,	,,	28	,,	9	,,	·08790	,,		·08802
,,	,,	42	,,	9	,,	·15644	,,		·15633
,,	,,	45	,,	8	,,	·15295	,,		·15292
,,	,,	46	,,	6	,,	·11792	,,		·11693
,,	,,	46	,,	9	,,	·17912	,,		·17903
,,	,,	49	,,	7	,,	·15160	,,		·15170
,,	,,	57	,,	10	,,	·27641	,,		·27661
Page 71,	,,	18	,,	11	,,	·07473	,,		·07515
,,	,,	27	,,	11	,,	·10688	,,		·10685
,,	,,	33	,,	15	,,	·19861	,,		·19869

Page 71,	Age	34,	column	15	years,	·20823	should	be	·20640
„	„	40	„	15	„	·25587	„		·25570
„	„	47	„	15	„	·31533	„		·31690
Page 72,	„	18	„	18	„	·13989	„		·13886
„	„	38	„	17	„	·27355	„		·27555
„	„	43	„	16	„	·30101	„		·30200
„	„	45	„	18	„	·36400	„		·36299
„	„	47	„	20	„	·42633	„		·42607
„	„	60	„	20	„	·55258	„		·55247
Page 73,	„	16	„	24	„	·19126	„		·19114
„	„	20	„	22	„	·19790	„		·19691
„	„	30	„	25	„	·32679	„		·33603
„	„	33	„	21	„	·29812	„		·29892
„	„	42	„	24	„	·45435	„		·45426
„	„	56	„	23	„	·58011	„		·58021
Page 74,	„	22	„	26	„	·26914	„		·26895
„	„	25	„	28	„	·33969	„		·33069
„	„	38	„	28	„	·48452	„		·48440
„	„	40	„	26	„	·47039	„		·47027
„	„	41	„	26	„	·48294	„		·48194
„	„	44	„	30	„	·59198	„		·59098
„	„	52	„	26	„	·60163	„		·60082
„	„	53	„	27	„	·62943	„		·62964
„	„	58	„	28	„	·60871	„		·69871
Page 75,	„	14	„	32	„	·27231	„		·27398
„	„	22	„	33	„	·37638	„		·37628
„	„	27	„	35	„	·47242	„		·47169
„	„	35	„	35	„	·57323	„		·57320
„	„	43	„	31	„	·59765	„		·59776
„	„	49	„	32	„	·67836	„		·67805
„	„	50	„	34	„	·72049	„		·72079
„	„	52	„	34	„	·74176	„		·74170
„	„	54	„	34	„	·76182	„		·76382
„	„	57	„	31	„	·74441	„		·74461
Page 76,	„	28	„	38	„	·53658	„		·53568
„	„	31	„	37	„	·55124	„		·55724
„	„	31	„	38	„	·57043	„		·57402
„	„	32	„	39	„	·60311	„		·60301
„	„	34	„	37	„	·59460	„		·59469
„	„	34	„	40	„	·64426	„		·64326
„	„	39	„	36	„	·63740	„		·63749
„	„	39	„	38	„	·67019	„		·66942
„	„	41	„	38	„	·69098	„		·69113
„	„	46	„	38	„	·74212	„		·74222
„	„	47	„	39	„	·76285	„		·76755
Page 77,	„	22	„	43	„	·53005	„		·53995
„	„	26	„	42	„	·57504	„		·57594
„	„	27	„	41	„	·57245	„		·57255
„	„	33	„	44	„	·69312	„		·69212
„	„	33	„	45	„	·69646	„		·70646
„	„	34	„	41	„	·65980	„		·65878
„	„	34	„	45	„	·71733	„		·71739

Page 77,	Age	35	column	44	years,	·71524	should	be	·71424
„	„	41	„	44	„	·77667	„		·77656
„	„	42	„	45	„	·80076	„		·80065
„	„	45	„	42	„	·78552	„		·79040
„	„	45	„	43	„	·80480	„		.80474
Page 78,	„	22	„	47	„	·60449	„		·60357
„	„	22	„	49	„	·63498	„		·63414
„	„	23	„	46	„	·60080	„		·60089
„	„	23	„	50	„	·66130	„		·66139
„	„	26	„	46	„	·64703	„		·63880
„	„	30	„	47	„	·70160	„		·70100
„	„	32	„	47	„	·71674	„		·72319
„	„	34	„	46	„	·73150	„		·73112
„	„	34	„	49	„	·77046	„		·77041
„	„	35	„	46	„	·74267	„		·74166
„	„	37	„	50	„	·81826	„		·81402
„	„	40	„	46	„	·79303	„		·79321

MISS FLORENCE NIGHTINGALE'S "NOTES ON NURSING."*

To the Editor of the Assurance Magazine.

SIR,—The important bearing which all matters connected with social and sanitary science have upon the health and longevity of the people, will be my apology for offering a few remarks on the above work, which, though entitled by the estimable Author *Notes on Nursing*, is devoted, to a considerable extent, to household *hygiène*—a subject entirely cognate to the questions usually discussed in the pages of the *Assurance Magazine*, as affecting, in the first place, the question of sanitary reform, in which we are all alike interested, and, in the second, the deepest interests of the assurance community.

With respect to that portion of the *Notes* which refers to nursing *proper*, I have only to say that the experience of the Author most fully qualifies her to lay down rules on the subject, and that every word she utters teems with valuable advice to those who have the personal charge of the health of others entrusted to them.

In connexion with *hygiène*, Miss Nightingale lays great stress upon the importance of procuring in our houses a due supply of fresh external air, by means of proper ventilation, a matter far too little attended to in general in this country, and she shows how it is possible to secure efficient ventilation without necessarily producing draughts of cold air—by means of proper arrangements as to the opening of the windows of houses, which, even at night, she considers to be unobjectionable.

Referring to the popular idea that exposure to the night air is generally undesirable, she says, that "in great cities, night air is often the best and purest air to be had in the twenty-four hours," and quotes the opinion of one of the highest medical authorities on consumption and climate in confirmation of the fact, that "the air in London is never so good as after ten o'clock at night."

* *Notes on Nursing: What it is, and What it is not.* By Florence Nightingale. London, Harrison, 59, Pall Mall.

While giving Miss Nightingale credit for having fully considered any subject on which she expresses an opinion, I must admit that her views militate against all my preconceived notions on this subject. The fact is patent to most of us, that a change from a warm room to the cold air has a most determined effect in producing irritation of the air passages of the lungs, if there should chance to exist the least tendency to weakness in this respect; and though I do not pretend to deny that the air at night may, in large cities, be purer than in the day-time, owing to the fact that a less amount of carbon is likely to be floating in the atmosphere after the extinction of the greater proportion of the fires used for domestic purposes—still the air being at night unquestionably colder than it is in the day-time, and the irritation produced by it on the delicate passages of the respiratory apparatus being clearly in proportion to the temperature, we have to consider how far the greater purity of the night air may compensate for evils likely to be produced by its lower temperature, and we must then strike a balance between the advantages and disadvantages of exposure to it.

There are five essential points, we are told, to which due attention must be paid, if we desire to secure the elements of health in our houses. These are—pure air, pure water, efficient drainage, cleanliness, and light.

In connexion with this subject, Miss Nightingale refers to the interests of Life Offices in sanitary matters, and says—"The object in building a house is to obtain the largest interest for the money, not to save doctors' bills to the tenants. But, if tenants should ever become so wise as to refuse to occupy unhealthily constructed houses, and if Insurance Companies should ever come to understand their interest so thoroughly as to pay a Sanitary Surveyor to look after the houses where their clients live, speculative architects would speedily be brought to their senses."

And again: "In Life Insurance and such like societies, were they instead of having the persons examined by a medical man, to have the houses, conditions, and ways of life, of these persons examined, at how much truer results would they arrive!"

"Undoubtedly," she adds, "a person of no scientific knowledge whatever but of observation and experience in these kinds of conditions, will be able to arrive at a much truer guess as to the probable duration of life of members of a family or inmates of a house, than the most scientific physician to whom the same persons are brought to have their pulse felt; no enquiry being made into their conditions."

There is some truth in this; though, I fear, it would be impracticable to carry into effect our author's suggestions.

"Minute enquiries into conditions," she goes on to say, "enable us to know that in such a district, nay, in such a street—or even on one side of that street, in such a particular house, or even on one floor of that particular house, will be the excess of mortality, that is, the person will die who ought not to have died before old age."

"Now, would it not very materially alter the opinion of whoever were endeavouring to form one, if he knew that from that floor, of that house, of that street the man came."

There can, certainly, be no doubt that with reference to one large class of diseases—zymotic diseases—as has been clearly shown in communications in these pages, we hold in our own hands the key to the mode of conduct by which alone we are likely to be enabled to grapple with them—and this key is, sanitary reform.

The principal diseases of this class are, fever, small-pox, scarlatina, whooping-cough, measles, croup, and diarrhœa.

That these diseases are, to no inconsiderable extent, of a preventable nature, there is, I apprehend, no reason to doubt. I cannot, however, entirely coincide with Miss Nightingale in her views on this subject.

"It is commonly thought," she says, "that children must have what are commonly called 'children's epidemics,' 'current contagions,' &c., in other words, that they are born to have measles, whooping-cough, perhaps even scarlet fever, just as they are born to cut their teeth, if they live."

"'Now, do tell us, why must a child have measles?' 'Oh, because,' you say, 'we cannot keep it from infection—other children have measles—and it must take them—and it is safer that it should.' 'But why must other children have measles? And if they have, why must yours have them too? If you believed in and observed the laws for preserving the health of houses which inculcate cleanliness, ventilation, white-washing, and other means, and which, by the way, *are laws*, as implicitly as you believe in the popular opinion, for it is nothing more than an opinion, that your child must have children's epidemics, don't you think that upon the whole your child would be more likely to escape altogether?'"

I fear that this is going a little too far. Though the means suggested would, and do, no doubt, tend to diminish the malignity of certain types of disease, it can hardly be predicated that the most complete attention to carry out such means to the very utmost would necessarily have the effect of eliminating the diseases in question; and for this reason, among others —that we do not find immunity from them, or anything like it, even among those classes, the sanitary state of whose dwellings, and whose position generally, is as nearly perfect in such respects as it possibly could be.

While cordially recommending Miss Nightingale's work to the attention of the readers of the *Assurance Magazine*, and thanking her for a contribution to sanitary science of no mean value, I have felt called upon to express conscientiously in what respect I dissent from her opinions.

Miss Nightingale is, perhaps, a little enthusiastic in the subject which she has taken up so warmly; but, when we remember the extraordinary devotion with which this Lady has sacrificed herself to the interests of suffering humanity, we can hardly expect that she would be otherwise than an enthusiast in relation to any matters connected with the sacred calling she so nobly adopted at her Country's sorest need.

I am, Sir,

Your obedient servant,

H. W. PORTER.

Alliance Assurance Office,
12th December, 1860.

THE

ASSURANCE MAGAZINE,

AND

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On the Rates of Premium required to provide certain Periodical Returns to the Assured. By ROBERT TUCKER, ESQ., *Actuary to the Pelican Life Insurance Company, and one of the Vice-Presidents of the Institute of Actuaries.*

[Read before the Institute the 28th January, 1861, and printed by order of the Council.]

A WELL-KNOWN contributor to the *Assurance Magazine* has drawn attention, at pp. 167 and 168 of the Number for April, 1859, to "the incongruity existing between the rates of premium charged at certain ages on bonus policies, and the benefits to which they entitle the holder."

This question, "H. A. S." remarks, "hardly meets with the consideration it deserves among Actuaries." It has, nevertheless, not been altogether lost sight of; for, so far back as the year 1851, Mr. Jellicoe noticed this "incongruity";* and, so recently as the month of March, 1857, a paper was read before the Institute of Actuaries by Mr. Sprague, and published in the July Number of the *Journal* for that year, "On certain methods of dividing the surplus among the assured in a Life Assurance Company, and *on the rates of premium that should be charged to render them equitable.*" In this paper various plans are investigated and examples

* *See* that gentleman's paper on the determination and distribution of surplus, vol. i., p. 161; also his paper on the conditions which give rise to surplus in Life Assurance Companies, and on the amount of return or "bonus" which such conditions justify, vol. ii., p. 333.

given—amongst them a formula almost identical with that by "H. A. S."

I propose to discuss this subject a little more in detail, and to introduce illustrations of some of the plans adopted by Assurance Institutions for the distribution of what, as it appears to me, cannot with propriety, in all cases, be called their *surplus profits*, but which may be designated as periodical returns made to the assured.

The example given in the letter referred to supposes a reversionary bonus of P per £1 per annum declared every t years, which is added to the principal sum and forms the capital upon which the next bonus is computed, and so on.

The sum insured being £1, the annual bonus for the first period is $P=B_1$; for the second period $P(1+tP)=B_2$; for the third period $P(1+tP)^2=B_3$; and, generally, for the nth period $P(1+tP)^{n-1}=B_n$. Whence, by the ordinary commutation tables, the annual premium for such a benefit at age x is—

$$\frac{M_x+B_1R_x+(B_2-B_1)R_{x+t}+(B_3-B_2)R_{x+2t}+\text{ \&c.}}{N_{x-1}} \quad . \quad . \quad . \quad (1)$$

If we substitute for B_1, B_2, B_3, &c., their values, we have—

$$\begin{aligned}
B_1&=P,\\
B_2-B_1&=P(1+tP)-P=tP^2,\\
B_3-B_2&=P(1+tP)^2-P(1+tP)=tP^2(1+tP),\\
B_4-B_3&=P(1+tP)^3-P(1+tP)^2=tP^2(1+tP)^2,\\
\text{\&c.}&=\text{\&c.};
\end{aligned}$$

and by dividing the annual premium into two parts, we obtain for the sum assured—

$$\frac{M_x}{N_{x-1}},$$

and for the bonus—

$$\frac{PR_x+tP^2.\{R_{x+t}+(1+tP)R_{x+2t}+(1+tP)^2R_{x+3t}+\text{ \&c.}\}}{N_{x-1}}.$$

Mr. Sprague commences with R_{x+1}, which, by the ordinary notation, is the value of an increasing assurance at age $(x+1)$ years, and implies that the additions are made according to the number of years of life completed by the assured, and not upon the number of premiums paid, which is the assumption in the other case. Subject to this alteration, the expression last obtained would be identical with that given by him.

The following examples are added by "H. A. S.," showing, according to the Carlisle 3 per cent. Table—1st, the annual premium to insure £1,000, at the ages stated; 2nd, the annual

premium required to meet the increasing benefit above described; and 3rd, the ratio of the first to the second.

Age.	Premium for £1,000.	Premium for £1,000, with Bonus.	Ratio.
20	14·9358	24·3651	1·63
30	19·5192	29·6399	1·52
40	25·9932	36·8245	1·42
50	36·2236	47·7889	1·32
60	57·8955	70·1644	1·21

It is then noticed that if an addition of 30 per cent. were made to the ordinary premium, the entrants at age 50 "get a reasonable equivalent for their payments," whilst at other ages the benefits are greatly in favour of the younger lives. It is worthy of remark that those who advocate a constant addition to the net premium, in preference to a percentage on it, may here obtain a confirmation of their views; for it is observable that an addition of £10 to the annual premium, without bonus, would very nearly provide for this reversionary bonus of £15 per annum, with its accumulations.

If we make the comparison by the Northampton 3 per cent. Table—which, perhaps, more correctly represents the premiums charged by Offices adopting this mode of apportionment—the following table shows that the inequality above noticed nearly disappears. It also proves that members entering at the earlier ages of life do not contribute unduly in the shape of premium, as has been often alleged, when the ultimate benefit insured to them is taken into account.

Age.	Premium for £1,000.	Premium for £1,000. with Bonus.	Ratio.
20	21·794	24·365	1·12
30	26·672	29·640	1·11
40	33·975	36·825	1·08
50	45·301	47·789	1·05
60	63·661	70·164	1·10

In order to obtain a practical result from a comparison of our theoretical conclusions with the bonuses prevailing according to the practice of Assurance Offices in their declarations of profit, it is necessary in our illustrations to endeavour to adapt them to such practice. For instance, the reversionary addition of P per £1 per annum $=B_1$ does not usually date from the commencement of the

assurance, unless the life assured be living at the end of the first period of t years; in like manner the subsequent annual increments B_2-B_1, B_3-B_2, &c., do not vest unless the assured be living at the end of $2t$, $3t$, &c., years respectively. Thus, in fact, the assured has an *interval bonus* at the same rate as that he was entitled to on completing the last period of t years.

The value, therefore, of the periodical additions, and of the prospective or interim bonuses, will be

$$tP.M_{x+t}+P.R_{x+t}+(B_2-B_1).(tM_{x+2t}+R_{x+2t})+$$
$$(B_3-B_2).(tM_{x+3t}+R_{x+3t})+\&c.;$$

and the annual premium for the whole benefit

$$\frac{M_x+tP.M_{x+t}+P.R_{x+t}+(B_2-B_1).(tM_{x+2t}+R_{x+2t})+\&c.}{N_{x-1}} \quad . \quad . \quad (2)$$

After the expiration of t years, a bonus of P per £1 per annum being added for every future premium paid, the value of these additions is evidently $P.R_{x+t}$; and the values of the subsequent increments are $(B_2-B_1).R_{x+2t}$, $(B_3-B_2)\,R_{x+3t}$, &c.

This alteration will not materially affect the results already given, and, as the computations are somewhat laborious, it seems scarcely necessary to ascertain the exact difference at the ages enumerated.

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Here the assured has an *interval bonus* after surviving the first t years.

When the bonus is declared at each period of t years, and no addition is made in the event of death occurring between any two periods of division, the annual premium will be—

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-1 [illegible] at a given rate
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satisfactory conclusion can be arrived at; and the statement must be received as the mere expression of an individual opinion.*

The truth is, that the whole bonus system is a matter of agreement. The public are invited to pay certain premiums, manifestly higher than are necessary for the risks undertaken, and for which the assured are to receive certain periodical returns—not, be it observed, "guaranteed," but dependent on the success of the particular Office they have selected.

There is clearly nothing unfair in practice, nor unsound in principle, in such arrangement, provided the returns are made with a due regard to the premiums paid. That some plans are more attractive than others, without being really better, may be readily imagined; but that unfair means should be resorted to by *respectable Offices* to make them so is not so easily credited.

I confess I have read with regret Mr. Scratchley's assertion, that *various respectable Offices have taken to declaring bonuses so large as to be obviously not justified by their financial condition nor consistent with security.*

It is difficult to believe that any *respectable Office,* properly so called, would do anything of the kind; and it seems to me unjustifiable that so severe a censure should be passed upon *various respectable Offices,* when the author only produces one solitary example in support of his declaration, and this is given on the authority of an experienced actuary whose name is withheld.

The extent of the error committed when the premiums paid are represented as creating large profits, is pointed out by Mr. Scratchley in a table *showing the amount that a net annual premium of £1 will assure at the death of a person of any given age, or the amount to which an annuity due of £1 will accumulate by the end of the year of his death.*

Mr. Scratchley appears to have committed the same error that some other writers have done, by treating these questions as identical. Professor De Morgan has shown, in the *Companion to the British Almanack for* 1842, that the answer to the first question is

$$\frac{N_{x-1}}{M_x},$$

$$\text{or, } \frac{(1+r).(1+A)}{1-rA};$$

* *See,* however, the remarks on this subject at page 163 of the paper in vol. i., already referred to.

A being the value of a life annuity and r the rate of interest; and that the answer to the second is

$$\frac{a_{x+1}}{a_x}(1+r)+\frac{a_{x+2}}{a_x}(1+r)^2+\frac{a_{x+3}}{a_x}(1+r)^3+\ldots.$$

a_x denoting the number of persons living at any age x.

Mr. De Morgan also adds the following examples by the Northampton Table at 4 per cent., showing the average sums obtained in the two cases:—

Age.	1st Case.	2nd Case.	Age.	1st Case.	2nd Case.
20	49·4	98·3	45	27·2	38·0
25	44·7	82·2	50	23·2	30·7
30	40·2	68·6	55	19·7	24·5
35	35·7	56·8	60	17·0	19·2
40	31·3	46·7			

And Mr. Hardy enters into an elaborate investigation of *the proper method of determining the amount of an annuity forborne and improved at interest during the existence of a given life,* in a paper read before the Institute on the 26th January, 1857, and published in the April number of the *Journal* for that year, clearly demonstrating that the expressions above given are identical only when *the annuity is forborne and improved for a term of years certain,* or *when money bears no interest*—in fact, that one is the average of *present values,* the other of *amounts.*

Of Compound Interest. By Dr. Edmund Halley, *Royal Astronomer, Savilian Professor of Geometry, Oxford, and* F.R.S.*

A PRINCIPAL use of logarithms is to solve all the cases of compound interest which are not, without great difficulty, attainable by the rules of common arithmetic. But before we proceed to the practical part, it may, perhaps, not be improper to say something of the foundation or demonstration of the rules we are to give.

Therefore, let p be any sum of money forborne t times, r the rate of interest or produce of £1 and its interest in one time—that is, as 1 to r, so £1 to its amount after one year or other space of

* We republish this paper from *Sherwin's Mathematical Tables,* printed for W. and J. Mount, 1761; and, considering the celebrity of the writer, the ability displayed in the paper itself, and the comparative scarcity of the work from which it is taken, we believe we do no more than consult the wishes of our readers in causing it to reappear in the pages of this *Journal.*—Ed. *A. M.*

time; and let m be the amount of the sum p forborne t times. Now, because in one year or time unity becomes r, by the same reason r will in another time become rr, and rr in a third time become r^3, &c., it appears that r^t, or r raised to the power whose index is the number of times, will be the amount of £1 forborne t times; and, therefore—

I. The amount $m=pr^t$; therefore multiply the logarithm of r by t, and to the product add the logarithm of p; the sum shall be the logarithm of m.

Example.—What is the amount of £15. 17*s.* 6*d.* forborne 12 years, at 6 per cent. per annum compound interest?

The number $1{\cdot}06=r$ . . . . . its log.	0·0253059
Which log. multiplied by $t=12$, the years, produces .	0·3036708
The principal sum £15. 17*s.* 6*d.*=£15·875=p, its log.	1·2007137
The amount £31. 18*s.* 10½*d.*=£31·94362=m, its log.	1·5043845

II. $r^t=\frac{m}{p}$; therefore, if from the logarithm of m the logarithm of p be subtracted, and the remainder be divided by t, the quotient is the logarithm of r.

Example.—What is the rate of compound interest when the sum of £15. 17*s.* 6*d.*, forborne 12 years, amounts to £31. 18*s.* 10½*d.*?

The amount £31. 18*s.* 10½*d.*=£31·94362=m, its log.	1·5043845
The principal sum £15. 17*s.* 6*d.*=£15·875=p, its log.	1·2007137
The remainder is log. of r^t	0·3036708
Which, divided by $t=12$, quotes log. of ($1{\cdot}06=r$) .	0·0253059

Therefore the rate is 6 per cent. per annum.

III. Because $r^t=\frac{m}{p}$; divide the difference of the logarithms of m and p by the logarithm of r; the quotient is t, or the time wherein the sum p will amount to m at the rate r.

Example.—In what time will the principal sum of £15. 17*s.* 6*d.* amount to £31. 18*s.* 10½*d.* at 6 per cent. per annum compound interest?

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The remainder is log. of r^t	0·3036708

Which, divided by 0·0253059=log. of r, quotes 12 years for the time.

IV. $p=\frac{m}{r^t}$; therefore multiply the logarithm of r by t, and

subtract the product from the log. of m; the remainder shall be the log. of p, the principal sum.

Example.—What is the principal sum that in 12 years, at 6 per cent. per annum compound interest, will amount to £31. 18*s*. 10½*d*.?

The amount £31. 18*s*. 10½*d*.=£31·94362=m, its log.	1·5043845
The number 1·06=r . . . . . its log.	0·0253059
Which log., multiplied by t=12, the years, produces .	0·3036708
And subtracted from the log. of the amount—	
The remainder is log. of (£15·875=£15. 17*s*. 6*d*.=p)=	1·2007137

The four preceding rules are also readily deduced from the consideration of the rebate of money in this manner.

For if, in any time, r becomes 1, in the same time 1 becomes $\frac{1}{r}$, and in the second time $\frac{1}{r}$ becomes $\frac{1}{rr}$, and in the third $\frac{1}{rrr}$, &c.; so that, putting p the value or present worth of any sum m payable after t times, at the rate of r to 1:

I. The sum $m=pr^t$; therefore multiply the log. of r by t, and to the product add the log. of p, the sum shall be log. of m sought.

II. $r^t=\frac{m}{p}$; therefore from the log. of m subtract the log. of p, and divide the remainder by t; the quotient will be the log. of r.

III. Since $r^t=\frac{m}{p}$, divide the difference of the logs. of m and p by the log. of r; the quotient shall be t, the number of years.

IV. $p=\frac{m}{r^t}$; therefore multiply the log. of r by t, and subtract the product from the log. of m; the remainder will be the log. of p, which finds the value of any sum of money payable after any time assigned.

The logarithms are also serviceable to resolve all questions concerning the amount or present worth of annuities not paid as due, or purchased to be paid for in time to come.

Let a be an annuity or yearly pension whose successive amounts for times past are ar^t, and whose present values are $\frac{a}{r^t}$ successively, by what goes before; and the series, &c., ar^5, ar^4, ar^3, ar^2, ar, a, $\frac{a}{r}$, $\frac{a}{r^2}$, $\frac{a}{r^3}$, $\frac{a}{r^4}$, $\frac{a}{r^5}$, &c., will be a rank of mean proportionals continued infinitely in the ratio of r to 1. Now, the sum of all the consequents, or of the whole infinite series, will be to the said

sum increased by the next greater term (or the sum of all the antecedents) as 1 to r, by *Euclid*, v. 12; wherefore, putting y for the said sum of the consequents, ry will be equal to $y+ar^t$, the sum of the antecedents; and $ry-y=ar^t$; and, therefore, $\frac{ar^t}{r-1}$ will be equal to y, the sum of all our mean proportionals whereof ar^{t-1} is the greatest; and, by the same rule, $\frac{a}{r-1}$ will be the sum of all the terms whereof $\frac{a}{r}$ is the greatest. So that if we subtract $\frac{a}{r-1}$ from $\frac{ar^t}{r-1}$, the difference will be the sum of all the terms whereof ar^{t-1} is the greatest and a the least, their number being t, which sum we will call z; therefore z (the amount of the annuity of a forborne t times at the rate r) $=\frac{ar^t-a}{r-1}$.

I. *The annuity* (a), *rate of interest* (r), *and time* (t), *being given, to find* (z) *the amount.*

From the log. of a subtract the log. of $\overline{r-1}$, and to the remainder add the log. of r^t; from the number answering to this last sum, subtract the number answering to the remainder; the difference shall be z, the amount sought.

Example.—What will an annuity of £34·4, forborne 12½ years, amount to at 6 per cent. per annum?

$a=34{\cdot}4$	L. $a=1{\cdot}5365584$	L. $r=0{\cdot}0253059$
$r-1=0{\cdot}06$	L. $\overline{r-1}=8{\cdot}7781513$	$t=$ . . 12·5
$\frac{a}{r-1}=573{\cdot}3333$	L. $\frac{a}{r-1}=2{\cdot}7584071$	1265295
		506118
	L. $r^t=0{\cdot}3163237$	253059
$\frac{ar^t}{r-1}=1187{\cdot}7660$	L. $\frac{ar^t}{r-1}=3{\cdot}0747308$	L. $r^t=0{\cdot}3163237{\cdot}5$

$$z=£614{\cdot}4327=\frac{ar^t}{r-1}-\frac{a}{r-1}=\text{the amount.}$$

II. *The annuity* (a), *its amount* (z), *and the rate of interest* (r), *being given, to find* (t) *the time.*

By the foregoing, $\frac{ar^t-a}{r-1}=z$, therefore $z+\frac{a}{r-1}=\frac{a}{r-1}\times r^t$; wherefore, from the log. of a subtract the log. $\overline{r-1}$; to the number answering to the remainder add the given amount, and from the log. of the sum subtract the afore-found remainder; this

second remainder, divided by the log. of r, will quote the time required.

Example.—In what time will an annuity of £34·4 amount to £614·4327 at the rate of 6 per cent. per annum?

$$a=34\cdot4 \qquad \text{L. } a=1\cdot5365584$$

$$r-1=0\cdot06 \qquad \text{L. } \overline{r-1}=8\cdot7781513$$

$$\frac{a}{r-1}=573\cdot3333 \qquad \text{L. } \frac{a}{r-1}=2\cdot7584071$$

$$z=614\cdot4327$$

$$\frac{ar^t}{r-1}=1187\cdot7660 \qquad \text{L. } \frac{ar^t}{r-1}=3\cdot0747308$$

$$\text{L. } r=0\cdot0253059)0\cdot3163237(12\cdot5 \text{ years}=t.$$

III. *The amount* (z), *rate of interest* (r), *and time* (t), *being given, to find* (a) *the annuity.*

The former equation being reduced, $a=\frac{z\times\overline{r-1}}{r^t-1}$; wherefore, to the log. of the amount z, add the log. of $\overline{r-1}$, and from the sum subtract the log. of $\overline{r^t-1}$; the remainder is the log. of a.

Example.—An annuity forborne 12½ years amounts, at 6 per cent. per annum, to the sum of £614·4327, how much is that annuity?

$$z=614\cdot4327 \qquad \text{L. } z=2\cdot7884743 \qquad \text{L. } r=0\cdot0253059$$

$$r-1=0\cdot06 \qquad \text{L. } \overline{r-1}=8\cdot7781513 \qquad t=\ .\ .\ 12\cdot5$$

$$r^t=2\cdot071685 \qquad 1\cdot5666256 \qquad \text{L. } r^t=0\cdot3163237$$

$$r^t-1=1\cdot071685 \qquad \text{L. } \overline{r^t-1}=0\cdot0300671$$

$$\text{L. £34·4 the annuity}=\text{L. } a=1\cdot5365585$$

IV. *The annuity* (a), *time* (t), *and amount* (z), *being given, to find* (r) *the rate.*

In order to find r, the former equation is reduced to $\frac{z}{a}-1=\frac{z}{a}r-r^t$, or, in our present case, $16\cdot8614=17\cdot8614r-r^t$, which is so affected as not readily to be resolved by the general method for resolution of equations, unless we can first approach it by some other means; for which purpose take the following rule, which will suffice where great exactness is not required.

Let $\sqrt[t-1]{\frac{z}{at}}=1+y$, and let $\frac{6}{t+1}=b$. I say that $\overline{bb+2by}^{\frac{1}{2}}-b$ is exceeding near the increase of the rate, or $r-1$.

Wherefore take the log. of the amount, and the complements of the logarithms of the time and annuity; the sum (abating 2 in

the tens place of the index) divide by $\frac{1}{2} \times \overline{t-1}$; the quotient shall be the log. of $1+y$. Then divide 6 by $t+1$, and to b (the quotient) add twice y, and to the log. of b add the log. of $\overline{b+2y}$; half the sum shall be the log. of $\overline{bb+2by}|^{\frac{1}{2}}$, from which square root subtract b; the residue will be very near the increase, or $r-1$; and, adding 1, r is found. If great exactness be desired, let r thus found be assumed, and $\frac{z}{a}r-r^t$, compared with $\frac{z}{a}-1$, will always be greater than it; and dividing the excess by $tr^{t-1}-\frac{z}{a}$, the quotient added to r shall verify as many more figures in the rate as were true in the assumed r.

Example.—An annuity of £34·4, forborne 12½ years, amounts to £614·4328, required the rate of interest allowed.

$z=614{\cdot}4327$ L. $z=2{\cdot}7884743$
$t=12{\cdot}5$ co. L. $t=8{\cdot}9030900$
$a=34{\cdot}4$ co. L. $a=8{\cdot}4634416$

$\frac{1}{2} \times \overline{t-1}=5{\cdot}75)0{\cdot}1550059(0{\cdot}0269575=$ L. $\overline{1+y}$, and $1+y=1{\cdot}064039$

$b=\frac{6}{t+1}=\frac{6}{13{\cdot}5}=0{\cdot}4444444$ $2y=0{\cdot}128078$

$2y$ . . . . . $0{\cdot}1280710$ L. $b=9{\cdot}6478175$

$b+2y$ . . . . $0{\cdot}5725224$ L. $\overline{b+2y}=9{\cdot}7577936$

$19{\cdot}4056111$

$\overline{bb+2by}|^{\frac{1}{2}}$ . . . $0{\cdot}5044353$ L. $\overline{bb+2by}|^{\frac{1}{2}}=9{\cdot}7028055$

b . . . . . $0{\cdot}4444444$

$r-1$ . . . . $0{\cdot}0599909$

Therefore the rate of interest sought is 6 per cent. per annum.

After the same manner the four cases relating to the purchase of annuities are readily solved by logarithms, and the theorems discovered with the same ease; for a, $\frac{a}{r}$, $\frac{a}{rr}$, $\frac{a}{r^3}$, $\frac{a}{r^4}$, &c., being a scale of mean proportionals in the ratio of r to 1, put y for the sum of all the consequents infinitely continued, whereof $\frac{a}{r}$ is the first, and that sum will be to the sum of all the antecedents whereof a is the first, as 1 to r (that is, $1 : r :: y : ry$), so that $ry=y+a$, and $\frac{a}{r-1}$ will be equal to y, the value of the fee, or the sum of all the mean proportionals less than a. And, by the same rule, $\frac{a}{r^t \times \overline{r-1}}$ will be the sum of all the means less than $\frac{a}{r^t}$, or the value of the reversion; and, subtracting the one sum from the other,

$\frac{a}{r-1} - \frac{a}{r^t \times r-1}$ will be equal to z, the sum of all the means, whereof $\frac{a}{r}$ is the greatest and $\frac{a}{r^t}$ the least.

I. *The annuity* (a), *time* (t), *and rate of interest* (r), *being given, to find* (z) *the present value.*

The present value $z = \frac{a}{r-1} - \frac{a}{r^t \times r-1}$; therefore, from log. of the annuity subtract the log. of $\overline{r-1}$, and from the residue subtract the log. of r^t; the difference of the numbers answering to the two remainders is the present value sought.

Example.—What is £70 per annum, to continue 59 years, worth in present money, at the rate of 5 per cent. per annum?

$a=70$	L. $a=1{\cdot}8450980$	L. $r=0{\cdot}0211893$
$r-1=0{\cdot}05$	L. $\overline{r-1}=8\ 6989700$	$t=$ . . . 59
Fee=£1400	L. fee=$3{\cdot}1461280$	1907037
Reversion, £78·6972	L. $r^t=1{\cdot}2501687$	1059465
	L. rever.=$1{\cdot}8959593$	L. $r^t=1{\cdot}2501687$

z=£1321·3028, the present value sought.

II. *The annuity* (a), *present value* (z), *and rate of interest* (r), *being given, to find* (t) *the time.*

Now, r^t will be equal to $\frac{a}{r-1}$ (or the fee) divided by the value of the reversion—that is, by $\frac{a}{r-1} - z$; wherefore, from the log. of the annuity subtract the log. of $\overline{r-1}$; the number answering to the remainder will be the value of the fee. From the fee subtract the present worth, the residue is the value of the reversion. Take the log. of the reversion from the log. of the fee, and divide the residue by the log. of r; the quotient will be t, the number of years sought.

Example.—In what time will an annuity of £70 per annum pay off a debt of £1321·3028, allowing the creditor 5 per cent. per annum?

$a=70$	L. $a=1{\cdot}8450980$
$r-1=0{\cdot}05$	L. $\overline{r-1}=8{\cdot}6989700$
Fee=1400	L. fee=$3{\cdot}1461280$
z—1321·3028	
Reversion, 78·6972	L. rev.=$1{\cdot}8959593$

L. $r=0{\cdot}0211893)1{\cdot}2501687(59$ years$=t$.

III. *The present value* (z), *rate of interest* (r), *and time* (t), *being given, to find* (a) *the annuity.*

The former equation may be reduced to this proportion, as $1-\frac{1}{r^t}$ to z, so is $r-1$ to a, the annuity sought.

Wherefore, to the complement of the log. of $\overline{1-\frac{1}{r^t}}$, add the logs. of z and of $\overline{r-1}$, the sum shall be the log. of a.

Example.—What annuity, to continue 59 years, can be purchased for £1321·3028, at the rate of 5 per cent. per annum?

$$\frac{1}{r^t}=0{\cdot}0562123$$

$$1-\frac{1}{r^t}=0{\cdot}9437877 \qquad \text{co. L. } \overline{1-\frac{1}{r^t}}=0{\cdot}0251257$$

$$z=1321{\cdot}3028 \qquad \text{L. } z \;\ldots\; 3{\cdot}1210023$$

$$r-1=0{\cdot}05 \qquad \text{L. } \overline{r-1}=8{\cdot}6989700$$

$$\text{L. } (£70=a \text{ the annuity sought})=1{\cdot}8450980$$

$$\text{L. } r=0{\cdot}0211893$$

$$t= \;\ldots\; 59$$

$$\text{L. } r^t=1{\cdot}2501687$$

$$\text{L. } \frac{1}{r}=8{\cdot}7498313$$

IV. *The annuity* (a), *present value* (z), *and time* (t), *being given, to find* (r) *the rate of interest.*

This problem being more difficult than appears at first sight, and requiring the solution of this equation, $\frac{a}{z}=\frac{z+a}{z}r^t-\frac{t+1}{r}$, to which it is reduced, there must be applied some method of approaching the root r, which is by no means evident; and that approximation, as the number of years and rate are greater or less, cannot properly be obtained by one general rule, but rather by two, according as the value of the reversion is greater or less.

If the number of years be great (as suppose 40 or upwards), and especially if the rate of interest be high, $1+\frac{a}{z}$ will be nearly the rate; or, more accurately, $\frac{z+a}{z}-\frac{z^t}{z+a}\times\frac{a}{z}$. Call it r, and $\frac{a}{r^t\times\overline{r-1}}$ will be exceeding near the value of the reversion, which let be x; then $1+\frac{a}{z+x}$ shall approach the true rate sufficiently. But if greater exactness be desired, by repeating this process it will be obtained. Hence this rule:—from the log. of a, and also from the log. of $z+a$, take the log. of z; this latter remainder

shall be nearly the log. of the rate. Multiply that log. by t, and the complement of the product add to the first remainder; the decimal fraction answering to the sum taken from the former rate shall give a more correct rate. With this rate seek x, the reversion after the time given, which add to z; then to the complement of the log. of $\overline{z+x}$ add the log. of a; the sum shall be the log. of the increase, or of $\overline{r-1}$, sufficiently near.

Example.—If £1321·3028 is paid for an annuity of £70 per annum for 59 years to come, what is the rate of interest allowed the purchaser?

$a=70$	L. $a=1{\cdot}8450980$	L. $\overline{z+a}=3{\cdot}1434217$
$z=1321{\cdot}3028$	L. $z=3{\cdot}1210023$	L. z . . $3{\cdot}1210023$
$\frac{z+a}{z}=1{\cdot}052978$	$8{\cdot}7240957$	L. $\frac{z+a}{z}=0{\cdot}0224194$
	co. tL. $\frac{z+a}{z}=8{\cdot}6772554$	t 59
Log. of 0·00252	$7{\cdot}3013511$	2017746
First $r=1{\cdot}050458$	L. $r=0{\cdot}0213787$	1120970
	$t=$ 59	
$z=1321{\cdot}3028$	1924083	tL. $\frac{z+a}{z}=1{\cdot}3227446$
$x=$ 76·002	1068935	
$z+x=1397{\cdot}3048$ the fee	L. $r'=1{\cdot}2613433$	co. L. $r'=8{\cdot}7386567$
		co. L. $\overline{r-1}=1{\cdot}2970700$
	co. L. $\overline{z+x}=6{\cdot}8547088$	L. $a=1{\cdot}8450980$
	L. a . . $1{\cdot}8450980$	L. $x=1{\cdot}8808247$
$r-1=0{\cdot}05096$	L. $\overline{r-1}=8{\cdot}6998068$	

So that the rate of interest sought is 5 per cent. per annum.

If the number of years be small, the aforesaid rule will avail little. In this case it will be requisite to approach the rate thus:—Let $\frac{t+1}{2}$ be the index of a root of $\frac{at}{z}$; from which root subtract 1, and the remainder call y, and let $\frac{6}{t-1}$ be called b. I say that $b-\overline{bb-2by}|^{\frac{1}{2}}$ is sufficiently near to $r-1$, and will be still nearer the truth as the number of years is smaller; and that the error will be always in excess. Hence the rule:—divide the log. of $\frac{at}{z}$ by $\frac{t+1}{2}$, and from the number answering to the quotient subtract 1; double the remainder, and subtract it from b—that is,

from the quotient of 6 divided by $t-1$; to the logarithm of this remainder add the log. of b; then the number answering to half the sum of those logarithms taken from b will leave $r-1$, the increase of the rate sought.

Example.—An annuity of £20 per annum, to continue 21 years, is sold for £220: required the rate of interest allowed the purchaser.

$a=20$ L. $a=1{\cdot}3010300$
$t=21$ L. $t=1{\cdot}3222193$
$z=220$ co. L. $z=7{\cdot}6575773$

$\frac{1}{2}\times\overline{t+1}=11)0{\cdot}2808266(0{\cdot}0255297=\text{L. }\overline{1+y}$, and $1+y=1{\cdot}060546$

$2y=0{\cdot}121092$

$b=\frac{6}{t-1}=\frac{6}{20}=0{\cdot}3$ L. $b=9{\cdot}4771213$
L. $\overline{b-2y}=9{\cdot}2526298$

$2y$ . . . . . $0{\cdot}121092$ $18{\cdot}7297511$

$b-2y$ . . . . . $0{\cdot}178908$ L. $\overline{bb-2by}|^{\frac{1}{2}}=9{\cdot}3648755$
$\overline{bb-2by}|^{\frac{1}{2}}$ . . . . $0{\cdot}231673$

Now, $b-\overline{bb-2by}|^{\frac{1}{2}}=0{\cdot}068327=r-1$

Therefore $r=1{\cdot}068327$, the rate sought.

The rate r thus found is always some small matter too big, the true rate being 1·06814; but as the number of years are fewer, the error becomes insensible. If greater exactness be required, it will be easy by the general method for the resolution of equations having so near an approximation, to prosecute this inquiry as far as you please; but this seems abundantly sufficient for use, which is our principal design in this place.

Lastly, by way of corollary to the former: let it be required to find the rate of interest allowed the purchaser when he pays a sum z, for an annuity a, wherein he has already a term t, to have it prolonged for a certain time x.

Example.—An annuity of £20 per annum being in possession for the term of 21 years, and for £40 paid down it can be prolonged for 10 years more, or to 31 years; what is the rate of interest required?

Put $T=2t+x+1$, and $\frac{1}{2}T$ shall be the index of a root of $\frac{ax}{z}$. Let $\overline{\frac{ax}{z}}|^{\frac{1}{2}T}$ be equal to $1+y$, and $\frac{6T+6}{xx}=b$. I say $r-1$ is very near to $b-\overline{bb-2by}|^{\frac{1}{2}}$.

$a=20$ L. $a=1{\cdot}3010300$
$x=10$ L. $x=1{\cdot}0000000$
$z=40$ co. L. $z=8{\cdot}3979400$

$\frac{1}{2}T=26{\cdot}5)0{\cdot}6989700(0{\cdot}0263762=\text{L.}\,\overline{1+y}$, and $1+y=1{\cdot}062616$

$2y=0{\cdot}125232$

$b=\dfrac{6T+6}{xx}=3{\cdot}24$

L. $b=0{\cdot}5105450$
L. $\overline{b-2y}=0{\cdot}4934257$

$2y$ . . . . $0{\cdot}125232$

$1{\cdot}0039707$

$\overline{bb-2by}|^{\frac{1}{2}}=3{\cdot}176767$ L. $\overline{bb-2by}|^{\frac{1}{2}}=0{\cdot}5019853$

$b-\overline{bb-2by}|^{\frac{1}{2}}=0{\cdot}063233=r-1$

Therefore $r=1{\cdot}063233$, the rate sought,

as will be readily proved by seeking the value of the reversion of an annuity of £20 per annum for 10 years after 21, at the rate of 1·063233 per cent. per annum. (*See* the Work.)

$a=20$	L. $a=1{\cdot}3010300$	L. $r=0{\cdot}0266284$
$r-1=0{\cdot}063233$	co. L. $\overline{r-1}=1{\cdot}1990562$	21
	co. L. $r'=9{\cdot}4408036$	266284
Rever. £87·275	L. rev. $=1{\cdot}9408898$	532568
	co. L. $r''=9{\cdot}7337160$	L. $r'=0{\cdot}5591964$
Rever. £47·2722	L. rev. $=1{\cdot}6746058$	

£40·0028, the value sought.

Thus it appears that £40 and about three farthings is the true value of the difference of the reversions at the rate of interest before found; by which it may be judged how near an approximation the foregoing rule affords towards finding the rate of interest, when the value of an annuity for a term of years to commence after such a distant time is proposed.

*Sixth Annual Report of the Insurance Commissioners of the Commonwealth of Massachusetts.**

IN no year since the business of life assurance commenced in this country has it advanced so rapidly as in the one just closed. The nineteen Companies now making their returns to this Office show over 10,000 new policies issued, insuring more than 30,000,000 dollars, and the whole amount insured by them exceeds 150,000,000 dollars.

Ninety-four per cent. of all this business consists of whole-life policies, kept up by equal annual premiums, which are largely in excess of the annual risks during the earlier years of the insurance, but will be in defect on all the lives that continue through the later years. Hence, to cover the entire mass of risks which increase year by year, the earlier excess of the premiums, which do not increase, must be carefully accumulated to provide for the future deficiency. To apply a test to this accumulation, that the public may know, in regard to each Company, how near it comes to the happy medium between a too costly excess and a ruinous deficiency, was the object of the law requiring us to make an annual valuation of the policies of all the Companies doing business in the Commonwealth. Though it, in effect, prescribes a net valuation by not requiring of the Companies data sufficient for any other, it does not prescribe the rate of mortality or of interest that is to be assumed as the basis of the work. In adopting the "Combined Experience" or English "Actuaries" rate of mortality, and 4 per cent. interest, we supposed we were travelling most safely between extremes, and we have the pleasure to know that the fault found with our choice has been about equally divided, as much of it making our valuation too low as too high. Four per cent., as the proper rate of interest, was fully approved by the Convention of Officers of Life Insurance Companies which met in New York last May, and we believe no objection has been made by any of the American Companies to the rate of mortality adopted. It is worthy to be remarked, however, that Prof. C. F. McCay, of Georgia, in a series of valuable articles in *Hunt's Merchants' Magazine*, on the valuation of life insurance policies, has expressed the opinion that our method of valuation requires too small a premium reserve. His argument is certainly conclusive as a reply to the mathematical gentlemen, Prof. Pierce, of Cambridge, and Messrs. Woolhouse and Neison, the English actuaries, on whose authority we were charged with doing great injustice to the International Assurance Society of London, by requiring too high a reserve! It is doubtless true, as Prof. McCay proves by example, that by the combined experience mortality, the value of policies entered below the average age of insurance will be smaller for a few years than by Dr. Farr's table, and this, so far as it has any effect, makes our valuation rather more favourable to new Companies than to old ones. But the aggregate value of policies entered at all ages will not differ much from Dr. Farr's, and especially when the Company has attained a few years' standing. Prof. McCay's own figures tend to show this. To show how the "Actuaries" rate does not probably require less reserve than Dr. Farr's—the only rate which has any claim to a higher

* We have been favoured by Mr. Elizur Wright with a copy of this Report, just published, and we give a portion of it, which we have no doubt will interest our readers. It has been necessary to omit some pages of tabular matter, and some few passages of less importance than the rest.—ED. *A. M.*

scientific authority—we give below the values of five policies of 100 dollars each, entered at five different ages, at the time when the sixteenth premium falls due, according to four different rates of mortality, at 4 per cent. interest, and also add the values by using Carlisle premiums discounted by Neison's life annuities.

Age of Entry.	Com. Ex.	Dr. Farr.	Carlisle.	Neison.	Neison and Carlisle.
20.....	$11 85	$12 14	$11 99	$11 10	$12 33
30.....	17 64	17 14	15 39	15 96	16 33
40.....	25 57	24 51	23 48	23 14	23 36
50.....	34 41	34 05	32 89	32 18	33 02
60.....	43 21	42 03	41 49	42 06	41 12
Totals....	$132 68	$129 87	$125 24	$124 44	$126 16

This may serve to illustrate what is undoubtedly true, that the "Combined Experience" rate, as a means of ascertaining the proper premium reserve of an established Company, corresponds more nearly with Dr. Farr's than the other rates. For the purposes of our official valuation, it is not open to Prof. McCay's objection. And considering it as a means of adjusting the rights of different members in the same Mutual Company, it has perhaps an advantage which he does not take into account, by virtue of the probable error which he points out. When, in computing the premium, we assume, as we intentionally do, a rate of interest lower than that which will be actually realised, the effect of this reduction is to increase the premiums, but to increase those of the younger more than those of the older ages, because there is more of *time*—the food that compound interest lives on—in the case of the former. This injustice to the younger ages, from assuming an interest below the actual, may be happily compensated by a rate of mortality which makes the risk of those ages a little too small, the flexibility of one assumption correcting the excessive rigidity of the other.

It must not be inferred, from our adopting this rule of valuation as a test of the sufficiency of the actual premium reserve, that we disapprove of keeping a larger reserve. We only think it need not be left much larger at the time of distributing the surplus, or declaring dividend. The strongest believers in the deterioration of life in old Companies, by the lapse and surrender of policies on the healthier lives, have never pretended, so far as we know, that the reserve need be more than about 14 per cent. And their observations included Companies of poor credit as well as long standing, in which losses and surrenders had had a more damaging effect on the quality of the residual life than can ever occur in Companies well managed and kept in good credit. In the latter, we do not believe the deterioration of life is any reason for reserving even 3 per cent. beyond the reserve at 4 per cent. of the "Combined Experience" rate, which is founded on the mortality of Companies averaging a greater deterioration from the causes mentioned than is ever likely to occur in American Companies. Still, as a matter of general prudence and financial force, there may be ample reason for keeping a reserve 15 or 20 per cent. higher than is really needed, so that the institutions may be felt, even by the most timid, to be high and dry on the rock above any flood-tide of chance.

The valuation for the past year has been made with all possible care to avoid error, and the results are arranged in tables similar to those of last year. No doubt considerable inaccuracies have crept in among the calculations, extending to 55,360 single policies, not to speak of several thousand bonuses or reversionary additions; but we believe they do not appreciably affect the results. These show a most gratifying increase, both of the business and stability of the Companies. The ratio of the aggregate actual premium reserve to the computed is considerably greater, and the ratio of expenses to receipts is hardly 1 per cent. greater, though competition has been greatly increased by the creation of several new Companies. New Companies planted within the shadow of flourishing old ones, cannot be expected to get into successful operation without expending on their machinery more than the premium receipts for one, two, or perhaps three, of the first years. And it is, therefore, no wonder that some of the new Companies show the necessity of a guarantee capital.

This capital has in all cases been paid in cash, and invested in good securities, and if the new Companies' terms of insurance are just and liberal to the insured, there is no reason why energy, fidelity, and perseverance on its part should not soon place it beyond the need of guarantee funds. It is very true that the struggle may be arduous, and the present risk of the capital is considerable. The chance for the public-spirited capitalist to escape without loss is not such as to encourage the creation of new Companies. The worst, however, that can happen to the policy-holder with such a guarantee is, that he will not, quite so soon as he might, begin to receive back surplus from his premium; for if the capital should become too much impaired, its owners will save it from utter loss by getting their risks transferred to a strong Company. But the money spent in these experiments is by no means thrown away, even if the Companies do not succeed. The old Companies, since they probably catch the greater part of the birds started up when the bush is vigorously beaten, no matter who beats it, ought to treat the new ones with great kindness and even gratitude.

We have this year, as last, taken pains to give the ratio of our computed premium reserve to the amount insured for each year; because, by observing this ratio, it will be seen that an empirical rule, very simple and easy of practical application, may be established for ascertaining a *safe* premium reserve. It will, at any rate, do practically for all Companies that are large enough to dispense with a guarantee fund, and it is this:—

Divide the policies into groups, according to the number of annual premiums paid on each, then reserve of the amount insured in each group a percentage of twice the number of premiums paid on it. That is, on all the policies that have paid one premium, reserve 2 per cent. of the amount insured; on those that have paid two, 4 per cent.; on those that have paid three, 6 per cent., and so on.

This rule would give for all the Companies combined an aggregate reserve about 20 per cent. higher than our computed reserve, and somewhat less than the actual reserve, as will appear from the table. We do not by any means recommend this rule as a substitute for accurate arithmetical calculation, but as better than guessing without any guide.

In former Reports we have dwelt on an imperfection of life insurance, as usually practised, growing out of the nature of the contract. The insured in a whole-life policy pays in advance of the progress of the risk, and is bound to continue to pay the same premium annually during life, under the

penalty not only of releasing the other party to the contract, but of forfeiting all that he has advanced beyond the risk up to the time of non-payment. Theoretically, this bargain appears disadvantageous to the insured, for he may, among the uncertainties of life, find it desirable to retire from the contract. Practically, it has been made an objection to this system of insurance; and many practical minds have been employed to find a remedy for it as an evil. We have endeavoured heretofore to expose the nature of this evil, and suggest a remedy; but not till now have we been in a condition to ascertain and show its magnitude. The policies which were in force in 1858, and were returned as forfeited for non-payment of premium in 1859, might be reinstated in 1860, so that we could not last year fairly state them as forfeitures.

The returns of 1860 show how many were restored, and those that were not, will probably never be; or, at any rate, a very trifling proportion of them will be. We give the number and amounts forfeited in thirteen Companies, dividing them into classes according to the number of premiums paid. The net value of each class, when the premium was due, and not paid, is given, and shows what the Company gained—in addition to the policy-holder's share of accrued divisible surplus—by the forfeiture, supposing the life to have been of the average vitality. The additions or bonuses attached to policies as reversionary dividends, in some of the Companies, and forfeited with the policies, are also given in amount separately, and with their present value at the time of forfeiture.

The total of what all the Companies gained—exclusive of the relinquished share of the excess of the Companies' actual premium reserve over our net valuation—was 234,000 dollars. This, it is true enough, is a very inconsiderable sum compared with the annual income of the Companies, being only 3 per cent. additional to the receipts of that year; and it is to be reduced by the premium notes outstanding in some of the Companies against the forfeiting parties, which, of course, will never be collected.

Only two of the Companies—the Berkshire and the Massachusetts Mutual—return the amount of these notes, cancelled by the forfeiture of the policy. In the case of the Berkshire, the amount of cancelled notes was 4,002·06 dollars, reducing the gain by forfeiture to 2,119·09 dollars; in that of the Massachusetts, 5,351·34 dollars, reducing the gain to 1,429·12 dollars. In individual cases, the note cancelled was larger than the value of the policy, so that the retiring policy-holder got off without paying for more insurance than he received, or perhaps not paying for quite so much. For this reason the others lost the more. In other and older premium note Companies, where the amount of notes has been more reduced by dividend, the cancelled notes cannot bear so high a ratio to the value forfeited. We think it a very large allowance to estimate the premium notes on all these forfeited policies at 90,000 dollars, reducing the whole amount of profit from that source—aside from the proportion of surplus—to 144,000 dollars, or about 2½ per cent. addition to the revenue of the year.

This average gain, however, it will be seen at a glance, was shared very unequally by different Companies. Some got nothing to speak of, or, say one-fifth of 1 per cent., others increased their revenue more than 16 per cent. by this means, or eighty times as much as the former. We do not wish to make invidious comparisons, but those who seek information can easily cipher for themselves, between the present Report and the last, the degree of cohesion which exists in these several institutions. Whatever

may be its strength in any, it plainly does not depend on the magnitude of the penalty in case of non-payment. Forfeiture occurs where many premiums have been fully paid in cash, and large reversionary additions have been made to the policy. One all-cash Company has had nearly 8 per cent. of its liability annihilated by forfeiture, while the half-note Company, which shows the largest amount of forfeiture, has had its liability diminished by only 2·18 per cent., and since the assets are diminished by the cancelled notes, the relative gain is still less. There is every reason to believe, from the study of this table, and an acquaintance with the officers, agents, and modes of management of the various Companies, that the strength of cohesion depends rather on the fairness and honour of the agents who solicit business, and the intelligence of the people on whom they operate.

Though the evil we have been endeavouring to measure is of no alarming magnitude, when viewed in its proportion to the whole business of the vast beneficent institutions to which it adheres with such wonderful tenacity, it is still large enough to be worthy of profound consideration. Successful motion depends on taking advantage of friction in the right place and avoiding it in the wrong, and friction depends on the minute quantities of matter which make the difference between a rough surface and a smooth one. It is often a slight filing and polishing which determines the question whether a well-proportioned and powerful machine shall work well and produce immense results, or run unprofitably and wear itself out. Smoothness of finish, at any rate, always saves oil and motive power.

Hence we cannot but recal the attention of the legislature to the legal remedy which we have heretofore explained at large, and which we need not here repeat. It proposes no interference with past contracts, but simply a provision of law that in the case of future ones nothing shall be forfeited beyond the policy-holder's share of accrued divisible surplus and the right to be insured beyond the term already fully paid for in cash,—the establishment of the principle, in fact, that the policy-holder or his representatives shall be entitled to all the insurance which he pays for, whatever may be the terms of the contract.

Corporations are the creatures of the legislature, and must undoubtedly conform the contracts they make to its will, when that will is once expressed. In this case, by expressing its will against a bargain, which, in point of morals, is no better than a bet, and an unfair one at that, we believe it will benefit the Life Insurance Companies no less than those who would otherwise become the victims of their peculiar mode of obtaining pay for service never to be performed. We do not in the slightest degree question that this is done with the best possible intentions. But we have all read of a bad place paved with those good materials. Practically the law is not needed against the best Companies, which are altogether better than their bargains. But Companies, such as have been and may yet be, under dishonest, reckless, and mercenary management, can and will, under such bargains, make a good thing, in a financial sense, of their bad credit, by sending out highly magnetic and glib-tongued agents into quarters where their standing is not well known—and the world will always be too large to have it known everywhere—and alluring men to take policies, who, after several premiums are paid, will discover their error, and forfeit what they have overpaid as the best mode of escape from greater loss. By such gains, in the mother country, Companies of virtual swindlers, under the name of Life Insurance, wasting, in profligate expenditures, a full third of all the

funds entrusted to them, have managed to exist through perhaps an entire generation, and make a show of solvency and respectability. They always court secrecy as to the ratio of their premium reserve to the net value of their policies, yet its insufficiency cannot be exposed without really enriching them by frightening the old policies into forfeiture, and when this has made their assets again equal to their liabilities, they will be able to recover from the temporary check given to their new business, and go on as before. In reality, for ingenious rogues, a Life Insurance Company, with the forfeiture clause in its policies, seems to be an engine for plunder with a principle of immortality in its very constitution. It seems incapable of death except by great destitution of intelligence on the part of the operators.

It is very pleasant to believe that none of the men who are charged with the management of the Companies now under our supervision, are of the sort to make any dishonourable use of this fault in the structure of their system. They all have the disposition, but they are unable, altogether to prevent it from transferring one or two hundred thousand dollars per annum, actually paid by some two thousand men for the benefit of the first fifty or sixty of their own widows, into the pockets of some forty or fifty thousand other probably more fortunate men to help them provide for their widows. We ask the State to help them prevent it.

Claims by Death against Sixteen Life Insurance Companies doing business in Massachusetts, for the year ending November 1, 1860.

Companies.	No. of Claims.	Amount.	Ratio of Loss to Amount Insured.	Ratio of No. of Claims to No. of Policies.
Massachusetts Hospital ..	4	$15,000 00	11·75	8·33
New England	18	74,302 00	0·57	0·45
State Mutual	9	12,107 54	0·42	0·50
Berkshire	8	21,000 00	1·17	0·93
Massachusetts Mutual	14	36,400 00	0·86	0·69
Mutual Life, N. Y.....	105	342,438 19	0·92	0·90
Mutual Benefit, N. J.	66	227,000 00	1·01	0·98
Connecticut, Ct.	94	235,700 00	1·31	1·02
National, Vt.	8	14,635 02	0·84	0·71
Union Mutual, Me.....	25	61,900 00	1·42	1·35
Manhattan, N. Y.	20	75,698 00	0·73	0·62
Charter Oak, Ct.	23	47,317 51	0·74	0·69
American Temperance, Ct.	9	14,500 00	0·58	0·50
Knickerbocker, N. Y.....	3	8,000 00	0·43	0·42
Equitable, N. Y.....	3	14,000 00	1·73	1·72
Guardian, N. Y.	1	5,000 00	2·39	1·33
Totals.....	410	$1,204,998 26	1·09	0·84

It is pleasant also to turn from this view of the short-coming of the Life Insurance Companies, to their magnificent well-doing, as exhibited in the above table of the termination of policies by death. Here we see in a single year more than 400 families, smitten by death of their natural protectors and providers, shielded against the most dreaded consequences of such a bereavement, by the distribution of more than a million of dollars.

This substantial aid to the ones they most loved, in the hour of their greatest need, cost the deceased a comparatively small sum of money. It was the result of a little prudence and self-denial, availing itself of a great law that governs human life, and really gives to what is called chance the calculable certainty of granite rocks and iron bars. It will be seen, from the figures elsewhere, that very few claims remain unsettled. The whole amount is usually paid without dispute, as soon as proper evidence of the death is submitted.

Want of time has prevented us from ascertaining how this mortuary experience has been distributed among the various ages, and what is its significance as illustrating the law of mortality among the lives at risk.

When the registration has attained greater age, and the constantly increasing number of policies has been submitted to observation for a considerable series of years, the results that may be deduced will be of a practical value far more than to compensate all the expenses of this office. And it is our design, so long as we are connected with it, to use every exertion to make the registration as accurate and fruitful of good results as the nature of the subject and the powers conferred by the law admit.

The following table, prepared from the returns of the year preceding the last, may serve to show how nearly for that year the actual mortality corresponded to the rate we have adopted as the basis of our valuation.

Mortality compiled from the Returns of Sixteen Life Insurance Companies doing business in Massachusetts, for the year 1859, *compared with the Combined Experience, Dr. Farr's English Table, and the experience of the Gotha Life Insurance Company, of Germany.*

Periods of Age.	No. of Losses.	No. of Years of Life exposed.	Ratio per Cent.	Com. Ex.	Farr.	Gotha.
Under 26	7	2,086·70	0·3354	—	—	—
26 to 30.....	45	4,998·96	0·9002	0·81	0·97	0·87
31 to 35.....	54	7,664·45	0·7045	0·89	1·10	0·92
36 to 40.....	52	9,198·71	0·5653	0·99	1·25	1·00
41 to 45.....	68	8,561·59	0·7942	1·13	1·42	1·04
45 to 50.....	65	6,356·66	1·022	1·43	1·62	1·45
51 to 55.....	58	3,923·19	1·478	1·91	1·87	1·82
56 to 60.....	45	2,032·76	2·214	2·65	2·71	2·77
61 to 65.....	15	908·12	1·651	3·79	3·95	3·83
66 to 70.....	12	355·49	3·375	5·55	5·75	6·08
70 to 75.....	3	105·66	2·839	8·13	8·32	9·04
75 to 80.....	5	39·08	12·790	11·88	11·94	11·35
Over 80	1	5·00	20·000	17·22	16·90	23·94

Respectfully submitted.

ELIZUR WRIGHT.
GEO. W. SARGENT.

Boston, February 20, 1861.

CORRESPONDENCE.

MR. FINLAISON'S "REPORT AND OBSERVATIONS ON THE MORTALITY OF THE GOVERNMENT LIFE ANNUITANTS." *

To the Editor of the Assurance Magazine.

SIR,—At this period, when the decennial census is on the point of being taken, it may not be out of place to call attention to the above Report, just issued, which happens to touch upon certain points that have a peculiar interest at present, as bearing upon facts which may affect the correctness of any future life tables that may be computed upon the basis of the present census.

The *Assurance Magazine* is, therefore, an appropriate organ for directing the attention of actuaries, and through them of the public, to an interesting and most valuable document, which, being published in the shape of an unpalatable Blue-book, is not likely to come under general notice.

I propose to refer to such portions of the Report only as have a bearing on the census question, in relation to the false returns which the experience of previous censuses has shown to have been made with respect to the ages of females, to an extent probably hitherto little suspected, and to point out how the advance of science may be imperilled and retarded, and the pecuniary interests of the public sacrificed, as well by the false census returns as by the loose system which very generally obtains in the method adopted in practice of arriving at the ages of deceased persons, with the view to their being recorded in the books of the District Registrars.

With reference then to the first question, Mr. Finlaison, who from the nature of his appointment is peculiarly competent to form a correct opinion on such points, states that "it is the universal tendency of persons supplying under no particular responsibility information of their own age or of that of their relations, to understate the truth when the party concerned is young, or not past middle age. On the other hand, where the person to be accounted for is old, there is a disposition to claim or attribute an exaggerated longevity." He explains that though errors in the reported ages of the dead may possibly to some extent counterbalance each other, this is never true of the reported ages of the living.

Mr. Finlaison quotes from the Report of the last census returns, vol. i., part ii., p. 24, to show the extent of the results produced by individual mis-statements.

It is mentioned in that Report that "persons of the age of 20 in

* *Report and Observations on the Mortality of the Government Life Annuitants.* By Alexander Glen Finlaison, Actuary of the National Debt, 1860. Ordered by the House of Commons to be printed, 25th August, 1860.

1851 must have been 10 years of age in 1841, and persons of the age of 25 in 1851 must have been of the age of 15 in 1841; and as there is a certain number of losses by death, it is evident that, excluding the effects of migration, the numbers at age 20 to 25 in 1851 must be less than the numbers living at the age 10 to 15 in 1841, of whom they (20 to 25) are the natural survivors.

"What are the statements which the abstract of ages express?

"1841—The number of girls aged 10 to 15 was	1,003,119
1851—The number of young women aged 20 to 25 was, as stated in the returns .	1,030,456

"Now, as the first number could never have swollen in 10 years to the magnitude of the second, we are driven to the hypothesis, that in 1841 and 1851 the heads of families returned several thousand ladies of the higher ages at the age of 20 to 25; and the hypothesis is confirmed by comparing the diminished numbers returned at the age 30 to 35 in 1851 with the numbers returned at 20 to 25 in 1841, where it is evident that the latter number is in deficiency as much as the former number is in excess.

"1841—The number of young women of the age 20 to 25, as stated in the returns, was .	973,696
1851—The number of women of the age 30 to 35, as stated in the returns, was . .	768,711"

The Report goes on to show that notwithstanding an acknowledged disturbing influence caused by the immigration of Irish into Great Britain during the 10 years 1841–51, "upon comparing the above numbers with those for males at the corresponding ages, the conclusion appears to be inevitable, that some 35,000 ladies, more or less, who have entered themselves in the second age, twenty to forty, really belong to the third age, forty to sixty. Thousands have allowed themselves to be called twenty, or some age near it, which happens to be the age at which marriage is commonly contracted in England."

If we could succeed in giving extensive publicity to such facts as these, and in bringing home to the minds of those who are weak enough, under the influence, no doubt, of some misplaced feeling of shame, to act in this foolish manner, and, if we could convince such people of the fact that statistical acumen is unerring in its power of detecting the falsities in such returns, we might succeed in checking the practice to which females are so prone of concealing, or of attempting to conceal, their true ages. And though it would be difficult, in *individual* cases, to prove the truth or falsehood of the statement as to a lady's age, let it be remembered that power of discovery *in the gross* undoubtedly exists, and that, by this power of

detection, not only is discredit thrown upon the returns, but the whole female population is made the subject of ridicule, while a certain proportion only is to blame; and further—how unlikely it is, that any real concealment of age can ever take place among intimate friends; and as regards the census-enumerators, can they, by possibility, care one *iota* whether any particular lady of the thousands, the records of whose ages pass through their hands, is 'sweet eighteen,' or 'fair, fat, and forty'? Let such ladies read Horace, and learn that even flowers of spring do not for ever retain their pristine glory, and that the blushing moon does not always shine with unchanging aspect.

> "Nec Coæ referunt jam tibi purpuræ,
> Nec clari lapides tempora, quæ semel
> Notis condita fastis
> Inclusit volucris diea."

"Notis fastis," I may observe, may fairly be translated, "in the records of the Registrar-General."

We should, however, endeavour to make clear to the minds of the fair sex—who, in general, do not take much interest in statistical details—the exact nature and extent of the evils that are likely to result from the falsification of returns on which so many monetary schemes, in which they are more particularly interested, depend.

For whose benefit has the greater proportion of the enormous sum of two hundred millions sterling been assured upon lives in this country? Certainly for the benefit of the wives and daughters of England.

Of the 28,367 Government annuitants—which form the subject of Mr. Finlaison's Report, now under discussion—16,538 were females; and it is reasonable to suppose, that in the lives of the 11,829 males, which make up the first-mentioned number, several thousand more wives and daughters, and other female relations of the males, were pecuniarily interested.

Now, what must be the necessary result, as regards the use of recently constructed tables based upon what should be the proper elements for the calculations of Life Assurance and Annuity Companies in this country—that is to say, upon the mortality and censuses of the country in which the transactions take place? Why, clearly this, that if, census after census, females persist in making false returns as to their ages, such Companies must secure themselves against the risk of any loss which may arise from the use of incorrect *data*, and they must accordingly, in self-protection, take a sufficient margin for their own safety in offering terms for the different descriptions of benefits in which they deal.

The effect of this will be, that such increased premiums in the one case must be charged, and such reduced annuities granted in the other, in all cases where female lives are concerned, as shall compen-

sate for all risk of errors in the returns which form the basis of the computations, upon the correctness of which the power of granting life assurances and annuities absolutely depends, and thus the public will fail to derive any advantage from improvements in statistical science, and the Companies will be compelled to continue the use of notoriously unscientifically constructed tables.

I may point out that, even if the result of the class of errors under observation should be actually *in favour* of the Companies *in the gross*, it would nevertheless be only just that, so long as such serious errors are found to exist, public Companies should protect themselves in the manner described against loss in granting either life assurances or annuities. As statistical returns come, in course of time, to be more correctly made, and accordingly more to be depended on, there can be no reason why females should be deprived of any possible advantage that they might derive from the use of tables which represent the mortality among their own sex.

If it should be clearly proved that the duration of life among females is actually greater than among males, a less premium for life assurance should be charged where the female sex is concerned; on the other hand, the charges for annuities depending on the lives of females should be higher than those for corresponding benefits on the lives of males.*

It is shown by the Government Life Tables that the mortality among males is higher than that among females throughout the whole of life.

Mr. Finlaison refers to a remarkable peculiarity shown by those tables—which were computed by the late Mr. Finlaison in 1823—"a peculiarity at the age of 23 which is present in so many other observations, as to appear to be a law of nature. This feature consists in a marked increase of the mortality, at the particular age mentioned, which mortality afterwards diminishes to again re-augment to the same extent at an age long subsequent, viz., the age of 48. The occurrence of a similar climax in the life of the mariner at the age of 32, has also been shown by separate observations drawn by different observers from independent sources. This peculiarity, however, does not exist in the female table of mortality. Nor is this remarkable variation," adds Mr. Finlaison, "altogether unimportant, for it militates against a theory supported by one or two eminent

* The only existing Company that quotes special rates for assurances on the lives of females is, I believe, the National Provident Institution. It appears from the prospectus of this Society that for assurances on the lives of females a small *addition* is made to the premiums charged for male lives, while the rates for annuities depending on the lives of females are *less* than those for similar benefits on the lives of males—as is the case with the Government Office and with many other Companies granting annuities. It would appear, therefore, that this Institution considers female vitality to be at once both inferior and superior to that of males—an apparent anomaly. The practice, however, of thus "working the oracle" might probably be defended.

mathematicians, to the effect that mankind dies in the order of a symmetrical curve. Mankind, however, does not do so, as far as observation informs us."

Any opinion on this subject will naturally be interesting to your readers just now, considering the important discussion that has so recently taken place among the members of the Institute of Actuaries on Mr. Gompertz's law of mortality.

The result of the superiority of female over male life, as shown by the Government Life Table, is stated briefly as follows, viz.,—"In childhood—that is, at eight years of age—the female life is computed to possess 5½ years greater mean duration of life than the male; in womanhood—that is, at 28 years of age—4½ years; at the age of 58, 3½ years; and in old age—viz., 78 years of age—1½ years." In drawing any inferences, however, from tables constructed upon the experience of Government as regards annuities and tontines, we must always bear in mind the well-known effect of the benefit of self-selection exercised by the nominees against the Government.

On this point Mr. Finlaison remarks—"It may be doubted, indeed, whether any step in the way of selection will ever surpass the intuitive perception which reigns in the mind of the self-nominated annuitant at the time of purchase."

This influence, however, affects male no less than female lives.

Notwithstanding, then, the indications given by these tables, of the superiority of female over male life, and notwithstanding the fact that the mean future lifetime, or expectation of life, of females is greater than that of males, according to Dr. Farr's tables, still, considering the doubts that have been shown to exist as to the correctness of the *data* on which the latter tables were constructed, it will perhaps be admitted that the question of the superiority of female vitality is not yet set at rest, particularly, too, since the table founded on the experience of the seventeen Life Assurance Companies shows, that "the mortality amongst assured females, taking all ages together, is greater than amongst assured males."

It is probable, however, that even in the event of the vitality of females being positively proved to be superior to that of males, the female sex might not, *on the whole*, obtain any advantage from the use of tables of female mortality, since, no doubt, a considerably greater number of annuities than of assurances are granted upon the lives of females; but this is no reason why equitable charges should not be made in *individual* cases.

So long, however, as the records, on which the calculations of such descriptions of benefits depend, continue to be wilfully falsified by the return to the census-enumerators of incorrect ages, this will be impossible.

In connection with the question of the superiority of female over

male life, Mr. Finlaison institutes a comparison between the English Life Table and the Government Life Annuitants' Table for both sexes. At the younger adult ages according to the English Life Table, the mortality among females is shown to be greater than that among males. "This conclusion," he says, "is contrary to most previous experience. It is a result which is contrary also to nature. The sexes are not created in equal numbers. For every 20 females there are produced 21 males. But no fact is more thoroughly established than that, whenever the population is counted, the females are present in considerably greater number. Making every allowance for the temporary absence of a part of the male population, such a result could not take place unless the stronger sex were subjected to a higher rate of mortality, and died off much faster than the females. Were it not so, and did not the males by their more rapid departure from the world subsequently compensate for their appearance in greater numbers in the first instance, it is evident that they would more and more preponderate at every successive enumeration."

I may mention that at corresponding ages the mortality of the female Government life annuitants appears, according to the table, to be considerably less than that of the males.

From what we have seen with respect to the census returns, there is probably great reason to doubt the correctness of the English Life Table as regards female lives; and though the same cannot be said of the male lives, still an Assurance Company would hardly adopt a table of mortality except in its entirety. If, therefore, this table came to be used at all, it would probably be a table based upon a combination of the two sexes—unless indeed the object should be to quote distinct rates for female life, and in this case the English Life Table could not probably with safety be made use of at present. It is to be regretted, however, that any circumstances should exist to prevent the use of tables founded upon the mortality of the country in which the grant of life assurances and annuities is carried on to so great an extent.

The English Life Table, constructed by Dr. Farr some years subsequent to the establishment of the Registrar-General's department for the registration of births, deaths, and marriages, naturally came to be considered peculiarly adapted for the use of Companies dealing in life contingencies—in the first place because the greater proportion of the business transacted by such Companies depended upon the lives of persons resident, for the most part throughout life, in the district embraced by the returns—and in the second place, because the table was based upon the records of the mortality experienced at a very recent period, and upon the previous decennial census; the table in question, moreover, was due to a computer of the highest eminence.

Of course, as far as respects correctness of computation, the table may be fully relied on; but minuteness of calculation, and accuracy

of records—supposing the returns of the Registrar-General can be depended upon—is of no avail, unless the census-enumerators succeed in obtaining exact information; for on the numbers living at each age, according to the periodical enumerations of the people, no less than upon the number of deaths, do our mortality tables depend.

I shall presently recur to the question of the accuracy of the mortuary records.

To show, practically, the effect in actual money of the use of the English Life Table in the grant of life annuities and assurances, Mr. Finlaison has computed that if 1,000 persons of each sex, all aged 55 years—the average age of entry at the Government Annuity Office—were to purchase life annuities, there would be found to be living, according to the English Life Table, at the expiration of 20 years, 368 males and 410 females—or, in all, 778 persons—for the future payment of whose annuities it would be necessary to provide; but according to the experience of the Government Tables, at the expiration of the 20 years, the number surviving out of the 2,000 persons of both sexes would be as follows, viz., 403 males and 498 females—or, in all, 901 persons. There would therefore be, in fact, 123 persons to provide for in excess of the 778 for whom provision had been made. Assuming the annuities to have been £60 each—which Mr. Finlaison states to be the usual average—the Annuity Office would be liable to the extent of £7,380 per annum during the remainder of the lives of the surviving annuitants, for which no provision would have been made.

Again: supposing 1,000 males, all aged 27 years—which Mr. Finlaison considers to be a fair average age to assume*—were mutually to assure one another for the sum of £1,000 each, according to the mortality of the Government male annuitant lives, taking interest at 4 per cent., Mr. Finlaison computes that the annual premium payable, to assure the sum of £1,000,000, would be £17,447; while another 1,000 males, of the same age, would, according to the English Life Table for males, at the same rate of interest, have to provide the sum of £16,885 only annually, or £562 per annum less than the first set. At the expiration of 25 years there would be living, according to the Government Table, 712, and according to the English Life Table, 724, persons; and, in the case of the first society, Mr. Finlaison goes on to show that the sum of £193,920 would, according to its own table, buy up the 712 existing policies, while, in the case of the second society, the sum necessary to buy up the existing policies, according to the English Life Table, would amount to £213,877, to provide for which the second society would have been receiving annually, for 25 years,

* The experience of Life Offices would probably warrant the assumption of a much higher average age—possibly 40.

the sum of £562 a year less than the amount requisite to provide for the liability under the Annuitants' Mortality Table.

Upon reference to the first of these two illustrations, it will be seen that the differences between the numbers living at the age 75, according to the English Life Table and Government Tables, were respectively as follows, viz., males, 368 and 403; females, 410 and 498. The differences between these numbers are respectively 35 and 88. The variation between the tables was, therefore, considerably more than twice as great in the case of females as in that of males.

Now, without attributing the whole of the blame to false census returns—for, to do so, we must necessarily predicate the absence of all other errors and disturbing causes, as well as the absolute correctness of the Government Life Annuitants' Table—there can be no doubt, from the foregoing considerations and illustrations, that the most mischievous results might readily be caused by use of the tables in question.

The latter illustration is less apposite to the purposes of this communication, since the mortality of male life only, according to the English Life Table, is made the subject of comparison with the Annuitants' Table; and it is to the effect of the errors in respect of the ages of females upon the safety of the table, that I desire particularly to confine my attention on the present occasion.

It is referred to, however, with the object of showing how important, in a monetary sense, is the choice of a proper table for any particular description of business.

As regards that illustration, too, it may be mentioned that, in the case of an Assurance Company, many elements must necessarily enter into the question of the choice of a mortality table that need not be considered by an Annuity Company, still less by the Government, whose object in granting life annuities is simply to exchange perpetual for terminable annuities, and not to make a profit of the transactions. A table, therefore, which might be perfectly suitable for the former, would be quite unfit for the latter, whatever be the direction in which the errors tend. I am now assuming what must probably always be the case in practice, viz., that all tables of mortality necessarily contain errors. A table that might be perfectly correct, would, I apprehend, be equally suitable for an Assurance or for an Annuity Company.

It would seem that if we could arrive at such an impossible result, and if we could moreover predict the rate of interest that would be realized during a long period of years, it would be sufficient for actuaries to make such additions to the net charges for the different risks undertaken as would be required to provide for the necessary profit on the several transactions, without, as they are now obliged to do, being compelled either to make an additional charge to compen-

sate for errors in the *data* on which they base their computations, or to make use of tables which give a far too unfavourable view of life, and derive the required profit from the mortality falling short of that which the tables showed might be anticipated.

In the case of Assurance Companies, considering the unfortunate system of speculation in bonuses which now obtains to so great an extent, of course the further necessary addition would have to be made to provide for the bonuses which the cormorant-public now so eagerly demands. This abuse has crept into the system of assurance, owing, in a great measure, to the inaccuracy of the old mortality tables, and is one which, from what we have now seen of the possible flaws in Dr. Farr's *model* table, is not likely soon to be remedied.

Probably sufficient has been said to show how much it is in the interest of the entire community to assist in checking the tendency to falsify statistical records—for, in proportion as such returns become, in course of time, more correct, so will the public be more likely to obtain, on equitable terms, the benefits conferred by Life Assurance and Annuity Companies.

Another source of error, however, exists, which tends to jeopardise the value of mortality tables.

There is reason to believe that the returns of ages at death, made to the District-Registrars of Births, Deaths, and Marriages, are in many cases far from correct.

It is a matter fully within the cognizance of those engaged in the business of life assurance, and who have constantly occasion to test the truth of these records, by means of proper documentary evidence of the dates of birth and baptism, that the ages of persons at death, as stated in the certificates of the Registrar, are often one, two, or more years greater or less than the correct age. Upon inquiring into the cause of such discrepancies, we discover how loose the system of arriving at the age at death really is. Upon a death occurring, it becomes the duty of the medical attendant of the deceased to fill up the certificate prescribed by the Act. This certificate, which must be signed by the gentleman who last attended the deceased, states the age last birthday—when last seen—the date and the cause of death. The time from the attack till death—each form of disease, or symptom, being reckoned from its commencement *till* death—is also required to be stated. Both the primary and secondary cause of death has also to be inserted. From this certificate, the entry is made in the books of the District-Registrar. Upon the truth, therefore, of the information furnished to the medical attendant, the correctness of the return entirely depends. Probably, in the majority of cases, the medical man has no intimate acquaintance with his patient, and in very many cases he is called in towards the close of life, and after the patient has been ill for some time. His power, therefore, of forming a satisfactory

independent judgment as to the probable correctness of the age, as stated to him by the relatives, is very much diminished, owing to the change in the appearance of the patient which disease has caused. The medical attendant must therefore, in most cases, be wholly at the mercy of his informants, who have often but a very imperfect knowledge themselves of the fact they are required to certify, and in a great number of cases, therefore, the age is assessed altogether at hazard.

The ideas of medical men differ very much as to the importance of statistical returns. One class will set down, without exercising any judgment in the matter, any age that may be given to them. Another will endeavour to arrive at the truth, should it seem likely that the deceased were older or younger than the age asserted. Now both these classes aid in assisting the registration of incorrect ages—the first by taking no trouble in the matter, and the second by self-deception as to the age of a person they have perhaps only seen upon a bed of sickness.

We all know how difficult it is to judge correctly even of the ages of persons in health, whom we are in the habit of seeing every day, in consequence of the varying effect produced upon the constitution by the wear and tear of life in the ever-changing circumstances in which different people are placed. Although, therefore, medical men of experience are, no doubt, better qualified than non-professional men to form a correct estimate of the age of persons at death, and to make the necessary allowances for the effect produced by the antecedent illness, I think it will be admitted that according to the system now in operation for recording the age at death, the returns are liable to be often very far from the truth.

The simple remedy for this evil would be to require that the medical attendant should give information as to facts only of which he is actually cognizant—that is to say, of the date and cause of death and duration of the illness, leaving it for the legal representative or next of kin of the deceased to furnish the Registrar with the age at death, the statement of which should be supported by documentary evidence to be furnished within a certain time after the date of death.

It would clearly be inconvenient to require that this evidence should be procured within a few days of the event; and the execution of the medical certificate could not be deferred, because the clergy are very properly not allowed to proceed to the interment of a body without the production of the Registrar's certificate, which, as previously mentioned, is based upon that of the medical attendant.

In connection with this subject, and referring to the remark of Mr. Finlaison, that "where the person to be accounted for is old, there is a disposition to claim or attribute an exaggerated longevity," I may observe that in very many of those cases where the frequent remarkable statements as to the ages which persons are registered as having

attained come to be subjected to the test of actual documentary evidence, it has been found that such ages are for the most part ridiculously overstated.

It is a curious fact, and one which should influence us in denying credence to extravagant statements of this kind, unless supported by satisfactory evidence, that it is not found that members of the Peerage, whose ages at death we have instant means at hand for ascertaining, ever attain to fabulous old age. If, therefore, this favoured class do not present cases of extreme longevity, it is reasonable that we should receive with great caution such statements as to the ages of members of other classes of the community.

Probably, with very rare exceptions, the maximum age obtained in this country never really much exceeds 100 years.

The object I have had in view in this letter, has been to direct attention to certain facts, with the view to the remedy of any possible flaws that may be admitted to exist in the system of the census-enumeration and the registration of deaths in this country, as affecting the accuracy of any future tables of mortality which may be computed upon such *data*—tables on which all calculations connected with life contingencies depend.

The principal heads are as follow, viz.:—

1. The inordinate extent to which the ages of females between the ages of—say, 20 and 40—are understated in the census.

2. The over-estimate of age in very advanced life, both in the census and in the registers of death.

3. The loose system adopted in ascertaining the ages at death of all classes of people, at all ages.

4. The direct pecuniary loss sustained by females in the purchase from the Government, or from such Annuity Companies as adopt special scales for female lives, on the present assumption of the vitality of females being so much in excess of that of males.

5. The advantage the female sex might derive, were Life Assurance Companies to adopt special rates of premium for female lives.

6. The general discredit thrown upon the English Life Table, both for males and females, and the necessity that exists for Life Assurance and Annuity Companies to abstain from using it, in consequence of the incorrect returns, which both as regards the numbers living and the numbers dying, at each age, have been, no doubt, habitually made to the census-enumerators and to the District Registrars of Deaths.

The limits of a letter like the present will not allow of my even touching upon many other points which might affect the question of the suitability or otherwise, for the purposes of Life Assurance and Annuity Companies, of any life tables based upon the census returns, one of which is the fact that the labouring classes, as well as paupers, vagrants, criminals, and dissolute persons, comprise, by far, the larger

proportion of the whole population; and it is not upon such lives that assurances and annuities are granted.

The mortality, too, among these classes is undoubtedly greater than that among those who rank higher in the social scale—as I have recently had an opportunity of showing, in a paper published in the Transactions of the Institute of Actuaries, in the course of which this question was entered into at considerable length. On the other hand, it might be urged that a table containing so large a number of lives exposed to an undue rate of mortality would be, for that very reason, more suited to the requirements of public Companies.

A discussion, however, even of this one question, would lead to an inquiry into the respective merits and demerits of the existing tables of mortality, and would cause me to engross far too much of the available space in the pages of the *Assurance Magazine*. The effect, too, of the exclusion from the census returns—notwithstanding all the care to prevent it—of a considerable proportion of the nomadic population of the country, as well as of numerous other influences, would have to be considered.

In conclusion, I may state that Mr. Finlaison—who, in connection with his recent Report, has evidently investigated and considered this subject most fully—is clearly of opinion, that "at the present day, the census, and registration of births, deaths, and marriages, cannot with prudence be adopted as the bases of the true measure of the value of life."

I am, Sir,
Your obedient servant,
H. W. PORTER.

Alliance Assurance Company,
16th March, 1861.

ON MR. GOMPERTZ'S LAW OF HUMAN MORTALITY, AND MR. EDMONDS'S CLAIMS TO ITS INDEPENDENT DISCOVERY AND EXTENSION.

To the Editor of the Assurance Magazine.

SIR,—The remarks which appeared from the pen of Professor De Morgan in the number of the *Assurance Magazine* for last July, must have attracted the notice, not only of those who are interested in the history of the theory of life contingencies, but of all your readers who wish to see improvements in science attributed to their actual originators. It is no unusual thing for the title to a discovery to be contested; and it not uncommonly appears that different persons have made the same discovery independently. This is sometimes an extremely difficult point to decide; but in the present case, the means of arriving at a satisfactory decision are unusually ample. It must be clear to all who have read Professor De Morgan's remarks and Mr. Edmonds's rejoinder, that the charge brought by the former against the latter has been completely substantiated: viz., that Mr. Edmonds, following in the footsteps of Mr. Gompertz, and familiar with his writings, "has adopted his ideas without anything

approaching to a sufficient acknowledgment." On this point, nothing further remains to be said. But Mr. Edmonds, in the course of his defence, as it may be termed, has introduced many other matters, which seem to me to call for further notice, although dismissed by Professor De Morgan as having no bearing on the question raised by him.

Mr. Edmonds, then, has made a minute comparison of his processes and results with those of Mr. Gompertz; and has made many reflections to the disparagement of the latter (*vide* pp. 174–178), which, if allowed to remain unanswered, may have more weight than they deserve with some of your readers, and lead them to undervalue the labours of Mr. Gompertz. Having been led by Professor De Morgan's remarks to make, for my own satisfaction, a somewhat close comparison of the writings of Mr. Gompertz and Mr. Edmonds, I am induced by the above consideration to lay before your readers the conclusions at which I have arrived.

In the first place, I notice that, curiously enough, Professor De Morgan, who appears as the voluntary champion of Mr. Gompertz, does not do him full justice. Mr. Gompertz's paper in the *Transactions of the Royal Society* is full of misprints—many of them being of the most serious character. Two such are corrected by Professor De Morgan in the extract he has made (pp. 87, 88); but he has left several uncorrected, which give rise to the "obscurity" to which Mr. Edmonds refers (pp. 178, 181). In order to demonstrate this, and with the secondary object of making Mr. Gompertz's important theorem more generally known,* I subjoin its demonstration. It will be seen, that besides correcting these misprints, I have expanded the reasoning in some of the steps, and have in several instances substituted for the notation of Mr. Gompertz, that more generally used at the present time.

Let the rate of mortality at the age x be denoted by aq^x, where a and q are constant quantities, and let L_x be the number living at the age x; then the number dying in the small time dx will be $aL_x \times q^x dx$; so that—

(1) $$dL_x = -aL_x \times q^x dx;$$

(2) $$\therefore \frac{1}{L_x} \cdot \frac{dL_x}{dx} = -a \times q^x.$$

(3) Integrating both sides, $\log_e \frac{L_x}{d} = -\frac{aq^x}{\log_e q} = \frac{aq^x}{\log_e \frac{1}{q}}$;

d being the constant introduced by integration.

Now, $\log_e \frac{L_x}{d} = \log_{10} \frac{L_x}{d} \cdot \log_e 10$, $\log_e \frac{1}{q} = \log_{10} \frac{1}{q} \cdot \log_e 10$; so that the last equation gives

(4) $$\log_{10} \frac{L_x}{d} = \frac{aq^x}{\log_{10} \frac{1}{q} \cdot (\log_e 10)^2}:$$

* Since writing the above, I have found that I have been anticipated in this object by Mr. Peter Gray, who has fully explained the nature and application of Mr. Gompertz's method of interpolation in a paper (*Assur. Mag.*, vol. vii., p. 121), which is well worthy of every student's attention. In comparing Mr. Gray's form of the demonstration with that given in the text, it must be borne in mind that the constant added in the integration being arbitrary, may either be written $+C$, as Mr. Gray has it, or $-\log_e d$, which is Mr. Gompertz's form.

(5) or, putting $\log_{10}\frac{1}{q}.(\log_e 10)^2=\frac{a}{c}$,

(6) $$\log_{10}\frac{L_x}{d}=cq^x.$$

(7) Hence $$\frac{L_x}{d}=10^{cq^x}.$$

And if $10^c=g$, or $g=\log_{10}^{-1}c$, *i.e.* the number whose common logarithm is c,

(8) $$\frac{L_x}{d}=g^{q^x},$$

(9) and $$L_x=dg^{q^x}.$$

On comparing the above with Mr. Gompertz's demonstration, as quoted by Professor De Morgan, it will be seen that the mysterious constant (b), which has caused so much perplexity, is simply a misprint for the sign of multiplication ($\times$). This will be at once admitted, when it is noticed that Mr. Gompertz writes down the *equation*

$$aL_x\times q^x\dot{x}=-(L_x)^{\cdot};$$

and then concludes that $$abq^x=-\frac{\dot{L}_x}{L_x}.$$

Professor De Morgan seems to have overlooked this circumstance, which proves that a constant b could not have been introduced in the manner he suggests.

It will also be noticed that the constant d, which does not appear in Mr. Gompertz's process till step (6), correctly enters on the integration in (3). Again, in (5), Mr. Gompertz's c should be $\frac{a}{c}$.

These corrections having been made, all "obscurity and ambiguity" disappear from Mr. Gompertz's process. Nor does that process contain, as Mr. Edmonds asserts (p. 181), "two superfluous and useless indeterminate constants." I have already disposed of b, and I shall presently show that d is neither superfluous nor indeterminate.

The preceding corrections occurred to me on reading carefully Mr. Gompertz's process, as quoted by Professor De Morgan (p. 88). In order, however, to make the comparison already mentioned of Mr. Edmonds's writings with Mr. Gompertz's, I referred to the copy of Mr. Gompertz's papers in the library of the Institute of Actuaries. That copy, which was presented to the Institute by Mr. Gompertz, exhibits the precise corrections I have already indicated, in addition to those pointed out by Professor De Morgan, made in the margin of the volume—as I presume, by Mr. Gompertz himself. (There is, however, in step (5), an *over-correction* made.) A single glance at this copy will convince any person of the accuracy of my statement, that Mr. Gompertz's paper, as it appears in the *Transactions of the Royal Society*, is full of misprints. It is to be presumed that that gentleman, like so many others of scientific eminence, does not write a very legible hand. Perhaps also the Royal Society, at the time the papers in question were printed, followed the course adopted at the present time by a few Societies—happily not by the Institute of Actuaries—and did not allow the contributors to its *Transactions* the opportunity of correcting the proofs of their papers.

We have thus seen that there is no error in Mr. Gompertz's demonstration of the theorem that "if the rate of mortality follows the law aq^x, then the number living at any age x will be given by the formula dg^{q^x};" and that Mr. Edmonds's doubts as to its correctness have arisen from the fact of its being obscured by numerous misprints. Let us now examine some of Mr. Edmonds's remarks upon that demonstration, and it will be seen that they contain errors equally grave with those falsely imputed to Mr. Gompertz, and which cannot be similarly explained. In the first place, then, Mr. Edmonds speaks (p. 180) of "b being introduced as arising in the process of integration." It will be noticed that b makes its appearance in step (2), while the integration is not performed till (3); also that d is the constant introduced by integration: so that the only conclusion appears to be that the remark just quoted was written under a misconception as to the real nature of the process of integration. This conclusion is strengthened by the perusal of the remarks (p. 178) on the limits of integration, and by the suggestion made (p. 184) of "an error committed in the process of integration, of which b represents the correction." Here a confusion of ideas is betrayed, of which it is difficult, if not impossible, to trace the origin. A similar confusion of ideas pervades the remarks (p. 181) on the constants a, b, a. The truth seems to be that Mr. Edmonds, having concentrated his attention for a long time upon a particular problem, with which it must be confessed he has shown a considerable degree of familiarity; and having arrived at a correct solution in a particular form, in which he has been aided by a knowledge of Mr. Gompertz's paper (*vide* p. 177, bottom), is yet unable, in consequence of want of familiarity with the principles of the integral calculus, to follow Mr. Gompertz in his somewhat more general way of treating the same question.

We next come to Mr. Edmonds's remarks upon the constant d. He terms this (p. 178) a "superfluous quantity," and speaks of it as constituting a "defect" in the formula. Instead of being such, it is really an essential part of the formula, *and as such is employed by Mr. Edmonds himself.* He uses the letter g to denote the same quantity as Mr. Gompertz's d; and in arriving at his formula, determines g so that $y=1$ when $x=0$. On the other hand, Mr. Gompertz leaves d undetermined; since for his purpose, there was nothing to be gained by determining its value. (This, I conjecture, is what Mr. Edmonds means by calling d, in p. 181, an indeterminate constant; but mathematicians mean by an "indeterminate" quantity, one whose value cannot be ascertained.) If it were required to determine the value of d in the formula $L_x=dg^{q^x}$, the process would be as follows:—Suppose the number living at the age n* to be L_n; then $L_n=dg^{q^n}$, whence $L_x=L_n(g^{q^x-q^n})$, or $L_x=L_n(g^{q^n})^{q^{x-n}-1}$. It must be noticed that in Mr. Gompertz's formula x denotes the age measured from birth, while in Mr. Edmonds's x is only measured from birth during the period of infancy, as will be seen by his example (p. 179). So far from d being a defect in the formula dg^{q^x}, the formula $10^{\frac{k^2a}{\lambda p}(1-p^x)}$ is, on the contrary, defective, because it assumes $y=1$ when $x=0$; and the defect in it is tacitly supplied by Mr. Edmonds himself in practice. Thus, in his example, he arrives at the result $y_{10}=1\div1{\cdot}076823$ ($=\cdot928658$); and x being measured from

* I use n here in preference to Mr. Gompertz's a, because the letter a has already been used to denote another quantity in the expression aq^x.

the age of 12, this should be the number living at the age of 22. Instead of this, Mr. Edmonds is compelled to introduce the quantity we have denoted above by L_n: he takes the number living at 12 to be 100,000 ($=L_{12}$), and deduces the number living at 22 to be 92865·8; as will be seen on reference to his Table of Mean Mortality. The above is not a point of great importance; but it shows that whatever defect does exist, is in Mr. Edmonds's formula, and not in Mr. Gompertz's. Mr. Edmonds, however, falls into a far more serious error (p. 180) in calling d a particular value of y. It is easily shown that y cannot possibly be equal to d. For, in the formula $y=dg^{q^x}$, if $y=d$, we have $g^{q^x}=1$; hence $q^x=0$, and $x=+\infty$ or $-\infty$, according as q is $<$ or >1; values which are of course inadmissible.

Mr. Edmonds's words in speaking of d are as follows (p. 178):—"The defect in Mr. Gompertz's formula, caused by the addition of (d), is the same as that which would exist in a table of discount of money at compound interest if any other basis were adopted than the value of the sum of £1 receivable (x) years hence." It would be generally considered that £1 was the *basis* of such a table,* and not the value of £1 receivable x years hence; but without dwelling on this minor inaccuracy, I notice that in the passage just quoted, two very different things are confounded with each other, viz., the *formula* for the value of a sum of money due at the end of x years, and the *table* by which that value would be practically found. It is true that a table in which any other basis than £1 were assumed, would be *inconvenient* in practice (though it could not be correctly described as *defective*); but on the other hand, a formula in which the sum to be received is taken as £1 (*i.e.* $(1+i)^{-x}$), would be defective; the correct formula being $s(1+i)^{-x}$.

Again, we read on the same page that Mr. Gompertz has changed the sign of c; whereas the true state of the case is that Mr. Edmonds has changed its sign for his own purposes. The quantity c, which does not appear in Mr. Edmonds's own investigation, is taken by Mr. Gompertz to be equal to $\dfrac{a}{\log_{10}\frac{1}{g}\cdot(\log_e 10)^2}$, which in Mr. Edmonds's notation would be $\dfrac{k^2a}{\lambda\frac{1}{p}}$, or $-\dfrac{k^2a}{\lambda p}$. The latter, when using the letter c for the first time on p. 178, takes it equal to $+\dfrac{k^2a}{\lambda p}$, and then says that Mr. Gompertz has changed the sign of c!

I now pass on to the remaining portion of my subject—a general comparison of the results obtained by Mr. Gompertz and Mr. Edmonds. Here at the outset I must direct attention to a grossly unfair misquotation by the latter of Mr. Gompertz's words. He wishes to give your readers the impression, that Mr. Gompertz's discovery is imperfect by reason of its not stating any limits of age, within which the formula is to be applied with the same constants; and for this purpose he professes to quote (p. 174, top) a passage from Mr. Gompertz's paper. *No such sentence is to be found in*

* Similar to this, is Mr. Edmonds's use of the word "formula" to denote an *equation*, as $L_x=dg^{q^x}$, instead of the *expression* (dg^{q^x}), which forms the second member of the equation.

that paper! Mr. Gompertz's words are as follows:—"*If* the average exhaustions of a man's power to avoid death were such that at the end of equal infinitely small intervals of time, he lost equal portions of his remaining power to oppose destruction which he had at the commencement of those intervals, *then* at the age x his power to avoid death, or the intensity of his mortality, might be denoted by aq^x." And again: "This equation [$L_x = dg^{q^x}$] between the number of the living, and the age, becomes deserving of attention, not in consequence of its hypothetical deduction, which in fact is congruous with many natural effects, as for instance, the exhaustions of the receiver of an air-pump by strokes repeated at equal intervals of time, but it is deserving of attention because it appears corroborated *during a long portion of life* by experience; as I derive the same equation from various published tables of mortality *during a long period of man's life*, which experience therefore proves that the hypothesis approximates to the law of mortality *during the same portion of life;* and in fact the hypothesis itself was derived from an analysis of the experience here alluded to." I have been thus particular in quoting the above passages (in which the italics are mine), because the question of misquotation is one upon which all your readers will be able to form an opinion, whereas it requires for the full appreciation of many of the points here raised, a greater knowledge of the higher mathematics than they can all be expected to possess. By comparing together the above two passages from Mr. Gompertz's paper, any person may judge of the "fairness" or the truth of Mr. Edmonds's assertion (p. 174) that "it might fairly be inferred from the above statement that the vital force of man, measured by the ratio of the living to the dying, is in a constant state of decay *from birth to the end of life* at one and the same uniform rate." The reason Mr. Gompertz has stated no definite limits of age is obviously because he did not believe them to be fixed in the same nearly invariable manner as Mr. Edmonds does. This brings me to the next point I have to notice—the improvements the latter claims to have made on the former's discovery.

Mr. Edmonds has convinced himself that in the formula for the mortality at any age, aq^x, q has three fixed values, invariable or nearly so, under all circumstances, and for all populations; and he endeavours, by reiterated assertions, to convince his readers of the truth of the proposition. I say *assertions*, because he has not advanced a single particle of *proof* in support of his proposition. In this single distinction is summed up the difference, or rather the contrast, between the writings of Mr. Gompertz and Mr. Edmonds. All that the former advances is *rigorously proved;* all that the latter claims to have discovered is *simply asserted.* Mr. Gompertz describes all his computations so as to enable a person with only slight knowledge of the subject to repeat them and test their acccuracy: he shows how he has found the values of his constants g and q; and he arranges his results so as to exhibit at a glance the degree of accuracy with which they agree with previously computed tables of mortality. Mr. Edmonds has done nothing of this kind: he simply states the results he has arrived at, and apparently expects them to be received without demonstration.

If Mr. Edmonds's proposition of three invariable values for q were established, it would no doubt be justly entitled to the name of a *discovery*, and it would be a great advance on what Mr. Gompertz has done. But in the absence of any proof, it is impossible to treat the proposition other-

wise than as an unproved hypothesis. If Mr. Edmonds possesses the means of proving his proposition, and publishes sufficient details of the extensive comparison of tables described by him on p. 172, the scientific public will then be in a position to judge of the truth or error of his proposition; and I for one shall not be slow to avow my reception of anything put forward by Mr. Edmonds when supported by sufficient proofs. As the case stands at present, it is impossible to admit the truth of Mr. Edmonds's proposition, or to countenance his use of the term "discovery" in describing the conclusions at which he has arrived.

It forms no part of my purpose to comment on the bad taste displayed by Mr. Edmonds in many of his remarks, nor do I think it necessary to allude particularly to his unjustifiable use of the phrase "*true* law of human mortality" to describe his hypothesis; for these are points which must force themselves on the notice of every intelligent reader. I will therefore conclude this part of my subject by pointing out two other inaccuracies into which Mr. Edmonds has fallen. He states (p. 177) that he believes Mr. Galloway to be the only person who has made a practical use of Mr. Gompertz's formula. This is by no means the case. Mr. Jellicoe, in the first volume of the *Assurance Magazine*, p. 166, has applied the formula to the adjustment of Mr. Neison's Table of Mortality deduced from the deaths among the officers of the Indian army. He has also used it in a modified form (*Assur. Mag.*, vol iv., p. 206) to graduate his own Table of Mortality derived from the experience of the Eagle Insurance Company; and the same method has been recently adopted by Mr. Dove to adjust the results deduced from the experience of the Royal Insurance Company. Mr. Farren has also employed it in his Life Contingency Tables. Again: Mr. Edmonds (p. 174) says that "nearly all he [Mr. Gompertz] offers to show is, how *interpolations* may be made for intermediate ages when the number of survivors at the beginning, and the number of survivors at the end, of a large interval of age are given." (*Conf.* p. 176, bottom.) Here Mr. Edmonds quite overlooks the fact, that it requires a knowledge of the number of the survivors at *three* ages at least, to apply Mr. Gompertz's method; which is an obvious deduction from the fact that the formula dg^{q^x} contains *three* disposable constants. When we remember that Mr. Edmonds erroneously rejects one of those constants as superfluous, it appears less surprising that he should have made this serious blunder.

We have seen that it is impossible to admit, on the evidence adduced hitherto, that Mr. Edmonds has *extended* the theory of Mr. Gompertz: it remains now to examine his claims to an independent discovery of what is common to them. Taking then Mr. Edmonds's own statement (pp. 172–178), it appears that he ascertained that the rate of mortality at different ages might be represented with sufficient accuracy by the formula αp^x, and he then learnt that Mr. Gompertz had shown that the number of survivors at any age would be given by the formula dg^{p^x}. In other words, Mr. Edmonds *very nearly* discovered what had been previously discovered by Mr. Gompertz; or, in his own phrase, if he had only had a little more time, he would certainly have discovered it. It is of no avail to make conjectures (*vide* pp. 175, 176) as to the course by which Mr. Gompertz arrived at his formula. The *fact* remains that he did arrive at it and publish it, and that Mr. Edmonds first became acquainted with the formula through Mr. Gompertz's writings. It was no doubt a great dis-

appointment to Mr. Edmonds to find that he had been anticipated in this manner, but it may be a consolation to him to reflect that his position is by no means a singular one. There has probably been scarcely a single discovery of any kind made, but some unfortunate man has been on the point of making it too, and would have done so, if he had only had sufficient time allowed.

In conclusion, I beg to state that in all I have said, I have been influenced by no personal feeling towards Mr. Edmonds. I wish to allow him all the credit justly due to him. I believe that in accomplishing the object he had in view—viz., that of forming a new hypothetical table of mortality—he has shown considerable judgment and skill; and I shall willingly admit the universality of his constants whenever I see sufficient proofs of their accuracy produced. My object has been simply truth; and if, in the course of my remarks, I have been led to use language which may seem severe, I regret the necessity I have been under. I feel I have trespassed rather unreasonably on your space; but trust that what I have said will enable others who have but little time to devote to such subjects, to come to a correct conclusion as to the claims put forward by Mr. Edmonds. My task would have been much lighter if Mr. Edmonds had confined himself to an explanation of his own theory; for then I should not have thought it necessary to dwell upon the numerous errors into which he has fallen. This course has been rendered necessary by his uncalled for criticisms of Mr. Gompertz. Your readers will probably be better able to form an opinion as to the proper weight to be assigned to those criticisms, the whole of which I have not thought it necessary to review, when they learn that the writer of them has himself fallen into many serious errors.

I am, Sir,

Your obedient servant,

25, *Pall Mall,*
7th February, 1861.

T. B. SPRAGUE.

EXPRESSION FOR THE VALUE OF A TERM ASSURANCE, LIFE AGAINST LIFE.

To the Editor of the Assurance Magazine.

SIR,—Perhaps the following brief expression for the value of a short period assurance on (x) against (y) may be worth a place in your *Journal.*

By the combined use of the ordinary Commutation Tables in Jones, and the tables of Gray, Smith, and Orchard, for such an assurance to be current for m years and paid for in n annual premiums, the formula becomes

$$\frac{D_{x.y}\,A^{1}_{\overline{x.y}} - D_{x+m,\,y+m}\,A^{1}_{\overline{x+m,\,y+m}}}{N_{x-1,\,y-1} - N_{x+n-1,\,y+n-1}}.$$

I am, Sir,

Your most obedient Servant,

Aberdeen, 7th May, 1861.

H. AMBROSE SMITH.

MR. GOMPERTZ'S PAPERS.

To the Editor of the Assurance Magazine.

SIR,—Mr. Gompertz has done me the honour to make me the medium of communication between himself and the members of the Institute of Actuaries, and has addressed the following letter to me for publication in the *Journal* of the Institute, in reference to the recent controversy on the subject of his celebrated paper, read before the Royal Society on the 16th of June, 1825, and published in the *Philosophical Transactions* for that year, "On the Nature of the Function expressive of the Law of Human Mortality."

The question as to the undoubted claim of Mr. Gompertz to be considered the sole and original discoverer of the theorem enunciated in that paper being now so definitively set at rest, by the complete analysis of the whole question by Mr. Sprague, in his able paper, read before the Institute of Actuaries on the 25th of February last, I may, perhaps, be allowed to express an opinion—in which I know I shall be joined by every gentleman who has the pleasure of Mr. Gompertz's acquaintance—that he would have been the very last person to fail to acknowledge any claim to an independent discovery, had such been made; and I cannot, perhaps, conclude better than by quoting Mr. Gompertz's own words on this point, which happen to occur in the course of a previous paper, read before the Royal Society on the 29th of June, 1820,—"To a true philosopher, it will ever be much more pleasing to grant even more praise than is actually due, than to pluck the laurel from the deserving brow."

This, Sir, is a sentiment, which I think you will agree with me is no less elegant in expression than it is indicative of the known amiability of disposition which has always characterised this distinguished mathematician and actuary.

I have the honour to be, Sir,

Your obedient servant,

H. W. PORTER.

Alliance Assurance Company,
12th March, 1861.

LETTER FROM BENJAMIN GOMPERTZ, ESQ., F.R.S.

Kennington Terrace, Vauxhall,
6th March, 1861.

MY DEAR SIR,—Not having been sufficiently in health since the gentlemen who are members of the Institute of Actuaries did me the honour to elect me an honorary member of their excellent establishment, to attend the meetings, nor to have been active in adding my mite to the papers which it publishes; with your sanction, I wish you to be the medium to express my thanks to several of the members for their frequent kind mention of my name in their valuable papers; but, in particular, I wish to allude to the distinguished and highly talented Professor De Morgan, for the two papers, cleverly written, to prevent the subject of my paper on the "Law of Human Mortality," published in the *Transactions of the Royal Society for* 1825, being lowered in the estimation of scientific men, in consequence of a claim made by a gentleman, who, with rather sharper criticism than I believe

ought to have been directed either to me or my paper, of having, I think about the year 1832, made the discovery independently of me; but I am not concerned in that claim; because, whether he did, or did not, make a discovery of a theorem, seven years after it had been honoured by the approbation of scientific men, after my publication of it, cannot be an injury to me. Still, I think it was not wise of him to try to use arguments to establish his claim, which would, I think, by their close resemblance to my own words, rather prove the reverse. I have no wish to rob him of any claim, which either he, or his friends for him, may consider his due, but, on the contrary, I wish well to everyone who may, virtuously and without jealousy, feel inclined to tread in a new path to promote the objects of science, or who, with a view to add information to knowledge acquired, should modestly follow the steps pointed out by an earlier labourer, who may have ploughed in the same field in which he may hope to grow his wheat, or who may have planned a garden in which myrtles may grow, and laurels thrive, to adorn the brow of some future labourer, who may, by arduous labour joined to humble pretensions, merit praise.

I am willing to own that there are many typographical errors which disfigure the paper, which may lead a school-boy from the direct meaning of the information I meant to convey; or which may so act on the mind of a lazy student as to confuse his judgment; but in all the mathematical papers I have published, I have found such errors constantly to occur, notwithstanding all the pains I have paid to prevent them; but I observe, that as long as mathematical papers are printed, without having a sufficiently clever mathematical superintendent of the press, that annoyance will intrude.

It is my intention, or at least my wish, to publish, either in the *Transactions of the Royal Society*, or elsewhere, a notice to correct those errors; yet, I observe, such errors may have the advantage of causing a reader to go more deeply into the meaning of an author, by obliging him to dig into the ground, instead of flying over the surface. I have, for two or three years, been endeavouring to add matter of importance, in my own opinion, to the subjects of my papers of 1820 and of 1825; but the state of my health has so far interfered with my object as to prevent my getting up my intended paper, to present to the Royal Society. And in a paper I wrote for the International Congress, held in July last, I gave hints—which, I flatter myself, will be interesting, when published by the Commissioners—relative to the subject of the paper I was writing for the Royal Society, on "Mortality and Invalidity," with some striking tables, corroborative of my new views on the subject, and hints of the very extensive service of the law of mortality I had further discovered in calculating values, with respect to all sorts of intricate cases and complications of intricacies; and in the paper which I am now endeavouring to induce my health to proceed with, I have improved parts of the paper I was then writing, but the hints do not much go into the abstruse part of my paper, as I did not consider that portion adapted to the intention of the Congress, and because I preferred also to make the Royal Society, if I were allowed that honour, the medium of the mathematical essence of the paper through which my views went forth to the public. In my paper of 1825, I showed, from tables and the law of mortality for portions of life, means of calculating, without much difficulty, the value of annuities on many joint lives; and the ingenious and worthy and highly talented Professor De Morgan has shown, in a paper he wrote,

published among the papers of the Institute, that if my formula $a.\overline{b}^{q^x}$ were absolutely true throughout, it would give the means of calculating the values of annuities on any number of joint lives; but as with the same constants a, b, q, the term of its applicability is limited, the method will not invariably correctly apply: and I think it proper to remark, that my investigation, published in the *Transactions*, ought not to have led the gentleman, who considers himself the second independent discoverer, to consider them absolutely constant, and my having pointed out that, at the ages between 60 and 100, they had different values, proves what my views with respect to those elements were; and I further observe, that my subsequent researches, hinted at in the paper I wrote for the Congress, and further to be explained in the paper which I am about writing, show the curious nature of the variability of those elements, and explain by what manner they put on the appearance, during long periods, of constancy. And the consequence of such slow variability of the elements which may be considered, for long periods, constant, together with the law of their variability, give the command of estimations with accuracy of most complicated cases of intricacy, which will be found useful. But by my improved statement of the law of mortality, by the methods which will be pointed out in the paper I am about to present, when sufficiently advanced, to the Royal Society, a great variety of complicated cases of contingencies will be very easily grappled with.

The tables of two joint lives, published by the Society of Useful Knowledge, are extremely useful. The famous tables of Barrett, on the Northampton Tables of Mortality of every combination of three lives, are too voluminous and too rare (if at all to be met with) to be generally useful, even where they apply; but my method, hinted at in the paper I wrote for the Congress, and which I hope shortly to lay before the Royal Society, will apply to any number of lives, whether they be all subject to the same law or to different given laws, pertaining to different individuals of the whole and through a very wide complication of conditions and contingencies. I therefore flatter myself, should my health last to enable me to proceed with, at least part of, my subject, the paper will be well received.

I wish you, my dear Sir, to present this letter to the Institute, as I wish in it to notify my thanks to Mr. Sprague, whom I have not the honour of personally knowing, for his kind and able paper in vindication of my claim to be the sole independent publisher of a theorem which, I am gratified to say, appears of importance to the scientific world — I say, the sole independent publisher, though the fact of my being the first independent discoverer has not been denied me, even by the gentleman who claims to be the second discoverer, because that claim, should it be ever proved to be a just one, would not interfere with me.

I am, my dear Sir,

Yours, with regard,

To H. W. PORTER, ESQ. BENJ^N GOMPERTZ.

AN APPROXIMATE EXPRESSION FOR THE VALUE OF AN ASSURANCE, LIFE AGAINST LIFE.

To the Editor of the Assurance Magazine.

SIR,—The following short investigation may, perhaps, be useful to some of your readers. It will be observed that the resulting formula is much more simple than the ordinary one for the single premium for a contingent assurance; and it has been found to be sufficiently accurate for practical purposes.

PROBLEM.

To find the present value of £1 to be received at the end of the year in which a life aged x may fail, provided that such event happen during the lifetime of another, aged y; the chance of both dying in the same year being neglected.

$$\text{Let } S = \text{value required};$$

$$\text{then } S = \Sigma v^n (p_{x\ n-1} - p_{x,n}) p_{y,n}$$

$$= \Sigma v^n (p_{x,n-1} p_{y,n} - p_{xy,n});$$

$$\text{but } \Sigma v^n p_{xy,n} = a_{xy}$$

$$\text{and } \Sigma v^n p_{x,n-1} p_{y,n} = \frac{l_x . l_{y+1}}{l_x . l_y} v + \frac{l_{x+1} . l_{y+2}}{l_x . l_y} v^2 + \frac{l_{x+2} . l_{y+3}}{l_x . l_y} v^3 + \&c.$$

$$= \frac{l_{y+1}}{l_y} v \left(1 + \frac{l_{x+1} . l_{y+2}}{l_x . l_{y+1}} v + \frac{l_{x+2} . l_{y+3}}{l_x . l_{y+1}} v^2 + \&c. \right)$$

$$= v p_y (1 + a_{x,y+1})$$

$$\therefore S = v p_y (1 + a_{x,y+1}) - a_{xy}.$$

I am, Sir,

Yours truly,

Equity and Law Life Office,
13th March, 1861.

ARTHUR H. BAILEY.

INSTITUTE OF ACTUARIES.

PROCEEDINGS OF THE INSTITUTE.

First Ordinary Meeting, Session 1860-61.—*Monday, 26th November,* 1860.

CHARLES JELLICOE, President, in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced various donations to the library.

Messrs. Alexander Burnett, George Thomas Ruck, James Stark, Jun., and Mark Symons, duly nominated at the last ordinary meeting, were unanimously elected Associates of the Institute.

Mr. J. Hill Williams read a paper—"On the theory of probabilities," by Robert Campbell, Esq., M.A.

Thanks were voted to Mr. Williams and to Mr. Campbell, and the meeting adjourned to the 31st December, 1860.

Second Ordinary Meeting, Session 1860-61.—*Monday,* 31*st December,* 1860.

CHARLES JELLICOE, President, in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced various donations to the library.

The undermentioned gentlemen, duly nominated at the last ordinary meeting, were unanimously elected members of the Institute:—

Official Associate—Henry D. Davenport, Esq.

Associates.

Mr. R. Clarke.	Mr. C. R. Saunders.
„ W. R. D. Gilbert.	„ T. Y. Strachan.
„ A. W. Mackenzie.	„ C. J. Wilkins.

The Chairman announced that out of eleven candidates for the matriculation examination (1860), nine passed, in the following order of merit:—

	Marks.		Marks.
Mr. C. R. Saunders	440	Mr. James Stark	332
„ R. P. Hardy	364	„ C. J. Wilkins	293
„ J. R. Knowles	344	„ T. Y. Strachan	278
„ Fredk. Harper	335	„ Jas. Henderson	271
„ H. W. Manly	331		

Mr. Hodge then read a paper—"On the stability of results based on average calculations," by Robert Campbell, Esq., M.A.

Thanks were voted to Mr. Campbell and Mr. Hodge, and the meeting adjourned to Monday, 28th January, 1861.

Third Ordinary Meeting, Session 1860-61.—*Monday,* 28*th January,* 1861.

CHARLES JELLICOE, President, in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced various donations to the library.

The undermentioned gentlemen, duly nominated at the last ordinary meeting, were elected members of the Institute:—

Official Associate—Frank McGedy, Esq.

Associates.

Mr. F. Harper.	Mr. J. R. Knowles.
„ Jas. Henderson.	„ Henry Wm. Manly.

Mr. Tucker read a paper—"On the rates of premium required to provide for certain periodical returns to the assured."

Thanks having been voted to Mr. Tucker, the meeting adjourned to Monday, 25th February, 1861.

Fourth Ordinary Meeting, Session 1860-61.—*Monday,* 25*th February,* 1861.

CHARLES JELLICOE, President, in the Chair.

Read and confirmed the minutes of last ordinary meeting.

Various donations to the library were announced.

The undermentioned gentlemen, duly nominated at the last ordinary meeting, were elected members of the Institute, viz.:—

Fellow—W. S. B. Woolhouse, Esq.

Associates.

Mr. C. F. Haycraft. | Mr. S. J. Shrubb.

Mr. Sprague read a paper "On Mr. Gompertz's law of mortality."

Thanks having been given to Mr. Sprague, the meeting adjourned to Monday, the 25th March, 1861.

ANNUAL EXAMINATIONS.

The members of the Institute will, no doubt, recollect that early in the last year some alterations were suggested by the examiners in the syllabus published by the Institute; and that their suggestions being approved by the Council, the amended syllabus was adopted and ordered to be thereafter acted upon.

The changes then made have necessarily affected, to a certain extent, the character of the questions; and as it is obviously desirable that the way in which these changes have operated should be understood, we now publish the questions given on the last occasion for the second and third years' examinations—those for the first year being but very slightly influenced by them. It will be seen that the third year's questions are those mainly affected by the alterations.

Second Year's Examination, 1860.

1. Assuming the formula for $\log_e(1+x)$, prove that

$$\log_e 3 = 1 + \frac{1}{3}\cdot\frac{1}{4} + \frac{1}{5}\cdot\frac{1}{4^2} + \frac{1}{7}\cdot\frac{1}{4^3} + \ldots.$$

$$\tfrac{1}{2}\log_e 10 = \log_e 3 + \frac{1}{19} + \frac{1}{3}\cdot\frac{1}{19^3} + \frac{1}{5}\cdot\frac{1}{19^5} + \ldots.$$

2. Explain what is meant by the modulus of a system of logarithms. By means of the formulæ in question, show that

$$\log_e 3 = 1{\cdot}098612$$
$$\log_e 10 = 2{\cdot}302585$$

Also show that the modulus of the common system of logarithms is ·434294.

3. When an event can happen in more ways than one, show that the probability of its happening is equal to the sum of the probabilities in respect of the different ways.

4. Determine (1) the probability that two persons now aged x and y respectively will both die in the nth year from the present time; (2) the probability that one or both of them will die in that year.

5. Find the present value of a sum of money due at the end of any time, assuming the operation of compound interest.

Prove that the value of £1 due at the end of $p+q$ years=value of £1 due in p years × value of £1 due in q years.

6. Describe a practical method of calculating a table of the value of £1 due at the end of any number of years from 1 to 100; and show how the

results may be employed to calculate a table of the values of annuities certain.

Ex.—The value of £1 due at the end of 10 years, at 4 per cent., is ·67556417: deduce the first ten terms of the value of an annuity for any number of years.

7. Explain what is meant by a table of mortality, and state the different methods in which the facts embodied in it may be exhibited.

8. Give some account of the origin and relative merits of the tables of mortality known as the Carlisle, Equitable, Experience, and English Life Tables.

9. State accurately what is meant by the expectation (or mean duration) of life at any age according to a given table of mortality.

Prove the formula for calculating the chance of living a year at any age from the expectation—

$$p_x = \frac{e_x - \frac{1}{2}}{e_{x+1} + \frac{1}{2}}.$$

10. Prove that if $e_{x-1} = e_x = e_{x+1}$, then will $p_{x-1} = p_x$; but that if $p_{x-1} = p_x = p_{x+1}$, e_{x-1} will not be equal to e_x unless $e_{x+2} = \frac{1}{2} \cdot \frac{1+p_x}{1-p_x}$.

11. The value of an annuity on the life of a person of a given age is frequently supposed to be equal to that of an annuity certain for a number of years equal to the mean duration of life at that age. Explain why this is not the case, and state upon what hypothesis it would be true.

12. If B represents the present value of a benefit of £1 upon a given life (x), B_1 the same upon a life one year older $(x+1)$, p the probability of a payment of B being received in the first year, and Π the probability of (x) surviving a year, prove that

$$\log B = \log v\,\Pi + \log\left(\frac{p}{\Pi} + B_1\right).$$

13. Describe the process of calculating a table of annuities by means of the formula $a_{m-1} = (1 + a_m) p_{m-1,1} . r$; and show that this formula is a particular case of the one in the last question.

14. Explain fully the construction and use of columns D, N, and M, in the columnar method of calculating the values of annuities and assurances; also state the superior advantages this method possesses over the old one.

15. Prove that the value of an annuity of £1 during the joint lives of x and y, and for t years afterwards, should x survive so long, is

$$a_x - \frac{D_{x+t}}{D_x}(a_{x+t} - a_{x+t,y}).$$

16. Give an expression for the annual premium for a contingent annuity to commence at the death of A and to continue as long as either B or C is living.

17. Find, in a convenient form for computation, the single and annual premiums for an annuity to commence at the death of y and continue payable during the remainder of x's life, but to be payable only if y dies within t years.

18. Prove that the single premium for an assurance payable on a life now aged n years attaining the age of $n+t$, or dying previously, may be

represented by the formula (Jones's notation), $\frac{1-ia_{\overline{t-1}|}}{1+i}$; and show that this formula is equivalent to $\frac{M_n-M_{n+t}+D_{n+t}}{D_n}$.

19. Give an expression for the single premium for an assurance on the life of A provided he die after B.

20. Prove that $A_{\underset{1}{(x,y)}\overline{|}t}=r^t.p_{(x,y)t}.A_{\underset{1}{(x+t,y+t)}}$.

21. Determine the value of an assurance on a life of 30, payable at the age of 60 or previous death, commencing at £a, increased by £p at the end of 5 and 10 years respectively, and thereafter increasing by £q *per annum*.

22. Prove the formulæ for the value of a policy of £1—

(1) $1-\frac{1+a_{m+n}}{1+a_m}$, (2) $(P_{m+n}-P_m)(1+a_{m+n})$, (3) $1-(d+P_m)(1+a_{m+n})$.

Show that the same forms apply when the policy is on the joint duration of two lives.

23. Find the value of an annuity on two successive lives, x and y, of which the second is to be nominated at the death of the first, and is supposed to be then y years of age. Also show that if I represent the value of a perpetuity of £1, the value of the annuity will be equal to $I-(1+I)A_xA_y$.

24. Find the single premium for an annuity to x after the death of y, with the condition that the premium is to be returned if x die before y.

25. A, aged x years, is entitled to the interest of £1 for life. If A die within t years, the interest is payable to B, or his representatives, till the expiration of the t years; when C, if living, is entitled to the capital. If, however, C, now aged y years, die either before A or within the t years, the capital reverts to B or his representatives. Determine the values of B's and of C's interests.

Third Year's Examination, 1860.

1. Describe Mr. Gompertz's method of graduating tables of mortality, and give his formula.

2. Describe the method proposed by Mr. Milne for that purpose.

3. Give a brief description of the other methods which have been devised with the like object.

4. Say which of these is, in your opinion, the most effective; and give your reason for thinking it so.

5. In what way do you consider that the surplus of an Assurance Company can be most equitably distributed amongst the assured?

6. When an assurance is effected by one person on the life of another, state the effect of an untrue statement by the life assured or the referees upon the validity of the policy.

7. Explain the difference in the constitution—(1) of Companies formed under the Act 7th and 8th of Victoria, c. 110; (2) of Companies formed before the passing of that Act, and having a special Act of Parliament; (3) of Companies so formed without a special Act; giving instances of each among the existing Companies.

8. How does this difference affect proceedings in the courts of law?

9. What qualifications are mainly necessary in a deed assigning a policy of assurance?

10. What legal remedy has an assignee, should a Company decline to pay under his policy?

11. Explain the process of forming a table of mortality from the burial registers of a stationary population, and the means of correcting for increase of population.

12. Point out the errors of the Northampton Table, and show how they arose from imperfection in the means of observation.

13. Describe the progress of an Insurance Company which receives, at the beginning of each year, a fixed number of new members of a given age, and in which the members only cease to be such by death.

14. Find the number of members at the end of any given number of years, and determine how long it will be before the number of deaths in a year is equal to the number of new members admitted.

15. What is the population of the United Kingdom at the present time, and what does the annual taxation amount to per head?

16. What is the effect at home and abroad of a depreciation in the value of the metallic currency of a country?

17. What consequences arise from the quantity of the coinage in circulation being excessive?

18. In what way, if at all, can the Bank of England influence the amount of circulation?

19. To what extent is the gold in the issue department of the Bank available, at any given time, in the banking department?

20. What is the precise nature of the security which a Bank of England note constitutes?

21. Describe the manner in which you would construct the accounts, or "open the books," of a Life Assurance Company commencing operations.

22. Explain the methods of determining the market value of a contingent reversion.

23. A wishes to buy an annuity for his wife, to commence at his decease; on what terms, as regards rates of interest and mortality, would he be likely to get it, and what regulates the terms? Both are in good health.

24. A has been presented with a policy of assurance on his own life, on which all the premiums have been paid; he wants to surrender this, in consideration of an annuity while he lives. In what way would you find the annuity to be given?

25. B is entitled to an estate at the death of his father, if he survive him. He has assured his life, against that of his father, for half the value of the estate, by a single payment; but he has to pay an annuity whilst he and his father are both living. Describe the way in which you would arrive at the market value of B's interest, and say what rates of interest and mortality you would use in the determination.

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INSTITUTE OF ACTUARIES.

On the Rate of Mortality prevailing amongst the Families of the Peerage during the 19th Century. By ARTHUR HUTCHESON BAILEY, *Actuary of the Equity and Law Assurance Society, and Fellow of the Institute of Actuaries; and* ARCHIBALD DAY, *Actuary of the London and Provincial Law Assurance Society, and Fellow of the Institute of Actuaries.*

[Read before the Institute the 29th April, 1861, and printed by order of the Council.]

IN a note in the introduction to Milne's *Treatise on Annuities*, the author remarks—"There can, I think, be no doubt but that the mortality is greater among the higher than the middle classes of society. They form too small a proportion of the population to have any sensible effect here; but it would be of importance to the Life Offices to determine the law of mortality among them." Since the publication of this work, forty-six years ago, some attempts have been made to test the accuracy of this assertion, and to supply the desideratum; but none with which we are acquainted are by any means conclusive.

In a tract, privately printed in the year 1832, entitled, *Observations on the Mortalities among the Members of the British Peerage*, by the late Mr. George Farren, the author investigates the average duration of the enjoyment of the title by each peer, from observa-

tions made in 740 cases. Out of these, the ages are stated to have been recorded in 447 instances,—of which 288 succeeded to the title between the ages of 10 and 40; from which the conclusion was deduced (and this is the only result stated), that, at the age of 25, the mean duration of life of the peers is very nearly 32 years, which corresponds with the mean duration by the Carlisle Table, at the age eight years older. The process adopted is not stated; but, independently of this and other objections, it will probably be considered that the number of facts observed was too small to warrant any general conclusions on the subject.

Mr. Edmonds contributed to the *Lancet* of the 10th February, 1838, and the 9th March, 1839, two papers—on the "Duration of Life in the English Peerage," and on the "Lineage of English Peers." His observations—extending, apparently, over a long interval of time—were made on 675 peers (32, whose deaths were violent or accidental, having been excluded), during the period of their possession of the title only. These 675 peers were the representatives of 109 titles, the first and last peer in the line of succession having been omitted. In 243 cases the ages were not stated, and this defect was supplied by an approximation made by observing the recorded dates of birth of other peers in the same line of descent. The results are given in three tables, from which it appears that the mortality among the peers is very much in excess of that of the Carlisle Table, but corresponds pretty nearly with a theoretical table constructed by Mr. Edmonds himself, which he designates "City Mortality," according to which the mortality at every age is 50 per cent. greater than in the Carlisle Table. The process by which these results are arrived at is nowhere stated, so that any verification of them is impossible. The number of facts seems to us altogether insufficient, and has been needlessly diminished by the exclusion of the first peer and the existing peer of every title, apparently for no other reason than that they were not required for another investigation which Mr. Edmonds had then in progress. The proportion of cases in which the ages were not recorded is very large—36 per cent. of the whole number; and the hypothetical method adopted for supplying this defect is, we think, open to considerable objection, especially as Mr. Edmonds seems to imply that, had he carried out his original intention of deducing the mortality from those peers only whose ages were recorded, the results would have been different. Also, the probable effect of confining the observations to the period of the occupancy of the title only, will have been, if hereditary ten-

dencies had their usual influence, to exaggerate the mortality of the younger ages, because the peers who succeeded to the title when young must, as a general rule, have been the sons of short-lived parents, and they would not enter into combination for several years with the sons of the long-lived parents, as the latter, for the most part, would not have succeeded to the title until older ages had been attained. Moreover, Mr. Edmonds's plan excludes altogether female and infant mortality, and takes but little account of that of the periods of childhood and youth. For these reasons, we cannot consider his investigations satisfactory or sufficient.

More recently, Dr. Guy made some observations on a much greater number of facts, the results of which he embodied in two papers read before the Statistical Society, and published in their Journal for March, 1845, and March, 1846. Dr. Guy's process is very clearly stated, and is sufficiently simple. He extracted from a Peerage and Baronetage in his possession the ages at death of 2,291 male lives who died above the age of 21 in a period of time extending from the 13th century to the year 1830, and having obtained the number of deaths at each age, a table of the mean duration of life was calculated from these materials. The result—somewhat unexpected, Dr. Guy says—being, that the mean duration of life among these classes is nowhere greater, and at all ages under 70 is materially less, than among the general male population of the country. Entertaining, as we do, a very sincere respect for Dr. Guy's scientific attainments, we cannot, at the same time, avoid remarking that his investigations upon the subject of mortality are unsatisfactory; all his observations having been made on the ages at death only, without any regard to the numbers living. This is not a fitting opportunity, nor can it be necessary here, to enlarge on this point. That observations made from deaths alone will always give erroneous results of the mortality, except in the case of a stationary population, is well known to all who have studied the subject: those who have not, may consult Milne's article in the *Encyclopædia Britannica* on "Mortality," and the 5th and 6th Annual Reports, with the Appendices, of the Registrar-General.

The results obtained by Dr. Guy have frequently been quoted both in newspapers and scientific publications—among others, by Mr. Neison, in his *Contributions to Vital Statistics*—and reasons attempted to be assigned for the conclusions arrived at. Now, as it is certain that, whether Dr. Guy's conclusions are correct or not, they cannot be deduced from the facts which he collected, and not

being satisfied with the investigations of his predecessors in the same path, we resolved to undertake the not inconsiderable labour of determining the mortality prevailing among the families of the peerage by proper technical methods, tracing each case through every year of the period of observation, and comparing the number of deaths with the number of living in each year of age.

Before commencing this task, however, the question arose—what reliance could be placed upon the accuracy of the peerage books? Having made some inquiries on this matter, we are informed, on good authority, that the process of their compilation is some such as the following:—The editors glean from the newspapers, from time to time, occurrences in the way of births, deaths, and marriages, and make the necessary alterations in the current editions of their works. A proof is then sent annually to each peer, of the portion relating to his own family, with a request that he will be good enough to correct and return it. And although the proofs are not all returned annually, the editors express the utmost confidence that, except in a few instances, communications are always made to them whenever corrections are required, and that the several dates are accurately recorded. Having satisfied ourselves upon this important point, the next matter was to select which of the different Peerages should be made use of. The first at hand, that of Burke, was quickly discarded. With a chivalrous spirit, worthy of the Ulster King-at-Arms, he invariably omits the dates of birth of the ladies; and his work, therefore, although probably well adapted for general circulation, was altogether unsuited for our purpose. We then turned to *Lodge's Peerage;* here the facts we required were fully and clearly given, even the still-born births being accurately recorded, and the only objection was, that it did not extend sufficiently far back in point of time. To remedy this, recourse was had to an older Peerage by Debrett; and from these two sources, with occasional reference to Burke, in cases of doubt or discrepancy, all the necessary information was obtained.

We had then to consider what should be the limits of our investigation. To have extended our researches back to distant generations would have been objectionable, because, as an impression commonly prevails that the rate of mortality has progressively improved, comparisons with other tables derived from modern data might have been considered inadmissible. Also, had we followed Mr. Farren and Mr. Edmonds, in restricting the observations to peers only, the number of facts would, in our judgment, have been insufficient, independently of other objections. On the other

hand, had we included all the collateral branches, the distinctive features of the class would have been lost. We therefore determined, that the observations should be limited in point of time to the present century, and should be made upon peers, sons and daughters of peers, and sons and daughters of the eldest sons of peers.

The observations commence with the anniversary of the date of birth in the year 1800 of those who were born in the last century, and with the actual date of birth for the remainder, and terminate on the 31st December, 1855; the only assumption introduced being, that when, as it occasionally happened, the year only of birth or death was given, the day being omitted, this latter was taken to be the 30th June. Those cases were altogether excluded in which the year of birth was not recorded. The number of facts obtained was as follows, viz.:—

	Males.	Females.	Both Sexes.
Deaths	1,938	1,253	3,191
Existing, 31st Dec., 1855 ..	2,283	1,999	4,282
Totals.....	4,221	3,252	7,473

When it is considered that the data from which the Carlisle Table of Mortality, which is probably the one in most general use, was derived, were two enumerations, made at an interval of about nine years, of 7,677 persons living on the first occasion, and 8,677 on the second; that the number of deaths was 1,840; and the ages obtained merely from information voluntarily given, we hope that in regard to time, numbers, and accuracy, the present materials may possess some value.

The extracts having been made separately for each sex, and afterwards combined, the results will be found in Tables I., II., and III., which are sufficiently explained by the headings of the columns. The only remarks needed being, that the number exposed to the risk was obtained by subtracting from the number who completed the age half of the number existing on the 31st December, 1855—that, as was unavoidable, the numbers at the oldest ages were somewhat arbitrarily dealt with—and that the mean duration of life was computed by means of the equation—

$$\text{Log. } e_x = \log. p_x + \log. (1 + e_{x+1}),$$

Where e_x represents the curtate mean duration at the age x. All

the processes, both of making the extracts and the computations, were performed by each of us independently of the other.

As was to have been expected, the column representing the annual mortality exhibits numerous irregularities, but we have purposely abstained from applying any method of graduation. Indispensable as this process is for many applications of tables of mortality, and great as has been the ingenuity expended upon it, the process itself can only be regarded as a necessary evil; and, without going so far as to say, with Professor De Morgan, that "the practice cannot be too strongly condemned," the force of his remark, that, "the tables thereby lose some of their value as representations of physical facts," is undeniable. And as the process was neither necessary nor desirable for one of the chief objects of this investigation—the comparison of the results with those of other tables—we have preferred to leave the observations unadjusted.

Examples of these results, compared with those of other tables of authority, will be found in Tables IV., V., and VI. The observations selected for comparison have been—of male lives exclusively, the English Life Table, representing the mortality of the population in general; the experience of the Equitable Society (chiefly males); that of the Government Annuitants, from Mr. Alexander Finlaison's recent Report; and the mortality of healthy districts, from a paper by Dr. Farr, lately read before the Royal Society. The latter, having been derived from the records of 63 districts of the country where the average annual mortality in 1,000 did not exceed 17, the general average being 22, is considered by Dr. Farr to be "the nearest approximation we can obtain to a table representing the human race in the normal state." For female lives exclusively, we have selected for comparison the corresponding tables of the English Life Table, the Government Annuitants, and the healthy districts; and for both sexes combined, the Carlisle Table, and the general table of the healthy districts.

Referring, first, to male lives only, it will be found that the average mean duration of life among the families of the peerage is, at all ages under 73, greater than among the general population of the country, greater even than among the selected lives of the Equitable Society (with the unimportant exception of the period from 15 to 21, where the number of cases in that Society was very small); greater, at all ages under 62, than among the Government annuitants; and, throughout, approaches pretty nearly to the standard table of the mortality of the healthy districts. At the

older ages, it very nearly coincides both with the English Life Table and the Equitable experience, but is somewhat less than that of the Government annuitants. Looking to particular periods of life, it will be observed that the advantage in favour of the families of the peerage is most remarkable in infancy and childhood—the mortality under the age of 10 years being little more than one-third of that of the general population. In the next decade, the two tables nearly coincide; but at the succeeding period, from 20 to 29, a very singular anomaly occurs. There the mortality among the families of the peerage is not only in excess of that of the English and Equitable Tables, but, contrary to our previous notions, is also materially greater than in the next decennial interval. A similar anomaly occurs in the Government annuitants, and also in the Society of Friends.* At all other ages, up to 80, the advantage is with the aristocratic class.

It is evident, therefore, that Dr. Guy's conclusions on this subject are erroneous; and, to make this more clear, we have thought it worth while to compute a table of the mean duration of life, deduced from the deaths alone, in the present observations, and have placed some examples of it in juxtaposition with the corresponding results deduced by the correct method, and also by Dr. Guy.

Males—Peerage Families.

Age.	Mean Duration of Life.	Do. deduced from the Deaths only.	
		Present Observations.	Dr. Guy.
20	41·46	36·42	38·48
30	35·51	31·63	30·88
40	28·33	25·36	24·45
50	21·40	19·38	17·92
60	14·56	13·27	12·57
70	8·77	8·03	8·15
80	4·58	4·20	5·09

The above table is, we think, deserving of some attention—partly, because the erroneous method of observation is very common with the medical profession; and partly, because the results of such

* In this Society, the annual mortality per cent. among the males has been found to be at the ages 20–29, ·881; and at the ages 30–39, ·782. Our information on the subject is derived from a most complete and interesting paper "On the Vital Statistics of the Society of Friends," by Joseph John Fox, read before the Statistical Society, 21st December, 1858.

observations are received with implicit confidence in some quarters where it could least have been expected.

Turning next to the tables for female life exclusively, it will be observed that the average mean duration of life among the families of the peerage is, throughout, materially greater than with the general population, coincides in a very remarkable manner, up to the age of 55, with that of the Government annuitants, and also of the inhabitants of the healthy districts, but surpasses them both for the remainder of life—indicating a more favourable mortality at the older ages than any table whatever with which we are acquainted. It will be seen that, as with the males, the contrast with the general population is most marked under the age of 10, but that in the next decade the mortality is somewhat in excess; the abnormal feature noticed in the males thus appearing among the females, but not to so great an extent, and occurring at a somewhat earlier period of life. The singularly favourable mortality above the age of 70 will not escape attention.

We believe that no previous attempt has been made to investigate the mortality prevailing among females of the higher classes in this country, and are not without hope that the present one may, owing to the trustworthiness of the data, help to elucidate the subject of female mortality generally. The known difficulty of obtaining accurate information on so delicate a subject as ladies' ages has, undoubtedly, tended to throw some degree of suspicion upon all results obtained from any general records of the female population. Even the compilers of the Census of 1851, laudably eager as they were to break a lance in defence of the veracity of their countrywomen, were reluctantly compelled to admit that this part of the returns required some awkward and troublesome corrections, and cited the case of an eminent French statist, who, after many persevering but fruitless attempts, had abandoned in despair a philosophical inquiry which had for its object to determine the ages of his wife and his cook.

Table VII. will illustrate the comparative mortality of the sexes. As this is a subject of some interest, and as the evidence respecting it is conflicting, we have thought it deserving of some further examination.

The superior longevity of the female sex having long been well known, one of the most striking results of the combined experience of the Assurance Offices was that the mortality amongst assured females was greater than amongst assured males. Several explanations of this apparent anomaly have been offered, none of them

very satisfactory, and some altogether absurd; but, after a careful consideration of the matter, we cannot help thinking that the anomaly does not really exist at all. According to Dr. Farr's Tables, the mortality in the period of infancy is greater among boys than girls, from 2 to 40 years of age the mortality of females is in excess, while for the remainder of life the females have the advantage over the males in a marked degree. And we find that the remarkable tenacity of female life in old age so far outweighs the greater mortality of the sex shortly before and during the childbearing period, as to give to the female sex an average mean duration of life greater than that of the male throughout the whole of the table. These results the present observations confirm in every respect—they exhibit the superior vitality of females in infancy, their remarkable tenacity of life in old age, and their greater mortality in youth and the prime of life; for, notwithstanding the abnormal mortality of the males between the ages of 20 and 30, it will be found that, taking the entire period from 5 to 45 years of age, the female mortality is in excess. Bearing these peculiarities in mind, it was certainly somewhat startling to find, on referring to the Combined Experience Tables, that the female mortality in the decade 70–79 was both so great absolutely, and so much in excess of the male, especially as between the ages of 45 and 70 the female mortality is less. But on looking more closely, and remarking that the objection often urged, that these tables represent the experience not of lives but of policies, is serious when the numbers are small, though not, perhaps, very important when they are large, we discover that the whole number of claims above the age of 70, under policies on female lives, in all the combined Offices, was exactly 60, and only 477 at all ages. And the actuaries themselves assign the paucity of the numbers as a reason for the apparent excess of the female mortality at the older ages, admitting that no importance is to be attached to this result. But as the facts were avowedly insufficient to bring out the peculiar tenacity of life of the female sex in old age, the result is perfectly consistent with other observations. Instead, therefore, of asserting that the "mortality amongst assured females, taking all ages together, is greater than amongst assured males," we think a more accurate statement would have been, that the experience of the Offices confirms the result of other observations, which show the greater mortality of females during the childbearing period, and affording little or no information for the periods of infancy and old age, it did not possess the means of exhibiting the general mor-

tality of the sex. In corroboration of this may be adduced the valuable information furnished by the experience of the Eagle Office, where the number of female deaths exceeded those of all the combined Offices. In the experience of the Eagle, the female mortality was lower than the male, taking all ages together; while, at the same time, under the age of 42, it was considerably in excess. And it must not be forgotten that the average period of observation of each life, although greater than with the combined Offices, did not exceed 8½ years; so that there is reason to believe that, if the observations were continued to the present time, the greater general vitality of the female sex would be still more apparent.

On the other hand, the late Mr. Finlaison, whose experience and authority on this subject were very great, stoutly contended that "the fact is undoubtedly certain, that the mortality of the female sex, at every period of life, is less than that of the male sex at the same ages, excepting only in infancy," and supported this assertion by numerous tables deduced from observations made on the nominees of certain tontines, and also on the Government annuitants from 1808 to 1825. These observations have been continued to the 31st December, 1850, by Mr. Alexander Finlaison, and the results have been recently published in a report by that gentleman. The later results confirming, in this particular, the former observations, Mr. Alexander Finlaison appears to consider that his father's views are now unimpeachable, and form a standard by which the accuracy of other tables may be tested. Applying this test to the English Life Table, he makes the following remarks:—"At the youthful and earlier adult ages, the mortality of the female is represented to be greater than that of the male. This conclusion is contrary to most previous experience. It is a result which is also contrary to nature. The sexes are not created in equal numbers. For every 20 females there are produced 21 males. But no fact is more thoroughly established, than that whenever the population is counted the females are present in considerably greater number. . . . Such a result could not take place, unless the stronger sex were subjected to a higher rate of mortality, and died off much faster than the females."

From these observations we must express our dissent. Dr. Farr's conclusion is not contrary to most previous experience. Not only in the English Life Table, but also in the observations on the inhabitants of the healthy districts, on assured lives, and on the Society of Friends, the mortality of the female in the youthful and earlier adult ages is found to be greater than that of the male;

and in *every* published table of mortality to which we have been able to refer, excepting only Mr. Finlaison's and the Swedish Tables, the mortality of the female is, at particular ages, in excess of that of the male. The peculiarity of Mr. Finlaison's results may, perhaps, be accounted for by the probability that a large proportion of the females in his observations are unmarried, and by the scantiness of his materials in middle life compared with their abundance at the older ages.

The English Life Table is not contrary to nature; for, concurrently with the greater mortality of the female at particular ages, we almost invariably observe a greater general mortality of the male. The two circumstances—the excess of male births and the greater general mortality of that sex—together cause that nearly uniform proportion of the sexes which successive enumerations disclose.

How far the long and elaborate attack on the English Life Table in Mr. Alexander Finlaison's report is consistent with the official etiquette usually observed by different departments of the State towards each other, we do not presume to determine. If the National Debt Office and the General Register Office are at variance, assuredly *non nostrum est tantas componere lites.* We only endeavour, amongst conflicting statements, to search for the truth; and the result of our inquiries on this particular subject—the comparative mortality of the sexes—has been to confirm the views of Dr. Farr, and not those of Mr. Finlaison. On the whole, if human life be divided into three great periods—of infancy, maturity, and old age—the weight of evidence is in favour of the general conclusion, that, at the two extremes, the mortality of the female sex is less, and at the intervening period greater, than that of the male; the probable after-lifetime being, at all ages, greater for the female.

The examples of the general table of mortality of the families of the peerage will suffice to dispel the previous views that have been propounded on the subject, and remove the erroneous ideas into which even so judicious and accomplished a writer as Milne has fallen, and which led Mr. Edmonds to make the unfounded assertion, that "the severest mortality is to be looked for in the poorest class of a city population, and in the highest class of the monied or nonlabouring portion of the community." It will be observed that the mean duration of life among the families of the peerage approaches nearly to that in Dr. Farr's standard table, and, with one slight exception, is throughout greater than in the

Carlisle Table. This exception occurs about the age of 80, and may be readily explained by the circumstance, that in the population of Carlisle, from which the mortality table was framed, the females were about 55 per cent. of the whole number—a much greater proportion than occurs in the general population. In our observations, on the contrary, the males outnumber the females—a greater number of cases in the latter sex having to be rejected owing to deficiency of dates. Had it not been for this circumstance, the results of the general table would have proved even more favourable.

The peculiar features of the mortality at different periods of life have already been sufficiently discussed.

One or two remarks on the application that may be made of the results of the present investigation, in the occupations in which most of us are engaged, may not be considered inappropriate. Adopting the common division of the different ranks of society—into upper, middle, and lower classes—it may be safely stated that the latter, although forming the great bulk of the community, have hardly any dealings with Life Offices. And we are inclined to think that the somewhat heterogeneous mass called the middle class does not resort to these Offices to the same extent, in proportion to its number, as the higher classes. It is, we are aware, a popular belief, that the extent of the practice of life assurance in this country affords strong evidence of the provident habits of the community, and that the success of the Offices in Great Britain is attributable to the greater degree of prudence and forethought prevailing here than among the nations of the Continent. But some of those who have had the most experience in the matter would probably be of opinion that the practice of life assurance affords quite as much evidence of improvident as of provident habits; that the Offices obtain as many supporters from those who exceed as from those who live within their incomes; and that their success is attributable, in no slight degree, to the extent to which the practice of making settlements of property prevails in this country, and to the consequent number and variety of life interests of a pecuniary nature arising therefrom. However this may be, there can be no doubt that those who are beneficially interested in these settlements, but, at the same time, have not usually much ready money at command (a description not unfrequently applicable to the class now under observation), are introduced to the Assurance Offices in considerable numbers. It cannot, therefore, be otherwise than satisfactory to find that the mortality prevailing among the class

in question is decidedly more favourable than any which the experience of the Offices has yet furnished.

The present investigation also indicates that the effect expected to be produced by selection of lives is much exaggerated; at the same time, it confirms an opinion occasionally expressed, and which seems to be founded on experience, that the best lives are those that are assured for large amounts.

It may, perhaps, be of some service in a department of our practice which urgently requires amendment—the system upon which extra premiums for foreign residence are charged. The male lives that have formed the subject of the present investigation have been found to experience an unusually favourable mortality; yet they enter the army and navy in large numbers, travel extensively, and are certainly more exposed to what the Assurance Offices consider extra risks, than the middle classes. It would seem, therefore, to be no unfair inference, that differences of climate have less effect on human mortality than differences of occupation and position in life; and as the Offices do not attach much importance to the latter—taking a butcher and a country clergyman on similar terms—they might perhaps relax somewhat in their estimate of the former. Considering the very unsatisfactory character of the present practice in this respect, it might be worth consideration whether any serious risk would be incurred by dispensing altogether, in the majority of cases, with the existing restrictions on foreign travelling and residence.

On the other hand, the results of this investigation afford some suggestions for the exercise of caution. The most painful comparison presented by the present tables is the remarkable difference in the mortality of the children when contrasted with the general population. Now, as it may be tolerably safely assumed that all children for whom endowments are purchased will be well cared for, it would seem that both the Carlisle and English Tables are unsafe data for the calculation of endowment premiums.

The exceptional mortality also of the period of early manhood, confirmed as it is by the experience both of the Government annuitants and of the Society of Friends, indicates that those assurances are not the most desirable that are effected under the age of 30 for terms of years, on an increasing scale of premium, or on that most inconsistent and odious method called the "half-credit system." The same circumstance will perhaps explain why, in the Economic Office, it has been found that the highest rate of mortality has been experienced on term assurances.

The information obtained upon the comparative mortality of the sexes leads to the conclusions, that tables of mortality for all purposes of life assurance should be derived from observations on male lives chiefly, or exclusively; that the greater vitality of the female will not justify any reduction of premium in contracts of assurance, because, on account of the large proportion of policies that are suffered to lapse, the greatest amount of risk will usually be incurred in middle life, at which particular period the female mortality is greater than the male. On the other hand, the distinction of sex is of serious moment in all contracts of annuities, immediate, deferred, and contingent; because in those cases the most important period is that of old age, where the distinction between the mortality of the sexes is most marked. The distinction is also important in another branch of our pursuits, where, we believe, it is frequently overlooked—we mean in the valuation of reversionary interests.

Finally, if this investigation should tend to encourage the belief that the mortality of each well-defined class has peculiar characteristics of its own, it must weaken the hold that the Carlisle Table has upon some of its votaries, who seem to consider that for all purposes, and under all circumstances, their favourite table is applicable. A consideration of the characteristic features, both of these and of other observations on persons in affluent circumstances, may suggest to another class of enthusiasts, that there are many other causes affecting the mortality of mankind besides the sanitary condition of their habitations; and that although ventilation, drainage, and water supply are all very necessary things, they are not "all the law and the prophets" notwithstanding. That the peculiar features of the present observations belong to the normal law of mortality of the human race, it would, we think, be very unwise either to affirm or deny. Notwithstanding all that has been written on that subject, we remain of opinion that that law is yet undiscovered, and that a much greater number and variety of observations than we at present possess will be required for its discovery. Such a law, if discovered, would be of high interest, both to the physiologist and the mathematician. But it will represent the law that really prevails among the living, moving, thinking men that inhabit the earth, much in the same way that the statue of the Apollo Belvedere represents their bodily form. Such a law will never supersede, in our pursuits at least, the exercise of that careful judgment and sound discrimination which it should be our study to cultivate, and without which the most varied talents will be useless and the greatest attainments vain.

TABLE I.—*Males.*

Age (x).	Completed the age x.	Existing 31st Dec., 1855, between the Ages x and $x+1$.	Died between the Ages x and $x+1$.	Number exposed to the Risk from the Age x to $x+1$.	Probability of Dying in the Year.	Probability of Surviving the Year.	Mean Duration of Life.
0	2534	31	197	2518·5	·07821	·92179	52·00
1	2358	38	38	2339·0	·01625	·98375	55·37
2	2326	27	20	2312·5	·00865	·99135	55·25
3	2328	42	9	2307·0	·00390	·99610	54·73
4	2315	42	10	2294·0	·00436	·99564	53·93
5	2302	27	11	2288·5	·00481	·99519	53·16
6	2293	45	4	2270·5	·00176	·99824	52·42
7	2276	34	6	2259·0	·00266	·99734	51·51
8	2271	49	10	2246·5	·00445	·99555	50·66
9	2250	30	8	2235·0	·00358	·99642	49·88
10	2249	42	9	2228·0	·00404	·99596	49·04
11	2238	36	13	2220·0	·00586	·99414	48·23
12	2230	26	8	2217·0	·00361	·99639	47·52
13	2231	28	11	2217·0	·00496	·99504	46·68
14	2227	36	5	2209·0	·00226	·99774	45·91
15	2225	34	9	2208·0	·00408	·99592	45·02
16	2223	33	16	2207·5	·00725	·99275	44·21
17	2211	30	14	2196·0	·00638	·99362	43·53
18	2197	27	16	2183·5	·00733	·99267	42·82
19	2188	36	17	2170·0	·00783	·99217	42·13
20	2174	48	18	2150·0	·00837	·99163	41·46
21	2135	42	35	2114·0	·01656	·98344	40·80
22	2089	32	20	2073·0	·00965	·99035	40·47
23	2077	31	24	2061·5	·01163	·98837	39·87
24	2053	36	18	2035·0	·00885	·99115	39·33
25	2035	31	23	2019·5	·01139	·98861	38·67
26	2010	38	25	1991·5	·01255	·98745	38·10
27	1985	24	15	1973·0	·00760	·99240	37·58
28	1978	40	17	1958·0	·00868	·99132	36·86
29	1949	27	18	1935·5	·00930	·99070	36·18
30	1938	36	24	1920·0	·01250	·98750	35·51
31	1899	35	11	1881·5	·00585	·99415	34·96
32	1882	30	19	1867·0	·01018	·98982	34·17
33	1867	34	14	1850·0	·00757	·99243	33·51
34	1851	31	12	1835·5	·00654	·99346	32·75
35	1835	26	10	1822·0	·00549	·99451	31·97
36	1815	33	25	1798·5	·01390	·98610	31·15
37	1773	49	11	1748·5	·00629	·99371	30·58
38	1738	31	17	1722·5	·00987	·99013	29·77
39	1720	28	15	1706·0	·00879	·99121	29·08
40	1697	30	23	1682·0	·01367	·98633	28·33
41	1657	36	16	1639·0	·00976	·99024	27·71
42	1626	26	22	1613·0	·01364	·98636	26·99
43	1604	37	17	1585·5	·01072	·98928	26·34
44	1565	38	10	1546·0	·00647	·99353	25·63
45	1537	39	22	1517·5	·01450	·98550	24·80
46	1489	26	22	1476·0	·01491	·98509	24·16

TABLE I. (continued).

Age (x).	Completed the Age x.	Existing 31st Dec., 1855, between the Ages x and $x+1$.	Died between the Ages x and $x+1$.	Number exposed to the Risk from the Age x to $x+1$.	Probability of Dying in the Year.	Probability of Surviving the Year.	Mean Duration of Life.
47	1464	37	19	1445·5	·01314	·98686	23·52
48	1419	30	19	1404·0	·01353	·98647	22·82
49	1385	26	17	1372·0	·01239	·98761	22·13
50	1355	29	21	1340·5	·01567	·98433	21·40
51	1317	29	18	1302·5	·01382	·98618	20·74
52	1280	31	15	1264·5	·01186	·98814	20·02
53	1244	33	21	1227·5	·01711	·98289	19·25
54	1201	21	27	1190·5	·02268	·97732	18·58
55	1161	28	19	1147·0	·01656	·98344	18·00
56	1128	35	24	1110·5	·02161	·97839	17·30
57	1074	27	11	1060·5	·01037	·98963	16·66
58	1042	33	27	1025·5	·02633	·97367	15·83
59	989	18	20	980·0	·02041	·97959	15·25
60	957	15	19	949·5	·02001	·97999	14·56
61	930	19	26	920·5	·02818	·97182	13·85
62	891	16	24	883·0	·02717	·97283	13·23
63	859	18	32	850·0	·03764	·96236	12·59
64	815	23	24	803·5	·02986	·97014	12·05
65	775	20	33	765·0	·04313	·95687	11·41
66	725	13	27	718·5	·03758	·96242	10·90
67	687	18	27	678·0	·03983	·96017	10·30
68	645	16	34	637·0	·05338	·94662	9·71
69	602	16	35	594·0	·05892	·94108	9·23
70	557	13	36	550·5	·06539	·93461	8·77
71	513	15	32	505·5	·06330	·93670	8·35
72	470	15	25	462·5	·05405	·94595	7·88
73	432	6	32	429·0	·07457	·92543	7·31
74	396	10	35	391·0	·08952	·91048	6·85
75	355	9	39	350·5	·11127	·88873	6·48
76	314	8	29	310·0	·09354	·90646	6·23
77	280	8	27	276·0	·09783	·90217	5·82
78	235	8	27	231·0	·11688	·88312	5·40
79	212	5	22	209·5	·10501	·89499	5·05
80	187	4	20	185·0	·10811	·89189	4·58
81	165	5	29	162·5	·17846	·82154	4·08
82	131	6	16	128·0	·12500	·87500	3·85
83	109	3	21	107·5	·19535	·80465	3·33
84	85	"	19	85·0	·22353	·77647	3·02
85	66	2	15	65·0	·23077	·76923	2·74
86	49	3	13	47·5	·27369	·72631	2·41
87	33	1	11	32·5	·33846	·66154	2·14
88	21	3	8	19·5	·41026	·58974	1·97
89	10	"	3	10·0	·30000	·70000	2·00
90	7	"	3	7·0	·42857	·57143	1·64
91	4	"	1	4·0	·25000	·75000	1·50
92	3	"	2	3·0	·66666	·33334	·83
93	1	1	"	·5			

TABLE II.—*Females.*

Age x.	Completed the Age x.	Existing 31st Dec., 1855, between the Ages x and $x+1$.	Died between the Ages x and $x+1$.	Number exposed to the Risk from the Age x to $x+1$.	Probability of Dying in the Year.	Probability of Surviving the Year.	Mean Duration of Life.
0	2152	33	127	2135·5	·05948	·94052	53·71
1	2029	30	33	2014·0	·01639	·98361	56·09
2	2003	36	14	1985·0	·00705	·99295	56·00
3	1987	35	11	1969·5	·00559	·99441	55·41
4	1979	35	11	1961·5	·00561	·99439	54·71
5	1963	36	5	1945·0	·00257	·99743	54·02
6	1945	34	11	1928·0	·00571	·99429	53·15
7	1926	33	11	1909·5	·00576	·99424	52·45
8	1912	31	8	1896·5	·00422	·99578	51·75
9	1901	35	7	1883·5	·00372	·99628	50·98
10	1889	34	16	1872·0	·00854	·99146	50·16
11	1861	34	18	1844·0	·00976	·99024	49·58
12	1835	29	11	1820·5	·00604	·99396	49·06
13	1834	40	4	1814·0	·00220	·99780	48·35
14	1822	33	12	1805·5	·00665	·99335	47·46
15	1813	35	8	1795·5	·00445	·99555	46·77
16	1794	34	10	1777·0	·00563	·99437	45·97
17	1776	30	20	1761·0	·01136	·98864	45·23
18	1745	37	11	1726·5	·00637	·99363	44·74
19	1722	38	18	1703·0	·01057	·98943	44·02
20	1698	26	12	1685·0	·00712	·99288	43·48
21	1679	34	12	1662·0	·00722	·99278	42·79
22	1656	23	11	1644·5	·00669	·99331	42·10
23	1643	32	16	1627·0	·00983	·99017	41·39
24	1621	36	14	1603·0	·00873	·99127	40·80
25	1596	29	14	1581·5	·00885	·99115	40·16
26	1575	32	11	1559·0	·00706	·99294	39·50
27	1560	29	13	1545·5	·00841	·99159	38·78
28	1538	25	18	1525·5	·01180	·98820	38·10
29	1518	27	11	1504·5	·00731	·99269	37·54
30	1495	30	9	1480 0	·00608	·99392	36·82
31	1479	27	12	1465·5	·00818	·99182	36·04
32	1461	24	13	1449·0	·00897	·99103	35·33
33	1434	32	18	1418·0	·01269	·98731	34·65
34	1405	22	10	1394·0	·00717	·99283	34·09
35	1383	22	20	1372·0	·01458	·98542	33·35
36	1358	27	10	1344·5	·00744	·99256	32·83
37	1335	20	12	1325·0	·00906	·99094	32·08
38	1315	29	11	1300·5	·00846	·99154	31·37
39	1287	31	12	1271·5	·00944	·99056	30·64
40	1257	23	20	1245·5	·01606	·98394	29·93
41	1225	32	13	1209·0	·01075	·98925	29·40
42	1190	24	8	1178·0	·00679	·99321	28·71
43	1164	28	16	1150·0	·01391	·98609	27·91
44	1123	34	15	1106·0	·01356	·98644	27·30
45	1079	28	13	1065·0	·01221	·98779	26·66
46	1049	35	14	1031·5	·01358	·98642	25·99
47	1006	16	9	998·0	·00902	·99098	25·34
48	986	33	8	969·5	·00825	·99175	24·56
49	952	25	13	939·5	·01383	·98617	23·76
50	924	29	16	909·5	·01759	·98241	23·08
51	883	21	10	872·5	·01146	·98854	22·49
52	857	19	5	847·5	·00590	·99410	21·74

TABLE II. (continued).

Age x.	Completed the Age x.	Existing 31st Dec., 1855, between the Ages x and $x+1$.	Died between the Ages x and $x+1$.	Number exposed to the Risk from the Age x to $x+1$.	Probability of Dying in the Year.	Probability of Surviving the Year.	Mean Duration of Life.
53	839	19	11	829·5	·01326	·98674	20·87
54	816	31	16	800·5	·01998	·98002	20·14
55	776	21	17	765·5	·02220	·97780	19·54
56	740	20	20	730·0	·02740	·97260	18·98
57	704	22	10	693·0	·01443	·98557	18·49
58	673	20	15	663·0	·02262	·97738	17·76
59	639	22	10	628·0	·01592	·98408	17·15
60	607	16	15	599·0	·02504	·97496	16·42
61	576	10	18	571·0	·03152	·96848	15·83
62	549	14	18	542·0	·03321	·96679	15·34
63	517	19	11	507·5	·02167	·97833	14·84
64	490	8	20	486·0	·04115	·95885	14·16
65	463	12	22	457·0	·04814	·95186	13·74
66	430	13	13	423·5	·03069	·96931	13·41
67	405	14	12	398·0	·03015	·96985	12·82
68	380	15	18	372·5	·04832	·95168	12·20
69	349	13	14	342·5	·04088	·95912	11·80
70	326	19	13	316·5	·04108	·95892	11·28
71	296	13	17	289·5	·05872	·94128	10·74
72	267	12	11	261·0	·04214	·95786	10·39
73	246	6	13	243·0	·05350	·94650	9·82
74	227	6	20	224·0	·08929	·91071	9·35
75	202	8	10	198·0	·05051	·94949	9·22
76	184	4	10	182·0	·05495	·94505	8·69
77	170	7	16	166·5	·09610	·90390	8·16
78	147	3	8	145·5	·05498	·94502	7·98
79	136	7	9	132·5	·06792	·93208	7·41
80	120	8	13	116·0	·11207	·88793	6·92
81	99	4	13	97·0	·13402	·86598	6·73
82	82	7	10	78·5	·12739	·87261	6·69
83	65	1	6	64·5	·09302	·90698	6·60
84	58	3	3	56·5	·05310	·94690	6·22
85	52	1	11	51·5	·21360	·78640	5·54
86	40	2	3	39·0	·07692	·92308	5·91
87	35	3	3	33·5	·08955	·91045	5·36
88	29	..	4	29·0	·13793	·86207	4·84
89	25	1	3	24·5	·12245	·87755	4·54
90	21	1	4	20·5	·19512	·80488	4·10
91	16	1	2	15·5	·12903	·87097	3·97
92	13	2	2	12·0	·16667	·83333	3·49
93	9	..	1	9·0	·11111	·88889	3·08
94	8	1	2	7·5	·26667	·73333	2·41
95	5	..	1	5·0	·20000	·80000	2·10
96	4	2	..	3·0	·33333	·66667	1·50
97	2	..	..	2·0	·50000	·50000	1·00
98	2	..	..	2·0			
99	2	..	..	2·0			
100	2	..	..	2·0			
101	2	1	..	1·5			
102	1	..	..	1·0			
103	1	..	..	1·0			
104	1	1	..	·5			

TABLE III.—*Both Sexes.*

Age x.	Number exposed to the Risk from the Age x to $x+1$.	Died between the Ages x and $x+1$.	Probability of Dying in the Year.	Probability of Surviving the Year.	Mean Duration of Life.
0	4654·0	324	·06962	·93038	52·62
1	4353·0	71	·01631	·98369	55·52
2	4297·5	34	·00791	·99209	55·44
3	4276·5	20	·00468	·99532	54·87
4	4255·5	21	·00494	·99506	54·13
5	4233·5	16	·00378	·99622	53·40
6	4198·5	15	·00357	·99643	52·60
7	4168·5	17	·00408	·99592	51·78
8	4143·0	18	·00434	·99566	50·99
9	4118·5	15	·00364	·99636	50·21
10	4100·0	25	·00610	·99390	49·39
11	4064·0	31	·00763	·99237	48·69
12	4037·5	19	·00471	·99529	48·07
13	4031·0	15	·00372	·99628	47·29
14	4014·5	17	·00423	·99577	46·47
15	4003·5	17	·00425	·99575	45·66
16	3984·5	26	·00653	·99347	44·85
17	3957·0	34	·00859	·99141	44·15
18	3910·0	27	·00691	·99309	43·52
19	3873·0	35	·00904	·99096	42·82
20	3835·0	30	·00782	·99218	42·21
21	3776·0	47	·01245	·98755	41·54
22	3717·5	31	·00834	·99166	41·05
23	3688·5	40	·01084	·98916	40·40
24	3638·0	32	·00880	·99120	39·83
25	3601·0	37	·01027	·98973	39·18
26	3550·5	36	·01014	·98986	38·58
27	3518·5	28	·00796	·99204	37·97
28	3483·5	35	·01005	·98995	37·27
29	3440·0	29	·00843	·99157	36·65
30	3400·0	33	·00970	·99030	35·96
31	3347·0	23	·00687	·99313	35·30
32	3316·0	32	·00965	·99035	34·54
33	3268·0	32	·00979	·99021	33·87
34	3229·5	22	·00681	·99319	33·20
35	3194·0	30	·00939	·99061	32·43
36	3143·0	35	·01114	·98886	31·73
37	3073·5	23	·00748	·99252	31·08
38	3023·0	28	·00926	·99074	30·31
39	2977·5	27	·00907	·99093	29·59
40	2927·5	43	·01469	·98531	28·86
41	2848·0	29	·01018	·98982	28·28
42	2791·0	30	·01075	·98925	27·57
43	2735·5	33	·01206	·98794	26·86
44	2652·0	25	·00943	·99057	26·18
45	2582·5	35	·01356	·98644	25·43
46	2507·5	36	·01436	·98564	24·77
47	2443·5	28	·01146	·98854	24·12
48	2373·5	27	·01138	·98862	23·40
49	2311·5	30	·01298	·98702	22·66
50	2250·0	37	·01644	·98356	21·95
51	2175·0	28	·01287	·98713	21·31
52	2112·0	20	·00947	·99053	20·58

TABLE III. (continued).

Age x.	Number exposed to the Risk from the Age x to $x+1$.	Died between the Ages x and $x+1$.	Probability of Dying in the Year.	Probability of Surviving the Year.	Mean Duration of Life.
53	2057·0	32	·01555	·98445	19·77
54	1991·0	43	·02160	·97840	19·08
55	1912·5	36	·01882	·98118	18·49
56	1840·5	44	·02391	·97609	17·83
57	1753·5	21	·01198	·98802	17·26
58	1688·5	42	·02487	·97513	16·46
59	1608·0	30	·01866	·98134	15·87
60	1548·5	34	·02196	·97804	15·16
61	1491·5	44	·02950	·97050	14·49
62	1425·0	42	·02947	·97053	13·91
63	1357·5	43	·03167	·96833	13·32
64	1289·5	44	·03412	·96588	12·74
65	1222·0	55	·04501	·95499	12·17
66	1142·0	40	·03503	·96497	11·72
67	1076·0	39	·03624	·96376	11·13
68	1009·5	52	·05151	·94849	10·53
69	936·5	49	·05232	·94768	10·08
70	867·0	49	·05652	·94348	9·61
71	795·0	49	·06163	·93837	9·15
72	723·5	36	·04976	·95024	8·72
73	672·0	45	·06696	·93304	8·15
74	615·0	55	·08943	·91057	7·70
75	548·5	49	·08934	·91066	7·41
76	492·0	39	·07927	·92073	7·08
77	442·5	43	·09718	·90282	6·65
78	376·5	35	·09296	·90704	6·31
79	342·0	31	·09064	·90936	5·91
80	301·0	33	·10963	·89037	5·45
81	259·5	42	·16185	·83815	5·06
82	206·5	26	·12591	·87409	4·93
83	172·0	27	·15698	·84302	4·57
84	141·5	22	·15547	·84453	4·33
85	116·5	26	·22317	·77683	4·04
86	86·5	16	·18497	·81503	4·05
87	66·0	14	·21212	·78788	3·86
88	48·5	12	·24742	·75258	3·77
89	34·5	6	·17391	·82609	3·84
90	27·5	7	·25455	·74545	3·54
91	19·5	3	·15385	·84615	3·58
92	15·0	4	·26667	·73333	3·14
93	9·5	1	·10526	·89474	3·10
94	7·5	2	·26667	·73333	2·41
95	5·0	1	·20000	·80000	2·10
96	3·0				
97	2·0				
98	2·0				
99	2·0				
100	2·0				
101	1·5				
102	1·0				
103	1·0				
104	·5				

TABLE IV.—*Males.*

Age.	Mean Duration of Life.				
	Peerage Families.	English Table. Dr. Farr.	Equitable. Morgan.	Government Annuitants. A. G. Finlaison.	Healthy Districts. Dr. Farr.
0	52·00	40·36	..	..	48·56
10	49·04	47·47	48·32	45·57	51·28
20	41·46	39·99	41·67	38·74	43·40
30	35·51	33·21	34·53	33·39	36·45
40	28·33	26·46	27·40	27·12	29·29
50	21·40	19·87	20·36	20·53	22·03
60	14·56	13·60	13·91	14·41	15·06
70	8·77	8·55	8·70	9·08	9·37
80	4·58	4·97	4·75	5·22	5·37
90	1·64	2·80	2·56	2·78	2·99

Age.	Annual Mortality per Cent.			
	Peerage Families.	English Table. Dr. Farr.	Equitable. Morgan.	Government Annuitants. A. G. Finlaison.
Under 5	2·227	7·072	..	..
5 to 9	·345	·926	..	·718
10—19	·536	·581	..	·742
20—29	1·046	·882	·749	1·315
30—39	·870	1·094	·928	1·216
40—49	1·227	1·487	1·243	1·368
50—59	1·764	2·275	2·111	2·269
60—69	3·757	4·654	4·304	3·971
70—79	8·714	10·012	8·994	8·685
80—89	23·836	21·370	20·786	18·600

TABLE V.—*Females.*

Age.	Mean Duration of Life.			
	Peerage Families.	English Table. Dr. Farr.	Government Annuitants. A. G. Finlaison.	Healthy Districts. Dr. Farr.
0	53·71	42·04	..	49·45
10	50·16	47·86	50·07	50·88
20	43·48	40·65	43·27	43·50
30	36·82	34·06	36·65	36·85
40	29·93	27·50	29·91	30·00
50	23·08	20·84	22·99	22·87
60	16·42	14·49	16·17	15·69
70	11·28	9·12	10·14	9·85
80	6·92	5·34	5·69	5·64
90	4·10	3·09	2·94	3·11

Age.	Annual Mortality per Cent.		
	Peerage Families.	English Table. Dr. Farr.	Government Annuitants. A. G. Finlaison.
Under 5	1·882	6·037	..
5 to 9	·440	·900	·668
10—19	·716	·639	·648
20—29	·830	·917	·850
30—39	·921	1·120	·995
40—49	1·179	1·389	1·149
50—59	1·708	2·107	1·621
60—69	3·508	4·079	3·063
70—79	6·092	9·095	7·119
80—89	11·601	19·461	16·724

TABLE VI.—*Both Sexes.*

Age.	Mean Duration of Life.		
	Peerage Families.	Carlisle.	Healthy Districts.
0	52·62	38·72	49·00
10	49·39	48·82	51·08
20	42·21	41·46	43·45
30	35·96	34·34	36·64
40	28·86	27·61	29·64
50	21·95	21·11	22·44
60	15·16	14·34	15·37
70	9·61	9·18	9·61
80	5·45	5·51	5·51
90	3·54	3·28	3·05

Age.	Annual Mortality per Cent.	
	Peerage Families.	Carlisle.
Under 5	2·069	7·324
5 to 9	·388	1·011
10—19	·617	·588
20—29	·951	·761
30—39	·892	1·053
40—49	1·208	1·423
50—59	1·742	1·863
60—69	3·668	4·082
70—79	7·737	8·801
80—89	17·514	17·262

TABLE VII.—*Peerage Families.*

Age.	Mean Duration of Life.	
	Males.	Females.
0	52·00	53·71
10	49·04	50·16
20	41·46	43·48
30	35·51	36·82
40	28·33	29·93
50	21·40	23·08
60	14·56	16·42
70	8·77	11·28
80	4·58	6·92
90	1·64	4·10

Age.	Annual Mortality per Cent.	
	Males.	Females.
Under 5	2·23	1·88
5 to 9	·35	·44
10—14	·41	·66
15—19	·66	·77
20—24	1·10	·79
25—29	·99	·87
30—39	·87	·92
40—49	1·23	1·18
50—59	1·76	1·71
60—69	3·76	3·51
70—79	8·71	6·09
80—89	23·84	11·60

On the Law of Human Mortality; and on Mr. Gompertz's new exposition of his Law of Mortality. By T. R. EDMONDS, B.A., *formerly of Trinity College, Cambridge.*

IN the last number of the *Assurance Magazine* (April, 1861), there appears a letter from Mr. Gompertz, in which reference is made to his and my claims to the discovery of the law, or part of the law, of human mortality. In this letter, Mr. Gompertz declares that my claim cannot interfere with his claim, because I had acknowledged the priority of his discovery. I admit the truth of this declaration, and can add thereto, that I also am able to declare, with equal truth, that his claim, as now described by himself, cannot interfere with any claim hitherto advanced by me. The law of human mortality, now claimed to be discovered by Mr. Gompertz, differs materially from the law which I have declared to exist. The two laws are in accordance with one another only at one period of life (extending, say, from the age of 15 to the age of 55 years), and then only in a partial degree. It is in this partial agreement, at this particular period of life, that the priority of Mr. Gompertz's discovery consists. The description of the law of human mortality, as *now* believed by Mr. Gompertz to exist, is contained in a paper, referred to in the letter above mentioned, and since published (page 454) in the Appendix to the *Report of Proceedings of the Fourth International Statistical Congress, held in London in the year* 1860.

The universal law of human mortality which I believe to exist (and have so believed for the last 33 years), may be described as consisting of three parts, all of great simplicity.

The first part is, that, from birth to extreme old age, human life is divided into three well-marked physiological periods, of growth, fruitfulness, and decay, which periods differ from one another in having distinctive rates of decrease or increase of their mortality, according to age. These periods may be described as those of infancy, fecundity, and senescence. The discovery of these three periods of different progressive mortality according to age, was first made public by Dr. Price, in the year 1769, as I have already stated in the *Assurance Magazine* (page 171) for October, 1860. According to Dr. Price, human life, from birth upwards, grows *gradually stronger* until the age of 10 years, then *slowly loses* strength until the age of 50, then *more rapidly loses* strength, until, at 70 or 75, it is brought back to all the weakness of the first month.

The second part of the law of human mortality is, that, from the beginning to the end of any one of the three stages of infancy, fecundity, or senescence, the rate of mortality varies with the age in a geometrical progression, the common ratio of which may, or may not, be different for different populations, or for the same population at different times. This law of increase or decrease is the simplest conceivable law. For example, suppose it to have been ascertained, by observation of the same population, that the annual mortality at the age 25 years was 1 per cent., and at the age 45 years, 1·80 per cent., and suppose it to be required to interpolate, according to some law, the mortality corresponding to intermediate ages. In that case, the assumption first made, as being the simplest, would most probably be, that the annual proportional increase according to age was constant throughout the entire period of 20 years of age. Adopting this assumption, it would ensue, that the common ratio of the geometric progression at which the mortality increases, is equal to the twentieth root of 1·80, or to 1·03. If every term is to the next preceding term as 1·03 to 1, then the twenty-first term will be to the first term as 1·80 to 1.

The third and last part of the law of human mortality consists in the permanency of each of the three common ratios of geometric progression of mortality, according to age, in all populations. The common ratios for the several periods of infancy, fecundity, and senescence, being known for one population are known for all other populations. The respective values of these three common ratios, for annual intervals of age, have been ascertained to be the following, nearly—viz., $\frac{1}{1{\cdot}479108}$, 1·0299117, and 1·0796923 (say $\frac{2}{3}$, 1·03, and 1·08), these being the numbers of which the common logarithms are (−·17), (+·0128), and (+·0333).

The public are indebted to Mr. Gompertz for the discovery (in 1825) of a portion of the second part of the law of human mortality. This portion is, that in one of the well-marked physiological periods of human life, the mortality increases with the age in a geometric progression, the common ratio of which may, or may not, be different for different populations. The period to which this discovery applies, is that of "fecundity," extending, say, from the age of 15 to the age of 55 years. The evidence offered of the existence of this portion of the law of human mortality was indirect. For Mr. Gompertz, without saying anything as to the division of human life into three, or any other number, of physiological stages or

periods, gave a general formula of geometric increase of mortality "*for a long period of man's life,*" without assigning any limits to such period. All the examples of the applicability of his formula, with one exception only, were founded upon observations of mortality made in the period of "fecundity." From these facts it may be fairly concluded, that Mr. Gompertz is entitled to the credit of the discovery, that in one of the three periods into which human life is divided, the mortality increases with the age in a geometrical progression, the common ratio of which may, or may not, be different for different populations.

In his paper, printed in the *Philosophical Transactions* of 1825, Mr. Gompertz expresses homely facts in transcendental language. Instead of supposing the mortality to increase in a geometrical progression with the age, he supposes the *power to oppose destruction* (which is inversely as the *mortality*) to decrease in equal proportions in equal infinitely small intervals of time. Instead of saying that the law of mortality, from the age of 15 to the age of 55 years, is such that the annual rate of mortality increases in a geometrical progression, of which the common ratio is (p), he states that the law is such that the survivors, or numbers left alive under the operation of this law for the term of (x) years, will be represented by the transcendental (dg^{p^x}). The law of mortality, however, is better and more simply expressed in terms of the mortality than it can be in terms of the survivors according to the same law. In the former case, we have (ap^x) to express the law; in the latter case we have the same law expressed by $y=10^{\frac{k^a}{\lambda p}(1-p^x)}$, which is the quantity (dg^{p^x}), corrected and reduced to its simplest terms. Both expressions indicate the same facts, or the mathematical consequences of the same facts. In either case, where (a) and (p) are given, the only variable quantity is (x), measured from the time when the mortality was (a). It is not stated by Mr. Gompertz which of the two equivalent expressions for the law of mortality in the period of "fecundity" was first discovered by him. His expression for the survivors according to age, might have been discovered by inspection of published tables of the logarithms of survivors, at yearly intervals of age, accompanying ordinary tables of mortality. It might easily have been gathered from such inspection, that the differences between the logarithms of the survivors, at successive decennial intervals of age, are in geometric progression, and of the form (Cp^x). From this fact, the equation $y=dg^{p^x}$ might easily have been deduced.

In supplying evidence of the truth of his law of mortality, Mr. Gompertz invariably omits to compare the resulting mortality as deduced from his theoretical tables with the observed mortality according to age, which is or ought to be the foundation of such tables. In my own case, I have always supplied evidence of this complete and most simple kind. Nevertheless, Mr. Gompertz and his two officious, but accepted, advocates, concur in saying, that I have not given evidence of the truth of my theory of mortality, or of the existence of any law of mortality beyond that discovered by Mr. Gompertz, as applicable "*to a long period of man's life,*" to which no limits are assigned. This concurrence of views between Mr. Gompertz and his two advocates is the more remarkable, because it is the only instance of such concurrence to be found in the recently published papers of the three parties. His advocates advance claims on his behalf, which Mr. Gompertz has never advanced himself. The statements of the advocates are not supported by any corresponding statements of their principal. The only support which Mr. Gompertz gives his two advocates, is, to praise them for the talent which they have displayed in the advocacy of a claim rightfully or wrongfully put forward for his benefit. Mr. Gompertz appears to consider himself free from any responsibility in lending his approbation to papers containing statements, which, from his superior knowledge of the facts, he could not, with any regard to truth, have uttered himself. Except in the present instance, Mr. Gompertz has written nothing against my claim, or in support of the claim made on his behalf, which I am at all interested in contradicting. But with respect to his two advocates, I deny that either of them has said anything in depreciation of my claim which is true in substance or even in form.

Mr. Gompertz's theory of mortality (like my theory) rested greatly for support on the observations of mortality according to age of the populations of Sweden and Carlisle, made in the last century by Wargentin and Heysham respectively. The simplest evidence of the soundness of his theory would have been to show, that, taking successive decennial periods of age, the mortality deduced from either of his theoretical tables was in near agreement with the mortality at the same intervals of age observed by Wargentin or Heysham, and that both the theoretical and observed mortality were in geometric progression from the age of 15 to the age of 55 years. Instead of doing this, Mr. Gompertz has merely compared his series of survivors according to age with the series of survivors formed by Price and Milne from the observed facts. He

had previously taken no pains to inquire, whether the tables of Price and Milne were in agreement with the facts in mortality on which they are professed to be founded, and whether Price and Milne had not deviated from the facts presented, in order to smoothe irregularities of decrement, according to some ideal law of mortality which they may have entertained without expressing.

In proof of the soundness of my theory of mortality, I know no fact more convincing or interesting than one which has already appeared in the pages of the *Assurance Magazine* (year 1855, page 144). It is there shown, for that part of the total male population of England which was contained in five decennial periods of age, commencing at age 25 and ending at age 75 years, that the mortality observed in these decennial intervals of age was almost in exact accordance with my theory of mortality published in 1832—being three years before there existed any complete observation of the mortality of any part of the general population of England, except that of Carlisle, in the year 1787. To show the closeness of the coincidence to the present reader, the following extract is made from the more extended table, of which it forms part.

Annual Mortality per Cent., during 10 *Years ending with* 1850, *of all the Male Population of England comprised between the Ages of* 25 *and* 75 *Years.*

Between Ages..	25-35.	35-45.	45-55.	55-65.	65-75.
Fact observed....	·97	1·25	1·78	3·14	6·61
Theory	·96	1·28	1·75	3·22	6·78

In the Appendix (page 455) to the *Proceedings of the Fourth International Statistical Congress,* there is contained a statement by Mr. Gompertz of the view which he *now* entertains of the "*one continuous and uniform law of mortality from birth to at least the age of* 100 *years.*" He there states his belief that such law is expressed by the following equation, wherein (L_x) represents the numbers surviving or living at any age (x) measured from birth:—

$$L_x = \text{constant} \times A^{c^x} \times B^{c^x x} \times C^{q^x} \times D^{p_x}.$$

The following are the remarks made by Mr. Gompertz with respect to the applicability of the above formula:—"In all the tables I have examined, all the factors but C^{q^x}, being between the ages 20 and 60, are so nearly constant that the difference from it may be neglected. This equation may be put into the form,

constant $\times C^{q^x}$." Afterwards, he goes on to say that, by the disappearance of the two first transcendental factors, the equation from the age 60, and ever after, is of the form L_x=constant $\times C^{q^x} \times D^{P_x}$.

From the above, it will be manifest that it is only at the interval of ages from 20 to 60 years that there is any agreement between Mr. Gompertz's theory and my theory of mortality; for, according to my theory of mortality, in either of the three stages into which human life is divided, the equation for the living or surviving at any age (x), from the beginning of that stage, is L_x=constant $\times g^{p^x}$, the quantity (p) having a fixed and distinct value in each of the three stages; whilst, according to Mr. Gompertz's theory of mortality, human life is divided into four stages, and in three of the four stages no more than two out of the four transcendentals which enter into the above general formula of mortality ever disappear.

Mr. Gompertz also gives the following as his general equation of mortality for the whole of life expressed in common logarithms—

$$\lambda L_x = \text{constant} + ke^x + k_{1\,1}e^x x - nq^x - P_x,$$

and makes the following observations thereon:—

"ke^x is at birth=k and decreases regularly as x increases, and becomes, before the age of 20, and ever after, in the table I have examined, perfectly insignificant.

"$k_{1\,1}e^x$ is of no value at birth, but increases in a very short time to a maximum of significant value; but in less than 12 months, in the table I have examined, decreases to perfect insignificance.

"nq^x is=n at birth, and continually increases, with the increase of x, to the remotest age.

"P_x is perfectly insignificant at birth, and till an age but a few years below 60, and it then continually increases with the age until far beyond the age of 100, and then decreases to perfect insignificance."

In speaking of the equation $L_x = AB^{q^x}$, used in his paper of 1825, Mr. Gompertz remarks—"That AB and q were supposed to represent constant quantities—or, at least, were shown to differ very little from constants—for a very long term of years (for instance, about 50 years), but differing a little for length of term and from one locality to another; and A and B and q being so related to one another, that, supposing them to be constant, we should have $A \times B = L_0$, the number at birth. But, in making the investigation, I did not pretend that A and B were absolutely constant. They were determined from a random selection from three distant periods of age, from a statement of the number of

persons who will be living at different ages out of a certain number of persons stated to have been born."

In neither of the two papers referred to as recently published in the name of Mr. Gompertz do I find any statement relative to our respective claims which impugns the correctness of any statement made in my *Life Tables*, or in my paper contained in the *Assurance Magazine* for October, 1860. I do not believe that Mr. Gompertz has ever complained of my having failed duly to acknowledge the share which he had in the discovery of the law of human mortality. Since the publication, in 1832, of my *Life Tables*, I have, at various times, been in free personal communication with gentlemen who had been, a short time previously, in free personal communication with Mr. Gompertz. From none of these gentlemen have I heard that Mr. Gompertz imagined himself to have been wronged by me. As Mr. Gompertz himself, during the space of 28 years, remained insensible to the supposed wrong, it may fairly be presumed that such supposed wrong had no existence, and that the charge of wrong brought against me by a third party (Mr. De Morgan) is entirely without foundation.

In the *Assurance Magazine* for July, 1860, Mr. De Morgan brought the charge against me of having, in the year 1832, in my *Life Tables*, "unfairly suppressed due acknowledgment to the writings of Mr. Benjamin Gompertz." After reading my paper in the next following Number of the *Magazine*, Mr. De Morgan writes as follows (page 214) in the Number for January, 1861:—"If Mr. Edmonds had given all the description which he has now given . . . . . there would have been no suppression." This admission appears to me to amount to the virtual abandonment of his original charge, although Mr. De Morgan denies this by saying that suppression existed in 1832 and did not cease until the year 1860. The erroneous conclusion of Mr. De Morgan is founded on the erroneous assumption, that, in 1860, I made any statement or admission more favourable to Mr. Gompertz's alleged claims than I had made in 1832. I do not think that Mr. De Morgan will find anyone (even Mr. Gompertz himself) to concur with him in such an erroneous assumption. In the year 1832 I wrote as follows:—"The honour of first discovering that some connexion existed between tables of mortality and the algebraic expression (a^{b^x}) belongs to Mr. Gompertz." These words appear to me to convey a sufficient intimation of the high value I then attached to the discovery, and of the credit due to Mr. Gompertz for making it. The same words conveyed a suggestion to the reader to inquire and judge for himself of

the comparative merits of my complete law and Mr. Gompertz's then incomplete law of mortality. The paper which I wrote in the *Assurance Magazine* of October, 1860, was written in self-defence (against an outrageous attack of Mr. De Morgan), for the purpose of circumscribing, within just and narrow limits, the favourable admission which I had made in general terms in the year 1832.

In a question like the present, when I have to meet an apparently unfounded accusation, it appears to me to be of importance that the reader should know that Mr. De Morgan and myself were not strangers to one another in our years of student life; that previous to publication, in 1832, my *Life Tables* obtained his approval; and that, without giving me any intimation of the withdrawal of this approval, he had been secretly writing and speaking against me (in relation to Mr. Gompertz's supposed claim), for the space of 28 years. Mr. De Morgan and myself were students of the same year of admission, of the same University, of the same College, and of the same class-room of the College. We attended the same mathematical class for three years—or, rather, we should have done so if I had not omitted to read mathematics during the whole of the second year. In our third year the present Astronomer-Royal was the mathematical lecturer of our class, which, near the termination of the year, was reduced to two students—Mr. De Morgan and myself. In the year 1832, before publication, I sent Mr. De Morgan a printed copy of my *Life Tables*. A few days afterwards, at a personal interview, he expressed his general approbation of the work, but objected to one sentence only, which had no connexion with Mr. Gompertz or his supposed claim. He recommended me to have this sentence (and the page containing it) cancelled. This I declined doing; and since that time I have had no communication with Mr. De Morgan, except on one occasion, when I discovered that he had ceased to be favourably disposed towards me—for no reason, that I could imagine, except my refusal to act upon his recommendation given in 1832. It appears to me that Mr. De Morgan, after expressing to me a favourable opinion of my *Life Tables* in 1832, was not justified in changing that opinion, and acting upon that changed opinion, without previously communicating with me, and calling for an explanation. If he had done so, I should have offered the explanation which is given in the *Assurance Magazine* of October, 1860—which explanation, as Mr. De Morgan himself admits, would have exonerated me from the charge of "unfair suppression" if it had been given in the year 1832.

The formula by which Mr. Gompertz represents the number surviving according to age in one of his four periods of human life is $y=dg^{p^x}$. This formula is defective in several respects. In the correct formula the exponent of (g) is (p^x-1), and not (p^x) as stated by Mr. Gompertz. The factor (d), in the formula of Mr. Gompertz, is superfluous; and, besides being superfluous, is defective as being a composite quantity, of which the other factor (g) forms part. I will proceed to place before the reader the successive steps by which the correct formula was obtained by me 33 years ago; and I will afterwards exhibit the course of investigation pursued, or intended to be pursued, by Mr. Gompertz in arriving at his defective formula, giving Mr. Gompertz the benefit of the most recent corrections made by himself and his two advocates.

The problem for solution is this:—Given the annual mortality (a) at the beginning of a period (x) years of age, given also the common ratio (p) of annual geometrical progression of mortality according to age; required to find an expression for (L_x), or the number living and surviving at the end of the (x) years of age, in terms of (a), (p), and other known quantities.

Previous to forming the differential equation, it is essential that the nature of the quantity (a), given to represent the proportional mortality for one year when the mortality is constant, should be determined. There are two different quantities by which this annual mortality may be expressed. In one case, (a) is the finite decrement for one year, and is such that the number living at the beginning of the year of age is to the number surviving at the end of the same year as $(1+a)$ is to 1. In the other case, the annual mortality at the age 0 is represented by a quantity α, which is the product of an indefinitely small decrement $\left(\frac{\alpha}{\nu}\right)$ multiplied by an indefinitely large number (ν). If the finite decrement for one year, when the mortality is constant, is known to be equal to (a), the value of (α) when $x=0$ may be determined from the equation $e^{\alpha}=1+a$, which gives $\alpha=$hyperbolic logarithm of $(1+a)$. If $a=\cdot0063845$, then will $\alpha=\cdot0063643$.

In Mr. Gompertz's investigations no statement is made of any algebraical or arithmetical value of (a) or (α), although the object of his investigation was to find an expression for the number living at any age (x) in terms of the annual mortality, whether (a) or (α). This omission of separate notice of the most important quantity in the formula affords ground for doubting whether Mr. Gompertz was acquainted with the nature of the quantity used by him to

denote the annual mortality when (x) was equal to 0. An independent table of mortality could not have been constructed from Mr. Gompertz's formula without the previous knowledge now indicated. It so happens that Mr. Gompertz has published no portion of a theoretical table of mortality which is independent of previously constructed tables. His examples show how interpolations may be made in existing tables—not how portions of new tables may be constructed from the mortality only. The differential equation of Mr. Gompertz is erroneous if he has used the finite decrement (a) to represent the mortality; for the differential equation is true for all values of (x) (whole number or fractional) only when the annual mortality is represented by (a), which is equal to the hyp. logarithm of $(1+a)$.

Beginning with the differential equation, we have—

$$dy=-yap^x dx;$$

integrating,

$$\log.\ y=\log.\ d-\frac{ap^x}{\log.\ p};$$

when $x=0$,

$$\log.\ L_0=\log.\ d-\frac{ap^0}{\log.\ p};$$

by subtraction,

$$\log.\frac{y}{L_0}=\frac{a}{\log.\ p}-\frac{ap^x}{\log.\ p}=\frac{a}{\log.\ p}(1-p^x);$$

whence

$$\frac{y}{L_0}=e^{\frac{a}{\log.\ p}(1-p^x)}=10^{\frac{k^2a}{\lambda p}(1-p^x)}=10^{c(1-p^x)}=g^{1-p^x};$$

whence

$$y=L_0\times g^{1-p^x}=L_0\times\frac{g}{g^{p^x}}=L_0\times\frac{1}{g^{p^x-1}}.$$

Instead of the correct expression for (y) or (L_x), just obtained, Mr. Gompertz gives the defective equation $y=dg^{p^x}$. In deducing the latter erroneous value, Mr. Gompertz stops at the second step of the investigation, and thus fails to discover that the constant (d) is a compound quantity, including (g) as one of its factors. According to the defective process of Mr. Gompertz—

$$\log.\frac{y}{d}=-\frac{ap^x}{\log.\ p},\ \text{so that}\ \frac{y}{d}=e^{-\frac{ap^x}{\log.\ p}}=10^{-\frac{k^2a}{\lambda p}p^x};$$

and, putting

$$+c=\left(-\frac{k^2a}{\lambda p}\right),\ \text{and}\ 10^{+c}=g,$$

he gets

$$y=d\times 10^{-\frac{k^2a}{\lambda p}p^x}=d\times 10^{+cp^x}=dg^{p^x}.$$

The latter equation is defective by reason of the constant (d) not having been determined by the aid of the equation $\log. L_0 = \log. d - \frac{a}{\log. p}$. If it had been so determined, (d) would have been found to be equal to $L_0 \times \frac{1}{g}$. If he had made the proper substitution, his equation would have become

$$y = L_0 \times \frac{1}{g} \times g^{p^x} = L_0 \times g^{p^x - 1}.$$

In addition to the defect just mentioned, there is the further defect of putting (g), whose exponent is positive, equal to $10^{-\frac{k^2 a}{\lambda p}}$, a quantity greater than unity whose exponent is negative, and thus transferring to the numerator a quantity which originally formed the denominator of a fraction. If Mr. Gompertz had not made this unnecessary change in the sign of the exponent of (g), his corrected formula, on substituting g for $\frac{1}{g}$, would have become

$$y = L_0 \times \frac{1}{g^{p^x - 1}}.$$

In addition to the defect last mentioned in Mr. Gompertz's formula, there is the further defect of using in the formula any such quantity as (L_0) at all; for (L_0) is the common multiple of all the terms of the series for the living at successive ages, and conveys no information whatever as to the proportions surviving at different ages, which is all the information sought by the formula. If, for (L_0), we substitute unity, we shall have the equation complete in its most simple terms and form, viz.:—

$$y = \frac{1}{g^{p^x - 1}} = g^{1 - p^x} = 10^{c(1 - p^x)} = 10^{\frac{k^2 a}{\lambda p}(1 - p^x)},$$

the last being the form in which it was published by me in the year 1832.

In my paper of October, 1860, I designated the quantity (d) in Mr. Gompertz's formula as superfluous, useless, and indeterminate. The new advocate of Mr. Gompertz objects to my use of the word "indeterminate," because the word is usually limited to the description of a quantity which cannot be determined. It appears to me, that my use of the term is in accordance with his own definition; for Mr. Gompertz and his two advocates have not hitherto been able to determine the value of (d) by decomposing it into its two constituent factors $\left(L_0 \text{ and } \frac{1}{g}\right)$, by doing which, they might have

simplified and rectified the original defective equation $y=dg^{x^2}$. Moreover, I would submit, that a quantity, whether intrinsically defective, as (d), or admitting an endless variety of arbitrary values, as (L_0) (none of which concern the object of the investigation), may with reason be called "indeterminate," because there is nothing to be determined, or worthy of being determined.

In the same paper I also termed (d) a particular value of (y), whilst, in strictness, I should have said that that part of (d) which did not contain (g) is a "particular value" of (y). I had previously said, that (L_0), or y when $x=0$, was equal to $\frac{d}{g}$, which is the same thing as saying that (d) was a particular value of (gy). In order to avoid circumlocution, I omitted to say that the argument was the same—whether (d) was a particular value of (y), or of (gy). On account of this omission, the new advocate of Mr. Gompertz has charged me with having committed "a far more serious error" than that of calling (d) an indeterminate constant. He says nothing of the really serious error of Mr. De Morgan, in the exposing of which I committed the pretended verbal error. In the *Assurance Magazine* of July, 1860, page 88, Mr. De Morgan gives a formula, designed to express the number living at any age in terms of the mortality, in which formula he omitted the most essential part, the quantity (a), representing the mortality. It may be said, on Mr. De Morgan's behalf, that he copied the error previously committed by Mr. Gompertz. On the other hand, it is to be stated, that Mr. De Morgan reproduced the investigation and formula of Mr. Gompertz, in order to prove that they were as good as mine, designed for a similar purpose. Having regard to the task voluntarily undertaken, it is clear that Mr. De Morgan was responsible for the error adopted, whether originally committed by Mr. Gompertz or his printer.

Mr. Gompertz has recently presented to the Institute of Actuaries a corrected copy of his paper, which was printed in the *Philosophical Transactions* of 1825. The corrections with which we are now concerned, relate to errors in the printed investigation of his formula $y=dg^{x^2}$, and especially to two admitted errors repeated—abq^x and abq^x, immediately following one another in two separate equations. The correction of the first error, abq^x, is stated to be $a\dot{x}q^x$, or $adxq^x$; and of the second error, abq^x, it is stated to be aq^x. I have had the opportunity of minutely examining the printed errors and their corrections, and do not concur, with the new advocate of Mr. Gompertz, in the opinion that either of the two

errors has its source in a mistake of the printer, (b) being substituted for ($\times$), the sign of multiplication. The first of the two corrections may be passed over as genuine and unobjectionable, although the (dx) occupies an unusual position, coming before, instead of after, (q^x). With regard to the second correction, I question its admissibility. For the equation, as corrected, is more defective, in form, at least, than was the equation intended to be corrected. A necessary result of the integration made, was the appearance of a new constant, (b) or (d), on one side or the other of the equation. The correction now made would leave no new constant on either side of the equation. In the absence of any satisfactory account of the second of the two admitted errors, I will now offer, for consideration, the explanation of this error which occurred to my mind before Mr. De Morgan gave it as his opinion that (b) was a constant, additional to, and independent of, the constant (d)—both constants being introduced (rightly or not) by Mr. Gompertz in his process of integration. The source of Mr. Gompertz's error, in my belief, was, that he had carelessly and erroneously concluded, that the constant (b) on one side of the equation, was the equivalent of the constant (d) on the other side of the equation.

In the same paper, I stated that the unknown constant (b) had been introduced by Mr. Gompertz through his incorrect process of integration; and that such constant was not only superfluous and useless (as Mr. De Morgan had declared it to be), but that it was erroneous also. I supported this statement by giving a new proof of my formula $y=10^{\frac{k^a}{\lambda p}(1-p^x)}$, and showed, without the aid of the differential calculus, that no quantity similar to (b) formed part of the formula. The new advocate of Mr. Gompertz does not call in question the truth of the substantial part of my statement, but accuses me of having committed an error in describing the mode in which the error was supposed to have been arrived at by Mr. Gompertz. He says, that the process of integration had not commenced when the erroneous (b) first appeared. In reply, I have to state, that there are two distinct errors of (b), both appearing after the completion of the differential equation. I have also to state, that the first error only has been satisfactorily corrected by the substitution of ($\dot{x}$), or dx, for (b); and the second erroneous (b) is, apparently, not a printer's error, but a new and erroneous constant, introduced by Mr. Gompertz immediately after the disappearance of (dy) and (dx) from the differential equation.

It is usual to consider the process of integration as commencing at the step immediately following the completion of the differential equation. In the present case, I will venture to offer the opinion, that the completion of the differential equation was the first and most important step in Mr. Gompertz's process of integration. For the integral sought was known, or believed to be known, by Mr. Gompertz, before the differential was known to him, as I have already shown, at page 176, in the paper before referred to. To know the correct differential and the correct integral, generally includes the knowledge of all the intermediate steps. It is doubtful, however, whether Mr. Gompertz possessed any exact knowledge either of the integral or differential sought. For his integral is erroneous, in exhibiting (p^x) as the exponent of the factor (g); and his differential equation would be erroneous, if the quantity (a) is intended to represent the finite annual decrement when the mortality is constant.

In the paper before referred to, I gave, as a quotation from Mr. Gompertz's paper in the *Philosophical Transactions* of 1825, a passage which has been repeatedly quoted as the essential part of his law of mortality. In doing this, however, I omitted the first word of the sentence, which is "if;" which word is used by Mr. Gompertz to signify "let it be assumed." In consequence of the omission of this insignificant word, I have been charged by the new advocate of Mr. Gompertz with quoting a sentence or passage which "has no existence in the writings of Mr. Gompertz." The writer does not describe the passage misquoted, although he refers to its place in the *Assurance Magazine* of October, 1860, nor does he state in what the alleged misquotation consisted. If he had done so, the charge (made in italics) would have appeared too ridiculous to be seriously entertained. I can truly say of this new advocate of Mr. Gompertz, whilst using his own expressive language, that his remarks "contain errors equally grave with those falsely imputed" to me.

The new advocate of Mr. Gompertz alleges, in opposition to the most obvious facts, that the defective quantity (d) "is really an essential part of the formula, and, *as such, is employed by Mr. Edmonds himself.*" There is not the remotest approach to truth in this statement, enforced by *italics*. The defective quantity (d) is obtainable from the equation $d = \frac{L_0}{g}$, and, when obtained, is worth less than nothing. Instead of attempting to prove that I had ever made use of such a quantity as (d), the new advocate of

Mr. Gompertz attempts to prove that I had used the correct but superfluous quantity (L_0) as distinct from unity. This he entirely fails to do, for he reproduces the identical figures which were given by me as obtained by the substitution of unity for L_0.

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*On the various methods pursued in the Distribution of Surplus among the Assured in a Life Assurance Company; with a comparison of the relative merits of such methods.** *By* Wm. Pollard Pattison, Esq., *of the London and Provincial Law Assurance Society.*

[The Author of this Essay obtained for it the Prize offered by the Institute for an Essay on the subject.—Ed. *A. M.*]

THERE is, probably, no department of an actuary's pursuits where there is such diversity of practice as that of which this paper treats. After the calculation of the premiums and the periodical valuations, the distribution of surplus is infinitely the most important duty that an actuary has to perform. The premium income of the various Life Offices now exceeds £8,000,000 a year, and probably not less than £1,000,000 is annually divided as surplus. Within the last 14 years, one Office alone has divided upwards of £1,200,000; and another, for many years past, has annually distributed as surplus more than £100,000. This will be sufficient to show the magnitude of the interests concerned; and yet, as will be seen in the sequel, these distributions are, in some cases, made in a most arbitrary manner—without reference to principles of justice and equity, and without the basis of accurate reasoning. After what has been written on this subject by Mr. Jellicoe and Professor De Morgan, it is not to be supposed that any actuary would defend the schemes here referred to as equitable; those who *do* follow them, content themselves with stating their advantages. In many cases, however, their conservatism arises from the difficulty of disturbing vested interests; and the maxim, "*Omnis innovatio plus novitate perturbat quam utilitate prodest,*" is very attractive to those in office, upon whom would fall the labour of carrying out a new system. Hence, in this, as in many other cases of old-established errors, all true reform must come from without. But the application of prin-

* By the title of this paper it will be perceived that the methods actually pursued are those which are the subject of investigation and comparison. I emphatically wish it to be understood that they are in no sense theoretical, but such as are adopted at the present time, and mostly, too, by Offices of high standing. The names have been carefully excluded, and everything that might lead to their identification.

ciples of reform is very difficult in the matter under consideration. It is a subject which can never become popular. The assuring public will never enter into the abstract merits of any particular scheme; but if the comparative numerical results of each one be clearly exhibited, the effect must be that the adoption of equitable methods would be forced upon the different Offices. Each would gradually lose the custom of any class of lives to which its method was known to be unfavourable, and the favoured classes would ultimately be drawing profits contributed only by themselves. In addition, therefore, to a statement of what I hold to be correct principles, and a description of the various methods in practice, I have appended a table of comparison, which, at a glance, will show the working of any particular scheme as compared with any other, in a manner that can be sufficiently understood even by persons who are not familiar with such matters.

The need for an equalization of the risks of death called into existence the plan of life assurance. The theory upon which this system depends, presupposes that the duration of life in a community can be accurately measured. Every year additional facts are acquired, and every year tends to confirm the soundness of the theory, though it modifies its application. The theory also presupposes that the rate of interest at which money will accumulate can be foreknown; a matter on which our predictions can never be more than approximately accurate. The uncertainty thus caused by modifications in the laws of mortality and the fluctuations in the rate of interest, apart from expenses of management, require that a marginal addition (technically termed a "loading") should be made to the estimate of the risk, sufficient to provide for the probable divergencies of the future from the experience of the past. Hence the premiums charged are greater than have been hitherto found sufficient for the risk; and this brings us to the consideration of the surplus, which may be traced to—

1. The loading, or margin for guaranteeing the engagements of the Company against the fluctuations of mortality and the rate of interest, for providing for the expenses of management, and for forming a bonus fund:
2. Interest—a higher rate being realized than was assumed: and,
3. Miscellaneous sources, lapses, surrenders, non-profit policies, office age over real age, fines.

The premiums charged to the public consist of two parts—(1), that which simply provides for the risk; and (2), the loading,

the whole or a portion of which is at intervals found to be surplus, and accordingly distributable among the assured. In respect of the former portion of the premiums, the assured form a community of assurance; and in respect of the latter portion, a community of profit. The only principle of the division of the surplus which has been advocated, is, that each of the assured should receive in proportion to his contribution to the profit fund. I have already pointed out that, as regards one portion of the premium, the assured may be considered as a community of profit—the criterion of all partnership; and the principle just laid down is that which guides all partnership contracts, whether the contribution to profits be property or labour. Now, the error of certain systems of distribution is owing, not to the denial of this principle, which no one has, as far as I know, ever attempted to controvert, but to the almost wilful refusal to connect the sources of surplus with the contributions of the members, without which connexion the principle of proportional distribution cannot be applied.

Let p = premium charged;
p' = premium for the risk, or true premium;
ϕ = loading, or $p-p'$;
a^n = amount of an annuity of £1 for n years;
s = total amount of divisible surplus; and
Σ be the symbol of summation.

The valuation being assumed to have been made by a true table of mortality and at a true rate of interest—and if not so based, the real surplus would obviously not be known at all—the principle would be correctly applied by the formula $\frac{s}{\Sigma\phi a^n}\phi a^n$,* to represent the share of the assured in the divisible surplus. These assumptions apply to the valuation alone; the table on which the premiums are based has nothing to do with the question. When p is not more than p', then there would be no bonus, as should be the case. The application of the principle by the above formula includes the assumption that the expenses of management are paid out of the miscellaneous profits which have not been contributed to by those who participate in the surplus; or, if these sources of profit will not pay the whole expenses, then it assumes that the management should be borne *pro ratâ* to the loading. I shall hereafter refer to this point, and, for the present, it will be convenient to exclude it from the general question which is the subject of this paper. Since the

* *Vide* Mr. Jellicoe's papers in the *Assurance Magazine*, vols. ii. and iii. I wish this to be considered as a general reference, applying to other portions of this Essay.

formula depends on the rigid fulfilment of the above conditions, it is necessary to inquire how far they can be obtained.

1*st. A true table of mortality.*—The English Life Table of the general population of the country may be safely considered as representing the mortality of the majority of Offices—certainly all with a large proportion of country business; and Mr. Morgan's Equitable Tables, as representing the mortality of the better class of assured lives. The valuation surplus would probably be derived very largely at the first and second periodical divisions, and afterwards, in a less degree, on account of suspended mortality, arising from recent selection of the lives; but this portion of the surplus ought not to be divided, inasmuch as the subsequent mortality cannot safely be assumed to follow the average. To ascertain the divisible surplus, deduction from the valuation surplus ought to be made for suspended mortality. Hence, then, no profits can be said to be made from that source; should they appear, inappropriate tables have been employed, and all certainty of profit gone. For these considerations, I would disregard this source of profit from the question of the true mode of distributing profits.

2nd Condition. A true rate of interest.—I would almost venture to say, that this cannot be fulfilled. While, on the one hand, we find 3½ and 3¾ realised by two Offices of the highest standing, so, on the other, there are Offices making 4¼ and 4½, and in one first-class Office very nearly 5 per cent. is made, with the life funds, through preference of investments being granted to that portion of a large accumulation of money. Four per cent. has in some valuations been assumed as the true rate. Now, it is to be observed, that with the accumulation of funds the difficulty of maintaining the rate of interest increases; and further, it depends upon good management. An Office may have now a skilful financier as its actuary; he would, doubtless, advocate a 4 per cent. valuation. But should the management change, and a few men, with prejudices for and against particular securities, become members of the Board, 3½ per cent. might be the rate of interest—and all this apart from the fluctuations in the value of money. Supposing £1,000,000 to be the liability by a 4 per cent. valuation, the change in the base to 3 per cent. would diminish the divisible surplus by upwards of £100,000; and this might cause considerable alarm among the Office connexions. I would, therefore, adhere to a lower rate of interest—3, or, at most, 3½ per cent.—and introduce, as a further element in the ratio of division, the profit from this source. From the total divisible surplus, the profit

so derived would be deducted, and would be composed of two parts—

1. To be divided among all that have paid premiums within the period; and,
2. to be divided among those who participate in the previous division.

It is clear that, if at the last valuation 3 per cent. were assumed and 4 per cent. have since been realised, the sum then reserved was more than sufficient, and the difference ought to be returned, as far as practicable, to those who would then have participated in a larger amount of surplus. This element in the distribution might be easily introduced where the premiums are based on the true table of mortality, and where the loading is wholly a percentage on the premiums or a percentage with a small constant for expenses. The following methods of distribution all omit a proportion of profit from interest to those who participated in a former division; and all but the first, the second (where the loading is a percentage), and the third (where the loading is a percentage and a constant), with respect to the increased interest obtained within the last period. In giving my opinion, therefore, that any one mode is equitable, it will be always subject to modification in the case of the interest obtained being more than the assumed rate.

The following are the principal methods at present pursued:—

1. In proportion to premiums paid since the last division, accumulated at compound interest to date of valuation.
2. In proportion to the loading—that is, the difference between true and Office premiums—accumulated at compound interest between the intervals of division.
3. In proportion to the difference between the pure and Office premiums, less a constant for expenses, accumulated at compound interest between the intervals of division. (This is adopted, with some modification.)
4. In proportion to the difference between the premiums paid, accumulated with interest, and the respective values of the policies.
5. In proportion to the values of the policies.
6. A reversionary bonus in proportion to the premiums paid, without interest, always reckoning from the commencement of the policy.
7. In augmentation of the sums assured by a percentage per annum on the sums assured and the bonuses already

declared, the number of years being reckoned back to the last distribution of profits only.

8. In augmentation of the sums assured by a percentage per annum from the date of the policy.
9. In reduction of the annual premiums by an uniform percentage to all members after payment of a fixed number of full premiums.
10. In proportion to the amounts assured under the policies which have become claims within the year.
11. Amongst those of the assured only whose premiums, accumulated with interest, exceed the amounts assured under their policies.

1st method.—In proportion to premiums paid since the last division, accumulated at compound interest to date of valuation.

This is the most general mode of distribution, and if the premium is formed from the true premium for the risk, and loaded by a percentage, it is equitable; and, which is a great recommendation, is a practically good mode. By "practically good," I mean that it produces that proportion between the bonuses at different ages which will give most general satisfaction among the assured.

Using the same notation, the working of this will be shown by the formula $\frac{s}{\Sigma pa^n} pa^n$; and if ϕ be a percentage on p', and, as before, $p=p'+\phi$, then $p : \Sigma p :: \phi : \Sigma\phi$. Hence this formula will give the same results as $\frac{s}{\Sigma\phi a^n}\phi a^n$.

But though this method of distribution, under the assumption of a percentage loading, is equitable, such a loading is very rare in practice; and according as the loading diverges from this formation, so does the method depart from what is just. I have known this method applied where Northampton premiums were charged. Its effect with such premiums will be readily seen from the following:—

Age.	Northampton. Premium per Cent. per Annum.	English Life Table. Premium per Cent. per Annum.	Δ.
	£ s. d.	£ s. d.	s. d.
25	2 8 1	1 15 9	12 4
55	5 6 4	4 16 9	9 7

If the English Life Table be held as the true rate, the respective shares in the surplus at ages 25 and 55 at entry would be as 24 : 53,

or about 4 : 9, though age 25 contributes annually to the surplus 2*s.* 9*d.* per cent. more than age 55. Were commission taken into account, the unfairness of the method applied to such premiums would be still more manifest.

If Mr. Morgan's Equitable be taken as the true rate, we have—

Age.	Northampton.	Equitable.	Δ.
	£ s. d.	£ s. d.	s. d.
25	2 8 1	1 17 2	10 11
55	5 6 4	4 14 7	11 9

that is to say, the ratios of division would be as 4 : 9, though the contributions were nearly equal, viz., 10*s.* 11*d.* and 11*s.* 9*d.* The Northampton rates exhibit the unfairness in its highest degree, but there are many others which to a less extent are unfair. Before passing on to the next method, I would notice that the ratio of the premiums paid would be a fair ratio for dividing that portion of the profit which has arisen from excess of interest derived on the premiums paid *within the period since the last division.* The profit from interest on the reserve for liabilities made at the former division would have to be divided among those only who then participated. I have remarked upon the expediency of this method of distribution, and upon this point it is to be further noticed that the ratio of division (premiums accumulated within the intervals of division) is, after the first division, a constant. The successive bonuses being equal, the share of each in cash will also be equal, and the reversionary bonuses with advancing years will continuously diminish. Now the reversionary bonus is the measure among the public, and the reason for the diminution is not understood. This result has, of course, nothing to do with the equity of the method, whatever it may have with its expediency. If the view of the public with respect to increasing bonuses were incompatible with equitable distribution, it would be necessary to dispel the notion, difficult as that might be. This, however, is not necessary. Mr. Sprague has pointed out that the cash bonus may be applied as an increasing assurance; and there is the plan pursued by some Offices of converting the cash into reversion by the table of single premiums given in the prospectus for participating policies, and then giving bonus on bonus in future divisions. When the Office is "Mutual," no objection can be urged on the ground of principle; but when the Office is "Mixed," the effect is to give to the shareholders a portion of a former division, although

they have already received their full share. This injustice is increased by the conversion of the cash bonus into reversion by the table above quoted, which is very rarely used when the assured have the option. The rates include a charge for Office expenses and commission; so that when the assured have a cash bonus, a portion of the two-thirds or four-fifths, as the case may be, to which they alone are entitled, that portion is taxed with a charge for the management of it, with commission which is never paid, and with a contribution one-third or one-fifth of which will go to the shareholders, though such contribution be a portion of the two-thirds or four-fifths of profits, to which they have no title whatever. If this system be advisable, deduction should at least be made of that portion of the loading, about one-half, which provides for expenses and commission.

2nd method.—In proportion to the loading—that is, the difference between true and Office premiums—accumulated at compound interest between the intervals of division.

It has before been stated, that any profit that may arise will be from ϕ; and if this be the measure of each one's profit, this method satisfies the only principle that has been laid down as equitable.

The application of this method is extremely easy, various as may be the premiums payable under the policies. The existence of twenty different rates of premiums would not be the least obstacle to its application.

*3rd method.—In proportion to the difference between the pure and the Office premiums, less a constant for expenses, accumulated at compound interest between the intervals of division.**

This is a modification of No. 2 method. Nos. 1 and 2 both assume that the expenses of management should be paid out of the general surplus fund, so that each one's share of expenses would be in proportion to his contribution to surplus. Now, if this be a constant, his share of the expenses is also a constant. This I hold to be correct. One £100 policy costs (disregarding commission, which is a percentage) as much per annum as another, whether the premium be £5 or £2. The charge for this purpose should therefore be the same at all ages. If this principle be admitted, the second method is erroneous to this slight extent, and the third is perfectly equitable. This constant need not be sufficient for the full amount of expenses, as the miscellaneous profits may be justly

* The working of this is, I believe, not exactly as described. The accumulation is made by a table of temporary annuities; and the former bonuses are taken into account in determining the ratios for subsequent divisions.

set off to diminish these. Could they be found sufficient, the second method would be equitable, and, as no constant would then have to be deducted, the third method would coincide with it.

4th method.—In proportion to the difference between the premiums paid, accumulated with interest, and the respective values of the policies.

The application of this at the first division of profits is very simple; and, supposing x to represent the ages at entry, the formula for working would be $\frac{s}{\Sigma(p_x a^n - l_x)} \times (p_x a^n - l_x)$ for the share of each policy. At the second and subsequent divisions, it becomes more complicated. All the policies that participated in the former division are, for the purpose of the second division, considered as new policies effected at the first division. Thus, if a man assure at age 25, and is 32 at the first division, the accumulated premiums paid, less the value of a policy 25 at entry, after seven years, will express the ratio of his share in the surplus at the first division. If the second division be seven years after the first, seven years accumulations of premiums payable at the age of 32—his age when the first division took place—less the value of a policy on the life of 32, after seven years, will express the ratio of his share at the second division. There are modifications of this in some instances, as regards the bonus. The bonuses declared at previous divisions are added to the amount assured, and the premiums are formed upon the augmented amount assured. Thus, if in the example before given £100 were the bonus at the first division on a policy for £1,000, the ratio for the second division would be the accumulated premiums on £1,100 at age 32, less the value of a policy of £1,100 seven years after inception at that age.

The formulæ for the working of this method at the second division, without and with the modifications as regards bonus, would be—

$$\frac{s}{\Sigma(p_{x+7}a^7 - {}^7l_{x+7})} \times (p_{x+7}a^7 - {}^7l_{x+7})$$

and

$$\frac{s}{\Sigma\{p_{x+7}(\mathrm{M}+\mathrm{B})a^7 - {}^7l_{x+7}(\mathrm{M}+\mathrm{B})\}} \times \{(p_{x+7}(\mathrm{M}+\mathrm{B})a^7 - {}^7l_{x+7}(\mathrm{M}+\mathrm{B})\},$$

M being the amount originally assured, B the bonus, and in the second formula p and l are the premiums and values per £1 assured. The first of these formulæ is to be preferred, as the working of it

is more simple, and it does not further augment the bonuses on policies of long standing, which are otherwise too large.

The principle on which this method seems to have been founded is this, that the accumulation of all premiums, less the value of all the policies, would represent the gross surplus, if there were no expenses and no claims, and the contribution of each to that gross surplus would be the accumulation of the premiums he had paid, less the value of his policy. The divisible surplus being the amount of such imaginary surplus, less the claims and expenses, the share of each in the divisible should be represented by the ratios in the imaginary surplus. This notion includes the fallacy, that if a man aged 55 lives over a period, say of five years, he has contributed more profit than one aged 25 living five years, apart from the question of the loading of the premium.

Now, it has been already shown that this is contrary to the very essence of life assurance. In a true table of mortality, no profit is derived from long life, except as regards that portion of the premium which expressly provides for profits and expenses. If this distribution between the two parts of the premium be not regarded, all consistency in our deductions will disappear.

Referring again to the formula $\frac{s}{\Sigma(pa^n-l)}pa^n-l$, if we substitute for p its equivalents $p'+\phi$, and $\therefore$ $(p'+\phi)a^n$ for Σpa^n and for Σl, the net liability on all the policies $\Sigma(p'a^n-c)$; for it is clear that if p' represents the true premiums, the amount which should be in hand at any time would be the sum of the amounts of every p' or $\Sigma p'a^n$ less the total amount in claims, or Σc; so also for l we may substitute $p'a^n-c$ (c being p' contribution to claims), we then have

$$\frac{s}{\Sigma(p'+\phi)a^n-\Sigma(p'a^n-c)}\times\{(p'+\phi)a^n-(p'a^n-c)\},$$

or

$$\frac{s}{\Sigma(\phi a^n+c)}\phi a^n+c;$$

that is to say, the profit is allotted in the proportion of the contributions to profits and the payment of claims, which has been shown to be contrary to principles of equity, and to include an assumption opposed to the fundamental principles of life assurance.

5th method.—In proportion to the values of the policies.

By this method the surplus is divided in proportion to the liability under each policy, and the working of it is expressed by the formula $\frac{s}{\Sigma.l}.l$.

How inequitably this system works will be seen by the table of comparison appended, and this is with a well-graduated premium upon the same base as the valuation. If applied in an Office where different rates are payable—some with a large margin, others with a small or no margin—those paying a large marginal addition will have the same bonus as those paying a small one (the ages of the lives and the dates of policies being the same), and even a less bonus than a policy with a premium inadequate for the risk. It is, however, to be observed, that if all the participating policies in an Office paid according to the same scale of premiums, and this were formed on a true base, with a percentage and small constant for loading, this method would provide to the new policies compensation in future years, since the value of every policy, whether effected on a young or old life, would, if the life attained the extreme age in the table, range from 1 to 99 per cent. The ratios of division, therefore, range between the same limits for all policies.

6th method.—A reversionary bonus in proportion to the premiums paid, without interest, always reckoning from the commencement of the policy.

The clause in the deed wherein this scheme was devised runs as follows:—

"The dividend to each member shall bear the same proportion to the total sum to be divided as the total premium received of the existing members insured for life, which sum allotted to each individual shall be added to the sums originally insured."

The Office, I believe, interprets this clause to mean, that the reversionary bonus is to be in proportion to the premiums paid, not the cash bonus, as it has been reasonably considered to mean, on the ground that the sum to be divided being cash, the fourth term of the proportion must be like, and also cash. There is no trace of principle in this. The profits are given to the long livers, and in greater proportion than by any other scheme.

7th method.—In augmentation of the sums assured by a percentage per annum on the sums assured and the bonuses already declared, the number of years being reckoned back to the last distribution of profits only.

This is unscientific; but, by a graduated loading, this plan could be made nearly equitable. It would not, however, admit of adjustment, should subsequent experience render it desirable to modify the table of mortality.

It will be seen in the table of comparison, that according to the

loading there made, it favours the young lives. How far the plan of giving bonus on bonus is inequitable, depends on the way in which the cash surplus is converted into a reversionary sum, and on the Office being "Mutual" or "Mixed."

8th method.—In augmentation of the sums assured by a percentage per annum from the date of the policy.

The origin of this is well known to have been the discovery that the experimental premiums charged to the early assured were greatly in excess of the risk, and this was devised to restore the equality. It could never be adopted again, and nothing but a great reputation for wealth could sustain an Office against the inexpediency of the method. The application of the principle varies in different Offices. The Office in which this method originated converts the cash bonus into reversion, and then divides this according to the above principle. Another Office, however, applies the principle of distribution to the cash surplus, and, though it is a *cash* share of profits, it is not payable until death. The effect of this in a proprietary Office, giving only, say two-thirds, of the profits, may be readily conceived. Assuming the surplus to be £300,000, the shareholders would get £100,000, and the policyholders £200,000; but since they would only come into this on the falling in of the policies, the present value would not be much more than £100,000. The surplus at the next division would consequently be increased by the remaining £100,000 accumulated, with interest. But here again the shareholders would get their one-third share, and of the policyholders' £100,000 accumulated as part of the general surplus.

The above-described methods are all that are included in the table of comparison before referred to. I append this table, which contains the different methods, arranged in the order I have described them.

The policies are all assumed to be effected in the commencing month of the quinquennial period in which they first participate. They are all for £1,000; and the ages at entry are 25, 40, and 55. The premiums are according to the 3 per cent. English Life Tables (published in the *Registrar-General's Report for* 1849), with loading of 10 per cent., and a constant of 5*s*. per cent. This loading has been adopted to obtain an average premium, and not from any reasons for that specific loading. The calculations have been made according to the same tables.

Table of Comparison.

Calculations based on the English Life Table (Males), 3 per Cent.

Age at Entry.	Premium payable.	Age at Valuation.	No. 1.		No. 2.		No. 3.		No. 4 (i).		No. 4 (ii).		No. 5.		No. 6.		No. 7.		No. 8 (i).		No. 8 (ii).	
			Cash.	Reversion.	Cash.	Reversion.	Cash.	Reversion.	Cash.	Reversion.	Cash.	Reversion.	Cash.	Reversion.	Cash.	Reversion.	Cash.	Reversion.	Cash.	Reversion.	Cash.	Reversion.
25	22·16	30	20·	48·6	25·4	61·5	19·	46·	21·4	51·9	21·4	51·9	18·3	44·4	14·2	34·5	25·5	61·8	25·5	61·8	..	33·3
40	32·70	45	29·5	56·	31·	58·7	29·4	55·6	29·2	55·3	29·2	55·3	30·	56·9	26·8	51·	32·6	61·8	32·6	61·8	..	33·3
55	55·80	60	50·5	74·4	43·6	64·3	51·6	76·	49·4	72·8	49·4	72·8	51·7	76·2	59·	87·	41·9	61·8	41·9	61·8	..	33·3
First Period			£100·		£100·		£100·		£100·		£100·		£100·		£100·		£100·		£100·		.. £	100·
25	22·16	35	20·	44·8	25·4	56·9	19·	42·5	20·2	45·2	20·7	46·3	25·8	57·9	19·5	43·6	27·3	61·1	34·9	78·2	..	44·4
40	32·70	50	29·5	51·4	31·	54·	29·4	51·2	32·4	56·4	30·	52·3	41·4	72·	36·9	64·4	35·1	61·1	45·	78·2	..	44·4
55	55·80	65	50·5	69·3	43·6	59·8	51·6	70·9	62·4	85·4	60·5	83·	66·7	91·4	80·1	110·	44·5	61·1	57·0	78·2	..	44·4
25	22·16	30	20·	48·6	25·4	61·5	19·	46·	18·3	44·4	19·	46·	12·1	29·6	9·	21·8	23·7	57·6	16·	39·1	..	22·2
40	32·70	45	29·5	56·	31·	58·7	29·4	55·6	24·8	47·1	26·	49·3	19·8	37·4	17·	32·2	30·4	57·6	20·6	39·1	..	22·2
55	55·80	60	50·5	74·4	43·6	64·3	51·6	76·	41·9	62·	43·8	64·6	34·2	50·4	37·5	55·3	39·	57·6	26·5	39·1	..	22·2
Second Period			£200·		£200·		£200·		£200·		£200·		£200·		£200·		£200·		£200·		.. £	200·
25	22·16	40	20·	41·2	25·4	52·1	19·	39·1	20·6	42 5	20·1	41·4	30·3	62·4	22·9	47·4	29·3	60·3	40·4	83·4	..	50·
40	32·70	55	29·5	47·2	31·	49·6	29·4	47·	32·1	51·3	31·2	50·	48·6	77·8	43·	69·	37·7	60·3	52·3	83·4	..	50·
55	55·80	70	50·5	65·	43·6	56·	51·6	66·4	78·6	101·3	73·	94·	72·4	93·3	91·5	117·9	46·8	60·3	64·9	83·4	..	50·
25	22·16	35	20·	44·8	25·4	56·9	19·	42·5	17·7	39·6	18·2	40·6	19·1	42·9	14·1	31·3	25·4	56·8	24·8	55·6	..	33·3
40	32·70	50	29·5	51·4	31·	54·	29·4	51·2	25·7	44·8	26·4	46·	30·6	53·1	26·3	46·	32·6	56·8	32·	55·6	..	33·3
55	55·80	65	50·5	69·3	43·6	59·8	51·6	70·9	52·6	72·3	53·	72·7	49·8	68·2	57·2	78·8	41·4	56·8	40·6	55·6	..	33·3
25	22·16	30	20·	48·6	25·4	61·5	19·	46·	15·6	37·8	16·8	40·7	9·	21·9	6·4	15·6	22·1	53·6	11·5	27·8	..	16·6
40	32·70	45	29·5	56·	31·	58·7	29·4	55·6	21·2	40·2	22·8	43·	14·7	28·	12·1	23·	28·3	53·6	14·7	27·8	..	16·6
55	55·80	60	50·5	74·4	43·6	64·3	51·6	76·	35·9	52·9	38·6	56·8	25·5	37·6	26·5	39·4	36·4	53·6	18·8	27·8	..	16·6
Third Period			£300·		£300·		£300·		£300·		£300·		£300·		£300·		£300·		£300·		.. £	300·

From the results here exhibited, it will be observed—

1. That according to the first three (which I will, for convenience, call first class) methods of division, the shares of each age in cash are constant—the same at one division as at another.

2. That the remaining methods—or second class—(omitting No. 6), give generally nearly similar results with one or other of the first class methods as regards the first period.

3. That at the second and third periods a rapid augmentation in the profits of the old policies takes place under the second class methods—omitting No. 7.

4. That this augmentation mainly goes to the old *lives* in No. 4 (i. and ii.); in the remaining portion of the second class methods it goes to the old *policies*, which retain very nearly the same proportion of profit, one with another, as at the first division.

5. That the bonus on policies of five years' standing rapidly diminishes with the increase in the number of old policies.

It has been observed, as regards No. 4 (i. and ii.), that "at each septennial division the same degree, or very nearly the same degree, of injustice is perpetrated as at the first"*—a little consideration of the *tendency* of the results will show that this is not so. The cash bonus of 25 at entry, taking the results of the third period under 4 (ii.) is, after 5 years, 16·8; after 10 years, 18·2; after 15 years, 20·1; which last is nearly equal to the bonus allotted at age 40 after 5 years. If this table had been worked out to a fourth period, the bonus of 25 would have been the same as 40 after 5 years. This result follows from the application of the principle as before described. The effect of this method, if continued to more distant divisions, will give, at the 11th period, to six policies taken out at the same age 25, but of different standing, the following proportions of bonus:—

	Years standing.						Years standing.
The policy of	20	would have about		$1\frac{1}{2}$	as much as	policy of	5
„	30	..	..	2	..	..	„
„	45	..	..	4	..	..	„
„	55	..	..	8	..	..	„

A policy taken out at an early age, and continuing in force to the extreme age, would have successively the ratios of division for all the interval ages up to the highest.

* *Assurance Magazine*, vol. iii., p. 191.

Of the methods not included in the table, I have but few observations to make—little more, indeed, than reasons for their omission.

From the peculiarity of the system of No. 9 (by which the annual premiums are reduced by an uniform percentage to all members, after payment of a fixed number of full premiums), no fair comparison could be instituted with the standard adopted in the table, and on this ground it was omitted. It is clear, however, that the bonuses are in proportion to the premiums, and therefore its equity mainly depends on the loading. The time of the postponement of the privilege of reduction being the same for the old as for the young lives, is also to be regarded in considering its equity. If the loading were a percentage, this rule would act very unequally, and would be unfavourable to the members entering at advanced ages.

The 10th method is neither more nor less than a pure lottery.

The 11th method, which assumes that those only are entitled to profits who have paid up their policies, is a system of distribution utterly opposed to the principles and intention of life assurance. As it would preclude profit to any policy within the period of those given in the table, it could not there be introduced.

The conditions upon which the first three methods in the table are equitable have been given. As regards the remaining methods (omitting No. 7, which I have before disposed of), if the principles laid down as to the sources of profit be borne in mind, the table of comparison establishes *that the profits contributed by the new policies are apportioned to the old, and when these new policies become old the methods provide compensation out of the contributions of the future new policies. How if these should not be forthcoming?*

I append a further table, based on Mr. Morgan's Equitable Tables. The loading in this is 15 per cent. on the pure premiums, with a constant of 5*s*. per cent. In the calculations of the bonuses according to No. 5 method, the values of the former bonuses have been included in the ratios of division. This has been done, as it is the practice of one of the Offices which has adopted this method. In other respects, the table explains itself.

Table of Comparison.

Calculations based on Mr. Morgan's Equitable Tables, 3 per cent.

Age at entry.	Premium payable.	Age at Valuation.	No. 1.		No. 2.		No. 3.		No. 4.		No. 5.	
			Cash.	Reversion.	Cash.	Reversion.	Cash.	Reversion.	Cash.	Reversion.	Cash.	Reversion.
25	21·55	30	19·5	49·4	23·9	60·5	18·5	46·9	20·2	51·15	18·6	36·8
40	32·36	45	29·2	56·7	30·6	59·3	28·9	56·	27·9	54·09	30·7	48·4
55	56·87	60	51·3	76·4	45·5	67·8	52·6	78·3	51·9	77·27	50·7	63·3
First Period			£100·		£100·		£100·		£100·		£100·	
25	21·55	35	19·5	45·2	23·9	55·4	18·5	42·9	19·8	45·9	26·9	49·4
40	32·36	50	29·2	51·6	30·6	54·1	28·9	51·1	28·5	50·4	44·5	64·7
55	56·87	65	51·3	70·9	45·5	62·7	52·6	72·5	62·2	86·	69·4	81·1
25	21·55	30	19·5	49·4	23·9	60·5	18·5	46·9	18·1	45·8	11·	21·7
40	32·36	45	29·2	56·7	30·6	59·3	28·9	56·	25·	48·5	18·2	28·7
55	56·87	60	51·3	76·4	45·5	67·8	52·6	78·3	46·4	69·1	30·	37·5
Second Period			£200·		£200·		£200·		£200·		£200·	
25	21·55	40	19·5	41·4	23·9	50·7	18·5	39·2	19·5	41·4	31·7	54·
40	32·36	55	29·2	47·2	30·6	49·4	28·9	46·5	30·7	49·6	52·3	70·3
55	56·87	70	51·3	66·4	45·5	58·7	52·6	67·8	74·6	96·5	77·3	85·1
25	21·55	35	19·5	45·2	23·9	55·4	18·5	42·9	17·3	40·1	18·6	34·1
40	32·36	50	29·2	51·6	30·6	54·1	28·9	51·1	25·	44·2	30·9	44·9
55	56·87	65	51·3	70·9	45·5	62·7	52·6	72·5	54·5	75·3	48·1	56·2
25	21·55	30	19·5	49·4	23·9	60·5	18·5	46·9	15·8	40·	7·6	15·
40	32·36	45	29·2	56·7	30·6	59·3	28·9	56·	21·9	42·5	12·6	19·9
55	56·87	60	51·3	76·4	45·5	67·8	52·6	78·3	40·7	60·6	20·9	26·1
Third Period			£300·		£300·		£300·		£300·		£300·	

CORRESPONDENCE.

MR. FINLAISON'S REPORT AND THE ENGLISH LIFE TABLE.

To the Editor of the Assurance Magazine.

SIR,—The letter of Mr. Porter, in your last number, contains so many questionable assertions, and does such injustice to one of the most valuable tables of mortality we possess, as to require some further notice.

The information contained in the tables of Mr. Finlaison's Report is of great interest and value; the accompanying remarks, embracing a variety of different subjects, as might be expected, afford room for considerable difference of opinion. But most of your readers will probably agree in thinking, that the greater part of the eleven pages devoted to criticising the English Life Table would have been advantageously omitted. That Mr. Finlaison should look with some disfavour upon actuarial investigations proceeding from another Government department than his own, is not, perhaps, very surprising; but that Mr. Porter, passing over so much that is valuable in the Report, should have chosen this particular passage for comment, is in every way unfortunate. What have the records of the National Debt Office to do with the census returns of 1851? The portion of the latter, referring to the ages of the population, was published in 1854, and the facts were generally commented upon at the time. Why, then, has Mr. Porter, with his strong opinion of their worthlessness, allowed them to pass for seven years unnoticed?

Mr. Porter has made a general summary of his remarks under six heads, upon each of which I will say a few words.

"1. The inordinate extent to which the ages of females between the ages of—say, 20 and 40—are understated in the census."

Could anyone, whose only information on the subject was derived from Mr. Porter's letter, suppose that the authors of the *Report of the Census for* 1851 expressed a decided opinion of the general accuracy of the returns of the ages both of males and females, and that the passage quoted (or rather misquoted) in Mr. Finlaison's Report, on which Mr. Porter lays so much stress, was actually written in support of this opinion? Such is, however, the fact; but, by the omission of eight words, the meaning of the passage is entirely perverted, and the Registrar-General and his colleagues are made to express an opinion diametrically opposite to the one they really entertain. On reference to the *Report of the Census for* 1851, part 2, vol. i., pages 23 and 24, it is stated, that—"The mean age of the females, as they are returned in England, exceeds the mean age of the males by *ten months;* so that the tendency in women to understate their ages has only operated on comparatively small numbers; and there is no doubt of their general truthfulness." Some further reasons in support of this view are then given; at the same time, we are informed that at certain ages there are some evident misstatements, which, however, admit of being corrected. Then follows the passage quoted in your *Magazine*, page 278, which is correctly given for the first portion, but which in the original proceeds as follows:—"The extensive immigration of the Irish into Great Britain during the 10 years, 1841–51, has exercised some disturbing influence on the proportions; but, upon comparing the above numbers with those for males at the corre-

sponding ages, the conclusion appears to be inevitable that some 35,000 ladies, more or less, who have entered themselves in the second age, 20–40, really belong to the third age, 40–60; *to which the body of delinquents are transferred in Table* 7. *Millions of women have returned their ages correctly;* thousands have allowed themselves to be called 20, or some age near it, &c." The words in italics are omitted in the quotation; and the meaning intended to be conveyed is, in consequence, entirely misrepresented. The plain signification of the passage is—that there was no doubt of the general truthfulness of the returns of the ages; that there were some inaccuracies at particular periods of life, but only in the proportion of thousands to millions; that these inaccuracies admitted of correction, and had been corrected accordingly.

Mr. Porter, having thus propounded a statement which is altogether untrue (inadvertently, of course, but which a very little research would have prevented), proceeds, in language which I must be allowed to characterise as neither courteous, nor manly, nor just, to declaim against the ladies for *their* supposed want of veracity. But the indignation which began to be aroused at this charge soon passed into a smile at the remedy suggested, viz., that ladies should add to their other accomplishments a knowledge of Horace! and that their especial attention should be directed to one of the odes, which, in my time at least, *young gentlemen* in the course of their studies were usually recommended to omit.

"2. The over-estimate of age in very advanced life, both in the census and in the registers of death."

Mr. Porter refers particularly to ages above 100, and remarks that it is a curious fact, that members of the Peerage never attain to fabulous age. Here, again, a little investigation into the real facts quite alters his view of the case. In the census of 1851, there were found 319 centenarians out of a population of 21,185,010, or only 1 in 66,411. I see no improbability in this, especially as in some researches on the families of the Peerage, recently made by Mr. Day and myself, we found 2 centenarians out of 7,473 cases, or 1 in 3,736.

"3. The loose system adopted in ascertaining the ages at death of all classes of people, at all ages."

This is a matter within the experience of everyone; the description given by Mr. Porter is borrowed from his own imagination. The simple fact is, that medical men exercise no judgment whatever upon the ages at death; the *cause* of death is another matter, but the age inserted in the certificate is derived from the information of relations and friends. As with the ages of the living, there are probably several errors of one or two years in both directions, but there is no reason to doubt the substantial accuracy of the returns. And in applying the results to the construction of life tables, it must be remembered that the changes made in the process of graduation are so extensive, that the individual errors in the returns are comparatively unimportant and probably would not affect any pecuniary results.

"4. The direct pecuniary loss sustained by females in the purchase from the Government, or from such Annuity Companies as adopt special scales for female lives, on the present assumption of the vitality of females being so much in excess of that of males."

This is quite new information. Mr. Hendriks, some years ago, contributed to the *Statistical Journal* an elaborate paper on the "Loss sustained

by Government in granting Annuities." As regards the mortality, the Government tables since 1829 have been deduced from their own experience; and the superior vitality of females, which Mr. Porter calls an assumption, is, at all events as regards annuitants, an incontestable fact, proved by many years' experience. The established Assurance Companies, with hardly an exception, have discontinued to grant annuities, because they have found the business unprofitable; and as the majority of annuitants are females, there can be no doubt that the "direct pecuniary loss" has been on the contrary side to what Mr. Porter alleges.

"5. The advantage the female sex might derive, were Life Assurance Companies to adopt special rates of premium for female lives."

Surely Mr. Porter cannot be ignorant that this experiment has been more than once tried and abandoned! For nearly a quarter of a century the Eagle Office adopted special premiums for female lives. A careful investigation into the results of the experience of this Office was made by Mr. Jellicoe, and published in this *Magazine* (vol. iv., page 199). In the course of his remarks, Mr. Jellicoe observes—"One inference is, at all events, fully supported by these data, viz., that the insurance of female life at less rates than that of male is scarcely justifiable."

"6. The general discredit thrown upon the English Life Table, both for males and females, and the necessity that exists for Life Assurance and Annuity Companies to abstain from using it, in consequence of the incorrect returns, which, both as regards the numbers living and the numbers dying at each age, have been, no doubt, habitually made to the census-enumerators and the District Registrars of Deaths."

With a passing remark, that all Mr. Porter's objections would apply with much greater force to the Carlisle Table, against which he says nothing, I will briefly give my reasons for taking exception to this sweeping censure. The English Life Table is based, not upon the census of 1851, but upon that of 1841, where the return of ages was made in quinquennial intervals only; and the first process was to obtain the annual mortality in quinquennial periods of age. Even when treated in this manner (vide *Registrar-General's 5th Annual Report*, page 347), it was found that, as with other similar observations, the series *for both sexes* exhibited great irregularities. Corrections were made by means of Mr. Gompertz's hypothesis, that the rate of mortality may be represented by the expression aq^x; and Dr. Farr accordingly reduced the annual mortality above the age of 15 to two geometrical series with a different common ratio, one for the interval from 15–55, and the other from 55–95. The result is, a table of mortality very convenient for practical use, upon the accuracy of which we rely—both because we believe the ages both of the living and dead to have been given with sufficient accuracy for pecuniary purposes (which is quite compatible with several individual errors), and also because it is consistent with the results of other trustworthy observations. To apply a most stringent test—let us compare the English Life Table with Mr. Finlaison's tables, taking the two identical examples quoted by him to establish the inaccuracy of the former. (1) Out of 2,000 persons living at the age of 55, there survive to 75 by the English Life Table 778, and by the Government Table 901, persons. Is not this precisely what was to have been expected? The tontine nominees, upon whom the bulk of Mr. Finlaison's observations in early life are made, are recruited in old age by a very select body of lives of a different class—the annuitants, properly so called. Can, then, anyone be

surprised at the Government tables indicating a more favourable mortality between the ages of 55 and 75, than that of the general population? Had a contrary result been arrived at, would there not have been much more reason to doubt the accuracy of the English Life Table? (2) At the age of 27, taking interest at 4 per cent., the annual premium to assure £1,000,000, is, by the Government Male Table, £17,447, and by the English Male Table, £16,885, *i.e.*, £1. 14*s.* 11*d.* per cent. by the former, and £1. 13*s.* 9*d.* per cent. by the latter, the difference being one shilling and two pence per cent. To my comprehension, this proves the accuracy of the English Life Table; and to confirm my impression, I referred to the prospectus of the Office from which Mr. Porter writes, and, comparing the tables with those of the Royal Exchange Office (the selection being made almost at random), made the discovery that at this identical age of 27, the rates of premium of these two Offices differed as much as two shillings and eleven pence per cent.

But, Mr. Porter remarks, the last example is derived from male lives only, and an Assurance Company could only employ a table based on a combination of both sexes. Now, as we know by experience that the proportion of the sexes amongst assured lives is about 9 males to 1 female, I think that it would be more judicious for an Assurance Company to employ a table of mortality deduced from male lives only. Otherwise, I do not think sufficient reason has been shown for distrusting the English Female Life Table; and it is curious, that neither Mr. Finlaison nor Mr. Porter should have discovered that the blow they aim at it recoils on themselves. If, as they would have us believe, between the ages of 20 and 40 the numbers living are enormously overstated, while at the same time the ages at death are more accurately given, it follows that the mortality during that period must be much greater than the table indicates. Yet, with strange inconsistency, one of their objections to the table is, that it represents the mortality of females at the younger adult ages to be so great as to be contrary to nature.

Having myself, for some years, been in the practice of using the Male English Life Table for Assurance calculations, I should have been glad to have given my reasons for thinking that, in the present state of our knowledge, this table is better adapted for the purpose than the others in common use. But having already trespassed too much on your space, I will conclude by commending the English Life Table to Mr. Porter's more careful study, requesting him, in the words of the author whom he wishes ladies to read for their moral improvement—

> "Si quid novisti rectius istis
> Candidus imperti, si non, his utere mecum."

I am, Sir,

Your obedient servant,

ARTHUR H. BAILEY.

Equity and Law Life Office,
11*th May*, 1861.

ON THE CALCULATION OF PREMIUMS FOR ASSURANCES ON LIVES AND SURVIVORSHIPS BY THE AID OF MR. GOMPERTZ'S HYPOTHESIS.

To the Editor of the Assurance Magazine.

SIR,—Your readers will have observed, with much gratification, the announcement made by Mr. Gompertz (in his letter to Mr. Porter, published in your April Number), that, during the last two or three years, that eminent mathematician has been engaged in adding to his important discoveries in connection with the law of mortality and the methods of computing the values of life contingencies; and they will most sincerely echo the wish that his health may permit him, ere long, to lay the complete results of his very valuable labours before the public, through the medium of the Royal Society.

The discovery of methods for shortening the labour required in computing correctly the values of intricate cases of survivorship assurances must be considered of the highest importance by all engaged in the business of life assurance; and I trust that, notwithstanding the announcement above referred to, which justifies the expectation of a full and comprehensive treatment of this important subject, the following brief description of a general formula (based upon what Mr. Gompertz has already given us), for the solution of the more usual cases of survivorships, may not be altogether devoid of interest.

In a paper headed "On the law of mortality and the construction of annuity tables," published in vol. viii. of this *Magazine*, I ventured to suggest a method which, by means of a slight modification of Mr. Gompertz's formula, appeared to me to possess some utility in facilitating the computation of the values of *annuities* on several lives. The alteration in question consisted simply in introducing into the formula for the probabilities of living an additional constant, in such way that it should combine with the constant representing the interest of money in the corresponding formula for the *values of sums* depending upon those probabilities; and, by this means, preserve an important property of Mr. Gompertz's formula, first observed by Professor De Morgan—viz., the power of substituting an equivalent single age, easily determined, for any combination of joint ages; with this difference, however, that, by the introduction of the additional constant referred to, the substitution consists of an *equal number of lives*, of a certain common age, in lieu of a *single life*.*

The subjoined tables, representing the decrements and the expectation of life at all ages, are constructed upon the principle explained in the paper before referred to. They are based upon the Carlisle Table; but, for the sake of convenience in calculation, I have slightly altered the values of the several constants yielded by that table—considering, for the object in view, a very close adherence to any particular table unnecessary. Nevertheless, upon a comparison of the expectation of life, it will, I think, be found that

* My paper having been drawn up previously to the publication of Professor De Morgan's article in the *Assurance Magazine* for July, 1859 (which is the only one I have seen on the subject), the enunciation of the property in question is given as *new*. In a letter to the Editor, which accompanied my manuscript a few days after the publication of the Number for July, 1859, I referred to Mr. De Morgan's prior discovery, but, by an oversight, omitted to insert a similar reference in the paper itself.

the difference does not exceed the licence usually allowed in the adjustment of mortality tables.

The first table, which I give entire, containing the decrements and expectation of life, needs no explanation, as the values correspond exactly with those formed by the usual methods. The annuity tables, however (of which extracts only are given), are constructed by the following formula—

$$\frac{1}{B_m^\mu}\int_0^\infty \left(B_m^\mu\right)^{q^x} \cdot \left(\frac{v}{a^\mu}\right)^x \cdot dx,^*$$

which denotes the value of an annuity *payable momently* during the joint existence of μ lives each aged m years, and consequently they differ in this respect from annuity tables in ordinary use. Various methods, more or less convenient, may be adopted for calculating the values of the above integral, but it is not my purpose to enter into this subject further than to state that in the tables in question the values are computed to the fourth decimal place. As, however, three decimal places are sufficient for most purposes, that number only is given in the extracts, and it will be observed that each annuity table for any given number of lives to three places could be comprised in the same amount of space as a table for single lives.

Adopting a table of annuities payable *momently* as the basis of calculation, let such an annuity on the joint existence of any given number of lives aged respectively m, n, r . . . be represented by $A_{m,n,r\ldots}$. To find the value of a similar annuity, payable t times a year, we have the following simple formula—

$$A_{m,n,r\ldots} \pm \frac{1}{2t},$$

the upper sign being taken when the payments are in advance, and the lower sign when in arrear.

By the aid of these *momently* annuities, a convenient general formula may be deduced for the *exact* solution of the following comprehensive problem in survivorship assurances.

Problem.—Required the value of £1 payable at the failure of the joint existence of μ lives aged respectively m, n . . . and r, provided that event shall happen before the failure of the joint existence of ν other lives aged respectively u, v . . . and z.

The formula by which this problem is solved is—

$$\frac{S_m}{S_m+S_u}\left\{1+\left(\log_e v-\log_e a\,.\,\frac{\nu S_m-\mu S_u}{S_m}\right)A_{m,n\ldots z}\right\} \quad . \quad [1]$$

where $S_m=q^m+q^n+\ldots+q^r$, $S_u=q^u+q^v+\ldots+q^z$ and $A_{m\ldots z}=$ the value of an annuity (payable momently) during the joint existence of all the lives involved.

In the case of an absolute assurance on the joint lives m, n . . . and r, the quantities ν and S_u vanish, and the formula becomes—

$$1+\log_e v\,.\,A_{m,n\ldots r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [2]$$

For calculating the value of an assurance on a single life (m) against ν other lives (u), (v) . . . (z), the general formula [1] may be put in the following more convenient form—

* The characters correspond with those used in my paper of January, 1860.

$$\frac{q^{m-y}}{q^{m-y}+q^{n-y}+\ldots+q^{s-y}}\left\{1+(\log_e v+C_{u-m}+C_{v-m}+\ldots+C_{s-m})A_{m,n\ldots s}\right\} \quad [3]$$

where $C_n=\log_e a(q^n-1)$, and y is the age of the youngest of all the lives involved.

When the problem refers to one life (m) against another life (u), the formula may be still further simplified, thus—

$$F_{u-m}+G_{u-m}.A_{m,n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [4]$$

where $F_n=(1+q^n)^{-1}$ and $G_n=F_n.(\log_e v+C_n)$.

It should be observed that the preceding formulæ give the value of the reversion payable at the instant of death, from which the value of the same, payable at the expiration of any time, t, after death, may be accurately determined by multiplying by v^t; and the value of the given sum payable at the expiration of the year of death may be considered equivalent to the same payable six months after death, and determined accordingly.

In taking as the basis of calculation the values of annuities payable momently, it would, perhaps, be preferable that the annual premiums should be supposed to be payable in the same manner—*i. e.*, by momently instalments. The general formula for the annual premium payable momently during the joint existence of all the lives would be—

$$\frac{S_m}{S_m+S_n}\left\{\frac{1}{A_{m,n\ldots s}}+\left(\log_e v-\log_e a\frac{\nu S_m-\mu S_n}{S_m}\right)\right\},$$

and the annual premium payable by $\frac{1}{p}$thly instalments (in advance) could be deduced from the value so found by simply deducting therefrom $\frac{1}{2p}$th of a year's interest, provided that the proportion of premium paid in advance for any period beyond the time at which the death takes place be returned to the assured. For instance, if π denote the annual premium payable momently, and t the unexpired fraction of the current year at the date of death, the amount of premium returnable to the assured at the instant of death will be $\pi t\left(1-\frac{rt}{2}\right)$ (r being the yearly interest on £1), for the Office may be supposed to owe the assured the sum πt payable by momently instalments during the time t.

A practice somewhat similar to the above is, I believe, generally followed in India with regard to half-yearly and quarterly premiums; and, independently of the facilities afforded in calculation, it is, perhaps, preferable to the plan adopted in this country. However, the annual, half-yearly, and quarterly premiums, according to the usual practice, may be found from the single premium by the aid of the formula for determining the value of an annuity payable p times a year.

The general formula [1] is the key to the exact solution of all cases of assurances on lives, whether absolute or contingent, treated in Baily's work on annuities and assurances, with the exception of those involving the problem of determining the value of a reversion on a given life, subject to the condition of a second life surviving a third during the lifetime of the first—a satisfactory solution of which has, I believe, never yet been *published.* And the term "exact solution" is to be taken in its fullest sense;

for the hypothesis of constant decrements for single years, or any similar hypothesis, is not resorted to in deducing the formula. The remaining cases of survivorship assurances, involving the problem before referred to, are also capable of exact solution on principles similar to those by which the formula [1] is deduced; but it is not my purpose to enter here upon the subject of these interesting but unusual cases.

A very important point in Mr. Gompertz's letter is the announcement that the method which he is now engaged upon admits of the solution of cases involving combinations of lives subject to different laws of mortality. This problem also admits of solution by tables constructed on the method suggested by me, provided one uniform value of q can be adopted in the construction of the several tables of mortality. By a slight arbitrary modification of the several constants, I have ascertained that this is perfectly practicable in the case of Indian and European lives, and in all probability it would be found equally so in other cases. Mr. Gompertz's promised paper will doubtless treat this subject in a manner worthy of its great practical importance.

I will not add to the length of this communication by inserting the demonstration of the solution of the general problem, which would occupy considerable space, although no particular difficulty is involved in it. I will, therefore, conclude by a single example of the most usual case of survivorships involving three lives.

Example.—Required the value of £1 payable at the death of (28), provided (30) and (33) shall be then both living.

(m) being the youngest life, the formula becomes—

$$\frac{1}{1+q^2+q^5}\left\{1+(\log_e v+C_2+C_5)A_{28,30,33}\right\}.$$

$$\begin{array}{r}
1\cdot \\
1\cdot1860 \\
1\cdot5317 \\
\hline
3\cdot7177\ \log.=\cdot57027^{*} \\
\log.\ 3=\cdot47712 \\
\hline
\cdot09315\times\left(27=\dfrac{1}{\log. q}\right) \\
\hline
2\cdot7945 \\
\cdot2795 \\
\hline
2\cdot515 \\
28\cdot \\
\hline
30\cdot515=\text{Eq. com. age.}
\end{array}$$

$$\begin{array}{rl}
A_{30.30.30}=12\cdot784 & -167\times\cdot515 \\
 & \overline{84} \\
-86 & 2 \\
\hline
12\cdot698 & \\
 & \\
\log_e v=\bar{1}\cdot96078 & \\
C_2=\ \cdot00132 & \\
C_5=\ \cdot00376 & \\
\hline
\bar{1}\cdot96586 &
\end{array}$$

$$\begin{array}{rl}
\bar{1}\cdot96586=-\cdot03414\ \log.= & \bar{2}\cdot53326 \\
\log.\ 12\cdot698= & 1\cdot10374 \\
-\log.\ (1+q^2+q^5)= & \bar{1}\cdot42978^{*}=\log.\ \cdot26899 \\
\hline
 & \bar{1}\cdot06673\ =\log.\ \cdot11661 \\
 & \overline{(\textit{Answer})\ \cdot15238}
\end{array}$$

I am, Sir,

Your very obedient servant,

5, *Lothbury.* W. M. MAKEHAM.

Age.	Living.	Decrements.	Expectation of Life.	Age.	Living.	Decrements.	Expectation of Life.
15	10000·000	75·323	44·86	60	5677·788	165·630	14·57
16	9924·677	75·183	44·20	61	5512·158	171·522	13·99
17	9849·494	75·070	43·53	62	5340·636	177·472	13·42
18	9774·424	74·997	42·86	63	5163·164	183·437	12·87
19	9699·427	74·958	42·19	64	4979·727	189·345	12·32
20	9624·469	74·959	41·51	65	4790·382	195·127	11·79
21	9549·510	75·001	40·83	66	4595·255	200·701	11·27
22	9474·509	75·093	40·15	67	4394·554	205·971	10·76
23	9399·416	75·226	39·47	68	4188·583	210·838	10·27
24	9324·190	75·418	38·79	69	3977·745	215·185	9·79
25	9248·772	75·660	38·10	70	3762·560	218·888	9·32
26	9173·112	75·964	37·41	71	3543·672	221·825	8·86
27	9097·148	76·332	36·72	72	3321·847	223·856	8·42
28	9020·816	76·764	36·02	73	3097·991	224·850	7·99
29	8944·052	77·272	35·33	74	2873·141	224·677	7·58
30	8866·780	77·860	34·63	75	2648·464	223·214	7·18
31	8788·920	78·530	33·93	76	2425·250	220·351	6·79
32	8710·390	79·286	33·23	77	2204·899	216·004	6·42
33	8631·104	80·142	32·53	78	1988·895	210·115	6·06
34	8550·962	81·094	31·83	79	1778·780	202·660	5·72
35	8469·868	82·158	31·14	80	1576·120	193·657	5·39
36	8387·710	83·336	30·44	81	1382·463	183·177	5·08
37	8304·374	84·640	29·74	82	1199·286	171·341	4·78
38	8219·734	86·070	29·04	83	1027·945	158·323	4·49
39	8133·664	87·642	28·34	84	869·622	144·355	4·22
40	8046·022	89·360	27·64	85	725·267	129·714	3·95
41	7956·662	91·233	26·95	86	595·553	114·715	3·71
42	7865·429	93·274	26·25	87	480·838	99·703	3·47
43	7772·155	95·483	25·56	88	381·135	85·029	3·25
44	7676·672	97·877	24·87	89	296·106	71·029	3·04
45	7578·795	100·465	24·19	90	225·077	58·013	2·84
46	7478·330	103·248	23·51	91	167·064	46·231	2·65
47	7375·082	106·244	22·83	92	120·833	35·869	2·48
48	7268·838	109·453	22·16	93	84·964	27·029	2·31
49	7159·385	112·887	21·49	94	57·935	19·730	2·15
50	7046·498	116·546	20·82	95	38·205	13·912	2·01
51	6929·952	120·444	20·16	96	24·293	9·448	1·87
52	6809·508	124·578	19·51	97	14·845	6·156	1·74
53	6684·930	128·947	18·87	98	8·689	3·838	1·63
54	6555·983	133·556	18·23	99	4·851	2·277	1·51
55	6422·427	138·396	17·60	100	2·574	1·283	1·41
56	6284·031	143·458	16·97	101	1·291	·681	1·32
57	6140·573	148·735	16·36	102	·610	·341	1·23
58	5991·838	154·204	15·75	103	·269	·158	1·15
59	5837·634	159·846	15·16	104	·111	·069	1·07

Annuities (4 per Cent.).

Ages. (n)	One Life. (A_n)		Two Lives. ($A_{n,n}$)		Three Lives. ($A_{n,n,n}$)		Ages. (n)	One Life. (A_n)		Two Lives. ($A_{n,n}$)		Three Lives. ($A_{n,n,n}$)	
		(−)		(−)		(−)			(−)		(−)		(−)
30	17·492	·164	14·668	·171	12·784	·167	35	16·620	·191	13·758	·199	11·899	·193
31	17·328	·169	14·497	·177	12·617	·171	36	16·429	·197	13·559	·204	11·706	·197
32	17·159	·174	14·320	·182	12·446	·177	37	16·232	·202	13·355	·209	11·509	·203
33	16·985	·180	14·138	·187	12·269	·183	38	16·030	·207	13·146	·215	11·306	·208
34	16·805	·185	13·951	·193	12·086	·187	39	15·823	·214	12·931	·220	11·098	·212

n	q^n	d_n	C_n	C_{-n}	F_n	F_{-n} $(=1-F_n)$	G_n	G_{-n}
1	1·089023	·511	·0006299	$\bar{1}$·9994216	·4786927	·5213073	$\bar{1}$·9815269	$\bar{1}$·9792524
2	1·185971	1·043	·0013158	$\bar{1}$·9988905	·4574626	·5425374	$\bar{1}$·9826599	$\bar{1}$·9781194
3	1·291550	1·596	·0020628	$\bar{1}$·9984028	·4363858	·5636142	$\bar{1}$·9837848	$\bar{1}$·9769944
4	1·406527	2·170	·0028763	$\bar{1}$·9979550	·4155366	·5844634	$\bar{1}$·9848976	$\bar{1}$·9758817
5	1·531740	2·765	·0037623	$\bar{1}$·9975438	·3949853	·6050147	$\bar{1}$·9859945	$\bar{1}$·9747849

ON THE SUPERANNUATION OF EMPLOYÉS IN ASSURANCE OFFICES.

To the Editor of the Assurance Magazine.

SIR,—May I solicit the favour of your allotting a small space in your *Journal* for the insertion of a few remarks upon the subject of superannuation of *employés* in Assurance Offices, in the hope that it may engage the attention that it certainly deserves, but which, I believe, it has not hitherto received.

There may be, and, no doubt, are, systems of superannuation in connexion with some of our public institutions, but they are not, I believe, general; indeed, I am only aware of one instance in which a scheme exists in connexion with a Joint Stock Company for granting retiring pensions after certain periods of service. The Company referred to is the National Provincial Bank of England, and the main features of the scheme are, the option of retirement, after 20 years' service, on one-third of salary; after 30 years, on half salary; or, after 40 years, on two-thirds salary; and in proportion for intermediate periods of service—one of the conditions being, that the age of 60 shall be attained before retiring.

It will be seen that there is here no inducement held out to withdraw from active duties, but there is an option given of doing so, that would be esteemed a boon by very many who yet might never avail themselves of it.

Habit, we all know, has a powerful hold upon men generally—probably upon no class is its influence greater than upon those engaged in official routine—and it may reasonably be supposed that few men would, if in health, readily sacrifice two-thirds, or a half, of their income, merely for the sake of living in idleness.

It may be suggested, that a person has no claim upon the Company by which he has been employed—whatever may have been the length of his services—when incapacitated, by sickness or other infirmity, for further duty; true, he has no *legal* claim, but I am happy to think that boards of directors of Assurance Companies are not usually composed of men who take this view of things.

Assuming, then, a willingness to entertain the question of superannuation, a difficulty may arise respecting the cost; this is, however, rather imaginary than real, as I hope to show.

Waiving the consideration of retiring pensions to heads of departments, I will take the general staff of an Office, and assume that all engagements commence with a salary of £60, rising £10 annually until a maximum of £250 is reached, which would be in 20 years. Supposing, then, an individual who has attained the age of 60, and has been engaged in the service of a Company for 30 years, should feel desirous to retire from the cares and

responsibilities that have for so long a period devolved upon him, what would be the pecuniary effect upon a Company, of allowing him to do so with a pension of £125 per annum, and appointing a junior to fill up the vacancy that would be caused? The cost of a Government annuity may be considered to be a fair basis upon which to form an estimate, and this, at the age of 60, would amount to about £1,300, against which has to be placed the present value of the annual differences between the salary paid to the newly appointed clerk and that which *would* have been payable had no change taken place; this will amount to about £1,500, and the result would be an actual gain to the Company of £200 in the 20 years that would elapse before the maximum salary of £250 would be attained.

True, it may be said that a person of 60 years of age is not likely to remain at his post another 20 years; but there is also the contingency of a junior not continuing 20 years. Indeed, I believe that the average length of time is likely to be as great in the former as in the latter case.

Did space permit, very many arguments might be adduced in favour of thus making provision for the retirement of officials of long standing; and I am not aware that any objections against it can be raised. However, I offer these few suggestions, trusting that an interest may be created which will lead to some more definite action in the matter.

Perhaps the Institute of Actuaries might give the subject their attention; it certainly is one that may fairly challenge discussion, involving, as it does, considerations affecting all grades in the profession, from the actuary or secretary to the junior clerk; and the adoption of some such scheme of superannuation as I have sketched out may not unfairly be considered as likely to influence the prosperity of such Companies as might be induced to entertain it; such a course being one that would appear to be eminently calculated to secure a continuance of the services of tried and efficient officers.

I am, Sir,
Your obedient servant,

London, June, 1861. H. A.

[Note.—The question of superannuation allowances by public institutions is one which we should be glad to see engage attention. The scheme recently adopted by the National Provincial Bank of England appears to be a very liberal one. This bank, moreover, several years ago, with the view to encourage provident habits among their *employés*, determined to pay one-half of the yearly premiums of assurances on the lives of their officers and clerks. We may refer our readers to a letter on this subject, which will be found at page 72, vol. v., of this *Magazine*. It is there suggested by a correspondent—Mr. Porter—that those engaged in life assurance, like those employed in all other descriptions of business, should be allowed to obtain the article in which their employers deal at *cost price*.—Ed. *A. M.*]

INSTITUTE OF ACTUARIES.

PROCEEDINGS OF THE INSTITUTE.

Fifth Ordinary Meeting, Session 1860-61.—*Monday, 25th March*, 1861.

Charles Jellicoe, President, in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced several donations to the library.

Mr. W. Newmarch, duly nominated at the last ordinary meeting, was elected a Fellow of the Institute on the recommendation of the Council.

The undermentioned gentleman, duly nominated at the last ordinary meeting, was elected a member of the Institute, viz.:—

Associate—Charles Evans Newbon.

A communication from Mr. Gompertz was read by Mr. Porter, and a paper by Mr. Sprague, "On the graduation of the series giving the expectation of life, and the nature of the corresponding curves," was also read by that gentleman.

Sixth Ordinary Meeting, Session 1860-61.—*Monday, 29th April,* 1861.

CHARLES JELLICOE, President, in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced several donations to the library.

The undermentioned gentleman, duly nominated at the last ordinary meeting, was elected a member of the Institute, viz.:—

Official Associate—John Messent.

Mr. Bailey read a paper "On the mortality amongst the families of the peerage during the nineteenth century," by Mr. A. H. Bailey and Mr. Archibald Day.

Thanks having been voted to Mr. Bailey and Mr. Day, the meeting adjourned to Monday, the 25th November next.

Fourteenth Annual General Meeting, Saturday, 1st June, 1861.

The circular convening the meeting having been read,

The minutes of the last ordinary meeting were read and confirmed.

Mr. J. Hill Williams, one of the Honorary Secretaries, then read the following Report of the Council on the progress of the Institute during the past year:—

"REPORT.

"The Council have again the pleasure of submitting to the members a Report as to the progress of the Institute during the past year.

"There has been a slight increase in the number of members, which is now 155, as compared with 147, at the date of the last Report. The numbers in each class are 46 Fellows, 21 Official Associates, and 88 Associates.

"The income of the year is £367. 4*s.*, and the expenditure £350. 12*s.* 4*d.* The assets amount to £428. 9*s.* 2*d.*, of which £192. 8*s.* 6*d.* is invested in the 3 per cent. Consols.

"The papers read during the Session have been of much interest; they are as follows:—

"'On the theory of probabilities.' By Robert Campbell, Esq., M.A.

"'On the stability of results based on average calculations.' By the same gentleman.

"'On the rates of premium required to provide for certain periodical returns to the assured.' By Robert Tucker, Esq., V.P.

"'On Mr. Gompertz's law of mortality.' By T. B. Sprague, Esq., M.A.

"'On the graduation of series, expressing the expectation of life, and on the curves connected therewith.' By the same writer.

"'On the rate of mortality prevailing amongst the families of the peerage during the nineteenth century.' By A. H. Bailey and Archibald Day, Esqs.

"In connection with the subject of the papers which have appeared in the *Journal* of the Institute, it may be interesting to state that the historical notice by Professor De Morgan, entitled 'Account of a correspondence between Mr. George Barrett and Mr. Francis Baily,' published in Vol. IV., led to the laborious calculations which Barrett left behind him in manuscript being brought under the notice of the members of the Institute. Professor De Morgan, in that paper, states that Barrett's manuscript tables were purchased,

a few years after his death, by Mr. Babbage; and one of the members of the Council having written to Mr. Babbage, requesting to know if the papers were still in his possession, that gentleman obligingly appointed a time for an interview, and exhibited all of these documents, which he still retains. After explaining the circumstances under which he purchased them of the executors, he most kindly allowed them to be sent to the Institute for the inspection of the members. The manuscripts comprise all the tables mentioned in Professor De Morgan's historical notice; and though it will be seen that, from the improved tables of population now in use, these volumes of calculations, the fruits of a laborious life incessantly employed upon them, are not at the present time of much practical use; yet they are worthy of examination as the work of an author whose columnar method is now so well known, and who, by means of it, has so greatly facilitated and improved assurance calculations.

"The Council have expressed, on behalf of the Institute, the sense which they entertain of the courteous attention of Mr. Babbage; and they are sure that every member will be fully sensible of his obliging consideration.

"The members are aware that the prize offered, in 1859, for an essay 'On the methods of distributing the surplus amongst the persons assured in a Life Assurance Company,' to be written by an Associate of the Institute, was again offered last year; and the Council have now the satisfaction of announcing, that essays on the prescribed subject have been sent in, and that one of them has been deemed worthy of the prize, and will be shortly published in the *Journal*. The author of it is Mr. William Pollard Pattison.

"Since the last annual meeting of the Institute, the International Statistical Congress has held its fourth session in this metropolis, with what success may be estimated from the Report which has been recently issued of its proceedings. That most remarkable and able production owes its existence, as is well known, to the labours of Dr. Farr and his coadjutors; and it is gratifying to the Council that members of the Institute have assisted so materially in the work. The papers and contributions of Mr. Samuel Brown, Mr. Hodge, Mr. Newmarch, and Mr. Hill Williams, are not surpassed in interest and importance by any to be found in the volume."

An abstract of the receipts and payments of the Institute, for the financial year ended 31st March last, was then read (*see* p. 372).

On the motion of the Chairman, the Report was unanimously adopted.

The election of a President, Vice-Presidents, and Officers, for the year ensuing, was then proceeded with.

Mr. Pattison and Mr. Strachan were appointed scrutineers.

On the result of the ballot being obtained, the following was declared to be the list, viz.:—

President.

CHARLES JELLICOE.

Vice Presidents.

SAMUEL BROWN.	WILLIAM BARWICK HODGE.
PETER HARDY, F.R.S.	ROBERT TUCKER.

Treasurer.

JOHN LAURENCE.

Honorary Secretaries.

JOHN REDDISH.	JOHN HILL WILLIAMS.

The following gentlemen were unanimously re-elected Auditors for the ensuing financial year, viz., John Coles, Edward Cutbush, and James Terry.

Mr. Jellicoe said—"I have to express the best thanks of my colleagues and myself for the compliment you have paid us, and for the confidence you have shown in our desire and ability to serve the Institute, by again electing us. It may be remembered, that at our last annual meeting I took occasion to refer to a few questions connected with our pursuits, and to remark on the desirableness of arriving at a determination of them. In the year which has since

elapsed, progress has been made towards the solution of some of these. There is reason, for instance, to believe, that the Report of the American Convention, on the rate of mortality of persons whose lives are assured in the United States, is now nearly completed, and will shortly be made public, and we may hope, therefore, soon to have the means of deciding the very interesting question, whether there really is, or is not, any material difference in the duration of human life thus circumstanced in the two countries; and it must, I think, be looked upon as fortunate, that all the materials for this investigation have been collected before the events which are calculated to create so great a disturbance in the results, and which are so much to be lamented, have had their origin.

"Another question to which I briefly alluded, has also, I am happy to say, received considerable illustration at the hands of one of our members. I refer to the inquiry as to the most accurate mode of dividing surplus in Assurance Companies. Mr. Pattison's essay on this subject cannot but be read with advantage by all who are desirous of forming correct ideas upon it; nor will the reader fail to appreciate the amount of useful information brought together, as to the various methods hitherto pursued in this process by those having the management of these important institutions.

"The members of the Institute have not addressed themselves during the past Session to the discussion of the vexed question of direct taxation; but the suggestions made at the last annual meeting have received remarkable confirmation in the measures of the Government with reference to the abolition of the duties on the manufacture and importation of paper, and in the efforts made by Mr. Hubbard to modify the system under which the income tax is at present levied. I cannot but regard the measures of the one and the efforts of the other as essentially in the right direction, and, having no other desire than the growth and prevalence of right principles, I am happy to think that a similar opinion is entertained by a large majority of those whom I have now the honour to address.

"Turning, however, to the consideration of what has been accomplished in the Institute during the Session, a source of much satisfaction arises from the contemplation of the many able and useful communications which have been laid before it. Apart from the importance of the practical results given in Mr. Campbell's last paper, the method therein suggested of eliminating quantities of small value, and thus reducing expressions, unmanageable from their intricacy and diffuseness, to a form easily tabulated, is likely very much to facilitate calculations in probabilities; the labour being by such means brought within reasonable compass, whilst the results are quite sufficiently accurate for all ordinary purposes.

"The mode, too, of investigation, which has of late been pursued, with a view to determine the relation between the premiums charged by Assurance Companies and the returns made by them, is useful and instructive. Heretofore, writers have sought to show what returns the premiums charged would justify. Of late, the object has been to determine what premiums are necessary to provide for such returns as are usually made; and as this latter process brings into forcible contrast the rates which are necessary and those ordinarily adopted, attention is more likely to be drawn to the absolute necessity of bringing the two into relations consistent with safety. Mr. Tucker has given formulæ with this object applicable to almost every case. I believe there is no system of division which he has not brought under investigation in this way.

"To Professor De Morgan and to Mr. Sprague we are indebted for a most thorough and complete exposition of the Gompertzian theory. (I do not speak here of Dr. Farr's masterly treatise.) The extent to which the logarithmic expression which arises out of it may be usefully applied has been clearly shown and well defined, and enough has transpired to prove that the expression itself must be handled with much delicacy and judgment, if we wish to adhere rigorously to the records of experience. As is the case with most other expressions for concatenated series, the series yielded by this is dependent in every part upon the whole; like a cane compressed into a curve and confined at the centre, we cannot give a direction to the one portion without influencing the

other, and thus our instrument is not quite of that degree of ductility which we require; whether it will better adapt itself to other kinds of series in use with us remains to be seen. Mr. Sprague seems to be directing his attention to this branch of our inquiries, and you will all admit that it cannot be in better hands.

"Prior to the investigation made by Mr. Bailey and Mr. Day into the rate of mortality prevailing amongst the families of the peerage, I believe the impression has been very general (derived from previous writings on the subject) that the duration of life in that class is considerably below the average. At all events, a great deal of doubt has existed as to the real facts of the case. The labours of Messrs. Bailey and Day have, however, set the question at rest. They have proved, beyond all controversy, that, so far as regards the experience of the present century, the male portion of the class in question enjoys a degree of longevity rarely known; and they have also shown, and are, I believe, the first writers who have done so, that the longevity of the female portion is at all ages equal to the greatest on our modern records, and at some ages even greater.

"These results are such as we might reasonably look for when the condition of the class in question is taken into account, and when we know that the great means it generally possesses are no longer dissipated, as they once may have been, in over-indulgence and dangerous excesses. A very satisfactory consideration also arises out of the deductions thus so conclusively arrived at by Mr. Bailey and his coadjutor, and one which I remember Mr. Newbatt ably adverted to at the reading of the paper; it is, that if the probabilities of living are so great amongst the highest classes, they cannot be very inferior amongst the classes immediately succeeding, since the habits and modes of life in all are much the same, making some allowance for the wear and tear of professional life.

"Hence there is reason to hope that the experience of our Assurance Companies will, at the least, be as favourable in future as it has hitherto been, since the names on the registers of those institutions are, for the most part, those of persons in the middle and upper ranks of the population. Whether the evidence which Messrs. Bailey and Day have brought forward will warrant the Offices in dispensing with the existing restrictions on foreign travel and residence I will not now inquire. The suggestion is, however, well worthy of consideration, and points to the adoption of an arrangement which, though at first somewhat startling, is but a trifle in advance of the practice almost universal with the Companies in the north, and not very unfrequent with those of this metropolis.

"Such, gentlemen, is a brief review of the subjects which have, for the most part, engaged our attention within the Institute during the last Session, whilst externally, during this period, an extraordinary call on the activity and intelligence of some of our colleagues has arisen from the advent hither of the International Statistical Congress. Amongst the statements and reports submitted to that body, organized with such extraordinary skill and ability by Dr. Farr and those around him, will be found Mr. Samuel Brown's programme on the statistical units of money, weights, and measures, and his report on the proceedings and progress of the Institute; Mr. Hodge's programme on military statistics; Mr. Newmarch's on prices and wages, and his report on the proceedings and progress of the Statistical Society; and Mr. Hill Williams' programme on the subdivision, transfer, and burdens of real property.

"How well these gentlemen have executed their several tasks—undertaken, I believe, in each case, at a very short notice—will be seen by the most cursory reference to them. Taken in connexion with those of which I have already spoken, they will at least serve to show that the labours of the gentlemen connected with this Institute are by no means light, and that no little consideration is due to them for devoting so freely to the common advantage, the time and ability already so fully absorbed by the duties of an arduous and engrossing occupation."

A vote of thanks to the President, moved by Mr. Lodge and seconded by Mr. Pinckard, was passed unanimously, and the proceedings terminated.

INSTITUTE OF ACTUARIES.

Dr. *Abstract of Receipts and Payments for the Year ending 31st March, 1861.* **Cr.**

RECEIPTS.

			£	s.	d.
April 1, 1859.	To balance brought forward		220	9	11
Subscriptions for 1859–60 (arrears)			7	7	0
Subscriptions due for 1860–61, from—					
47 Fellows	35 Town .. at £3 3 0	£110 5 0			
	12 Country „ 2 2 0 ..	25 4 0			
21 Official Associates	19 Town .. „ 3 3 0 ..	59 17 0			
	2 Country „ 2 2 0 ..	4 4 0			
87 Associates	69 Town .. „ 2 2 0 ..	144 18 0			
	18 Country „ 1 1 0 ..	18 18 0			
		363 6 0			
Subscriptions of Members not paid, viz.—					
1 Fellow, Town	at £3 3 0 ..	£3 3 0			
4 Associates, Town	„ 2 2 0 ..	8 8 0			
1 „ Country	„ 1 1 0 ..	1 1 0			
		12 12 0			
			350	14	0
Examination Fee			5	5	0
Sundries			3	18	1
			£587	14	0
March 31, 1861.	To Balance brought down		£236	0	8

Note.—The Assets of the Institute, on the 31st March, 1861, consisted of £198. 16*s.* 2*d.* Consols, say 183 19 4
Cash 236 0 8
Books in Library, say 300 0 0
Total £720 0 0

12, St. James's Square, London,
April, 1861.

PAYMENTS.

1860.	£	s.	d.	£	s.	d.
By Rent	75	0	0			
Salaries	99	5	10			
Journal	88	0	0			
Library	10	14	5			
Stationery and Printing (2 years)	29	12	9			
Postage and Receipt Stamps	3	17	6			
Lighting	6	7	3			
Ordinary Meetings	15	3	0			
Advertising Examinations	3	9	6			
Statistical Congress expenses	5	9	0			
Miscellaneous	13	13	1			
Total of general expenses				350	12	4
March 31, 1861.						
Subscription partly repaid				1	1	0
Balance carried forward				236	0	8
				£587	14	0

Examined and approved:—

John Coles,
Edward Cutbush,
James Terry, } *Auditors.*

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ERRATA.

In Diagram No. 2 of Mr. Spens' paper—For 1861, per Table II., read 1861, per Table XI.

Page 74, last line of figures—For ·928 and ·912, read 2·160 and 2·200.

Page 76, heading of last column, Table VIII.—For Table II., read Table VI.

Page 297, line 18—For ·002758, read ·01379.

Page 298, line 5—For ·006784, read ·033919.

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OF THE

INSTITUTE OF ACTUARIES.

Seventh Census of England and Wales, April, 1861. *Summary of General Results.**

THE Act for taking the Census of England required that the 31,000 Enumerators employed should copy into as many books the householders' schedules and other particulars collected by them in their several districts. These books were to be placed, with the schedules, in the hands of the 2,197 Registrars, who were to subject them to a strict examination, and make all necessary corrections. This being accomplished, the books and other documents were to be transferred before the 30th April to the custody of the 631 Superintendent Registrars, who were required to test the accuracy of their contents by a further process of revision.

The Superintendent Registrars were allowed a short time for the purpose of returning the revised books to the Census Office, where they have still to undergo strict and minute revision, before any *detailed* and *exact* statement of results can be given. As this

* This abstract contains the substance of one laid before Parliament on the 7th June last, and prepared in the Registrar-General's office by Dr. Farr and Mr. Hammack. It may be thought by some inexpedient to occupy the pages of this *Journal* with matter so easily accessible elsewhere; but we believe most of our readers will consider it only fitting that some brief record should appear in it of the results of an inquiry of such importance as this undoubtedly is.—ED. *A. M.*

essential labour must, however, of necessity, engage much time, it has been thought desirable not to withhold from the public such an approximation to the general results as might be obtained, without waiting for the entire completion of this series of checks. The Registrars, therefore, were desired to forward to the Census Office summaries of the population and houses within their respective districts.

From these summaries the following tables have been chiefly compiled, and they must be taken to represent the results of the Census according to the statements of the *local officers*, previous to the revision now in progress at the central office. And although minute accuracy is not to be looked for, yet it is apprehended that the alterations which a careful revision of the original documents may render necessary will not be of importance sufficient to lessen the value of the figures as materials for whatever *general* inferences may fairly be drawn from them.

Table I. comprises the *Population enumerated* on April 8th, 1861, in England and Wales and the islands in the British seas, amounting to 20,205,504. The portion of the Army, Royal Navy, and Merchant Seamen *out of the country* at the time of the Census is not included; and as it appears from official returns that the *Army abroad* amounted to 137,000, the *Royal Navy and Marines absent* from the United Kingdom to about 42,900, and the *Merchant Seamen absent* on voyages to about 96,000, it may be assumed that the numbers of these classes *belonging to* England and Wales were collectively not less than 162,021, mostly adult males.

Table II. exhibits the number of *Houses* and the *Population enumerated* in England and Wales in 1851 and 1861, with the *increase* in the interval.

Table III. shows the population of England and Wales at each of the Censuses, 1801-61, *including the Army, Navy, and Merchant Seamen abroad* as well as at home, with the increase in each decennial period. The *actual increase* of population (2,169,576), between 1851 and 1861, was greater than in any previous decennial period, but the *rate of increase*, owing to active emigration, had somewhat diminished.

Table IV. presents a comparative view of the number of houses enumerated at each of the Censuses.

Table V. shows the *Houses* and *Population* enumerated in the Counties (proper) in 1851 and 1861, together with the increase or decrease of persons in each county in the period between the enumerations.

In Table VI. the numbers of *Inhabited Houses* and of the *Population* in 1851 and 1861, with the increase or decrease of persons in the interval, are presented for each of the 631 Superintendent Registrars' Districts into which England is divided. These districts are for the most part co-extensive with single Poor Law Unions, or two or more unions combined, but in some instances they consist of places in which the relief of the Poor is regulated under local Acts or otherwise than by the Poor Law Amendment Act. A column is added in this table, showing the excess of *Registered Births* over *Deaths* in the ten years 1851-60; but, in comparing these numbers with the *ascertained* increase or decrease, it is necessary to bear in mind that the whole of the births are not registered. No birth can be recorded after six months, and registration is not compulsory; so, in the present state of the law, many births are not entered in the registers, especially in London and the large towns.

Table VII. is framed from the details in Table VI. The Superintendent Registrars' Districts are grouped together in eleven divisions, each comprising the whole, or nearly the whole, of the several counties named. In the columns showing the ascertained increase of population, and the excess of births over deaths in these divisions, may be traced the powerful stream of immigration into the principal centres of trade and the seats of mining and manufacturing industry. Thus in London (Division I.), where the excess of registered births over deaths was 253,989, the influx of persons from other parts had raised the actual increase to 440,798; in the Eastern Division (IV.), consisting of the counties of Essex, Suffolk, and Norfolk, the ascertained increase was only 28,220, while the natural increase or excess of births over deaths was 129,726.

In Table VIII. the *Houses* and *Population* in the principal *Cities* and *Boroughs* in 1861 are stated, chiefly on the authority of the Superintendent Registrars in whose districts the places are wholly or partly comprised. The municipal and parliamentary boundaries are frequently not conterminate with those of parishes

or other local subdivisions, and great care is required in assigning the population within their exact limits. It is probable that sufficient attention to questions of boundary has not been given in some cases by the local officers in preparing these statements within the limited time allowed them. The numbers, must, therefore, be received as approximations only, subject to revision hereafter at the Census Office. This table includes only corporate towns and boroughs returning members to Parliament; several large towns, consequently, do not appear in it.

Tables IX. and X., referring to the Channel Islands and the Isle of Man, require no explanation.

Table XI. relates to *Emigration.* According to the returns of the Emigration Commissioners, 2,249,355 emigrants sailed from the ports of the United Kingdom in the interval between the Census of March 31st, 1851, and the Census of April 8th, 1861. But 194,532 of the number were probably of foreign origin, leaving 2,054,823 emigrants from the population of the United Kingdom; of whom about 640,210 were of English origin, 183,627 were of Scotch origin, and 1,230,986 were of Irish origin.

TABLE I.—*England and Wales and Islands in the British Seas. Population Enumerated in England and Wales, and in the Islands in the British Seas, on April 8th,* 1861.

	Persons.	Males.	Females.
Population enumerated on April 8th, 1861*	20,205,504	9,825,246	10,380,258
England and Wales	20,061,725	9,758,852	10,302,873
Islands in the British Seas....	143,779	66,394	77,385

* "This number includes the part of the Army in England and the Channel Islands; also the Navy, Merchant Seamen, and others on board vessels in the ports, rivers, and creeks, on the 8th April. The part of the Army, Navy, and Merchant Seamen out of the country, is *not included.*"

TABLE II.—*England and Wales. Houses and Population enumerated in England and Wales in* 1861 *and* 1851.

1	2	3	4	5	6	7
Census Year.	Houses.			Population.		
	Inhabited.	Uninhabited.	Building.	Persons.	Males.	Females.
1861 (April 8th)	3,745,463	182,325	27,580	20,061,725	9,758,852	10,302,873
1851 (March 31st)	3,278,039	153,494	26,571	17,927,609	8,781,225	9,146,384
Increase in the interval between 1851 and 1861.....	467,424	28,831	1,009	2,134,116	977,627	1,156,489

TABLE III.—*England and Wales. Population at each of the Censuses* 1801-61, *including the Army, Navy, and Merchant Seamen Abroad, as well as at Home, belonging to England and Wales, Actual Increase, and Rates of Increase, in the Decennial Periods.*

	1801.	1811.	1821.	1831.	1841.	1851.	1861.
Population enumerated	9,156,171	10,454,529	12,172,664	14,051,986	16,035,198	18,054,170	20,223,746
Actual Increase in the Decennial Periods	1,298,358	1,718,135	1,879,322	1,983,212	2,018,972	2,169,576	
Decennial Rates of Increase per cent.	14	16	15	14	13	12	

TABLE IV.—*England and Wales. Number of Houses Enumerated at each of the Censuses* 1801-61.

	1801.	1811.	1821.	1831.	1841.	1851.	1861.
Houses—							
Inhabited ...	1,575,923	1,797,504	2,088,156	2,481,544	2,943,945	3,278,039	3,745,463
Uninhabited .	57,476	51,020	69,707	119,915	173,247	153,494	182,325
Building	No return	16,207	19,274	24,759	27,444	26,571	27,580

TABLE V.—*England and Wales. Houses and Population*

1	2	3	4	5	6	7	8
		1851 (March 31st).					
		Houses.			Population.		
Counties (Proper).	Area in Statute Acres.	Inhabited.	Uninhabited.	Building.	Persons.	Males.	Females.
England & Wales	37,324,915	3,278,039	153,494	26,571	17,927,609	8,781,225	9,146,384
ENGLAND.							
Bedford	295,582	24,673	661	127	124,478	59,941	64,537
Berks	451,040	33,481	1,397	197	170,065	84,927	85,138
Buckingham	466,932	33,196	1,206	98	163,723	81,074	82,649
Cambridge	523,861	37,226	1,629	195	185,405	92,699	92,706
Chester	707,078	85,260	4,341	845	455,725	222,386	233,339
Cornwall	873,600	67,987	4,544	347	355,558	171,636	183,922
Cumberland	1,001,273	36,763	1,545	239	195,492	96,244	99,248
Derby	658,803	59,371	2,498	453	296,084	147,737	148,347
Devon	1,657,180	98,387	6,014	751	567,098	269,583	297,515
Dorset	632,025	36,138	1,587	215	184,207	89,204	95,003
Durham	622,476	64,977	2,794	570	390,997	196,700	194,297
Essex	1,060,549	73,530	3,569	381	369,318	185,399	183,919
Gloucester	805,102	86,359	5,318	441	458,805	218,187	240,618
Hereford	534,823	23,890	1,191	77	115,489	58,114	57,375
Hertford	391,141	32,573	1,188	207	167,298	82,831	84,467
Huntingdon	230,865	13,285	632	64	64,183	31,933	32,250
Kent	1,041,479	107,748	5,460	1,267	615,766	307,041	308,725
Lancaster	1,219,221	349,938	17,420	3,463	2,031,236	991,090	1,040,146
Leicester	514,164	48,953	1,629	211	230,308	112,937	117,371
Lincoln	1,776,738	81,335	3,450	592	407,222	205,083	202,139
Middlesex	180,168	239,362	11,874	3,392	1,886,576	882,823	1,003,753
Monmouth	368,399	28,939	1,353	152	157,418	82,349	75,069
Norfolk	1,354,301	93,143	3,505	452	442,714	215,254	227,460
Northampton	630,358	43,942	1,538	227	212,380	105,984	106,396
Northumberland	1,249,299	47,737	2,064	386	303,568	149,515	154,053
Nottingham	526,076	55,019	1,502	250	270,427	132,263	138,164
Oxford	472,887	34,398	1,334	105	170,439	85,524	84,915
Rutland	95,805	4,588	153	14	27,983	11,801	11,182
Salop	826,055	45,648	2,062	116	229,341	114,340	115,001
Somerset	1,047,220	85,054	4,912	393	443,916	211,045	232,871
Southampton	1,070,216	75,238	3,543	613	405,370	202,014	203,356
Stafford	728,468	116,273	4,668	958	608,716	310,032	298,684
Suffolk	947,681	69,282	3,107	449	337,215	166,308	170,907
Surrey	478,792	108,822	5,770	1,540	683,082	325,041	358,041
Sussex	934,851	58,663	2,247	606	336,844	165,772	171,072

enumerated in the Counties (proper) in 1851 *and* 1861.

9	10	11	12	13	14	15	16	17
1861 (April 8th).						Increase or Decrease in the Number of Persons between 1851 and 1861.		Counties (Proper).
Houses.			Population.					
Inhabited.	Uninhabited.	Building.	Persons.	Males.	Females.	Increase.	Decrease.	
3,745,463	182,325	27,580	20,061,725	9,758,852	10,302,873	2,134,116	—	England and Wales
								ENGLAND.
27,419	705	142	135,265	63,780	71,485	10,787	—	Bedford
35,880	1,335	202	176,103	86,657	89,446	6,050	—	Berks
34,680	1,287	285	166,597	82,148	84,449	2,878	—	Buckingham
37,677	1,847	76	175,950	86,304	89,646	—	9,455	Cambridge
97,952	5,305	751	505,153	243,960	261,193	49,428	—	Chester
73,243	3,389	469	369,323	176,107	193,216	13,765	—	Cornwall
40,579	2,061	251	205,293	100,218	105,075	9,801	—	Cumberland
69,404	3,318	531	339,377	170,509	168,868	43,293	—	Derby
101,406	5,459	678	584,531	279,216	305,315	17,433	—	Devon
37,745	1,531	280	188,651	91,544	97,107	4,444	—	Dorset
84,877	4,164	588	509,018	258,343	250,675	118,021	—	Durham
81,220	4,092	538	404,644	202,713	201,931	35,346	—	Essex
93,900	4,711	565	485,502	228,312	257,190	26,697	—	Gloucester
25,271	800	171	123,659	62,753	60,906	8,170	—	Hereford
34,899	1,535	188	173,294	84,242	89,052	5,996	—	Hertford
13,733	497	53	64,297	31,713	32,584	114	—	Huntingdon
126,246	5,138	1,055	733,675	368,450	365,225	117,909	—	Kent
439,634	19,831	3,703	2,428,744	1,171,322	1,257,422	397,508	—	Lancaster
51,909	2,454	230	237,402	115,179	122,223	7,094	—	Leicester
86,688	4,104	523	411,997	204,200	207,797	4,775	—	Lincoln
279,831	13,407	3,240	2,205,711	1,020,191	1,185,580	310,195	—	Middlesex
33,101	2,031	264	174,670	89,533	85,137	17,252	—	Monmouth
96,951	4,929	354	435,422	208,910	226,512	—	7,292	Norfolk
48,547	2,114	283	227,727	112,963	114,764	15,347	—	Northampton
55,900	2,635	683	343,028	170,359	172,669	39,460	—	Northumberland
62,557	4,432	500	293,784	141,027	152,757	23,357	—	Nottingham
36,309	1,314	220	172,266	85,386	86,880	1,827	—	Oxford
4,652	175	17	21,851	10,899	10,960	—	1,124	Rutland
48,155	1,572	221	240,876	120,154	120,722	11,535	—	Salop
87,561	4,009	559	444,725	209,301	235,424	809	—	Somerset
86,494	3,707	637	481,495	246,076	235,419	76,125	—	Southampton
147,244	8,871	1,109	746,584	376,464	370,120	137,868	—	Stafford
73,067	3,662	219	336,271	164,239	172,032	—	964	Suffolk
130,563	4,611	1,384	830,685	392,786	437,899	147,603	—	Surrey
65,471	2,754	430	363,648	174,782	188,866	26,804	—	Sussex

Table V.—*England and Wales.*

1	2	3	4	5	6	7	8
Counties (Proper).	Area in Statute Acres.	1851 (March 31st).					
		Houses.			Population.		
		Inhabited.	Uninhabited.	Building.	Persons.	Males.	Females.
Warwick	563,946	96,731	4,596	992	475,013	232,411	242,602
Westmoreland	485,432	11,217	533	87	58,287	29,079	29,208
Wilts	865,092	51,667	2,250	176	254,221	125,728	128,493
Worcester	472,165	55,639	2,723	337	276,926	136,956	139,970
York (*East Riding*)	768,419	44,363	2,964	385	220,983	109,443	111,540
„ (*City*)	2,720	7,077	415	91	36,303	16,977	19,326
„ (*North Riding*)	1,350,121	44,446	2,343	224	215,214	106,710	108,504
„ (*West Riding*)	1,708,026	264,302	10,970	2,507	1,325,495	659,619	665,876
WALES.							
Anglesey	193,453	12,124	545	134	57,327	28,101	29,226
Brecon	460,158	12,221	731	74	61,474	31,314	30,160
Cardigan	443,387	14,978	544	70	70,796	32,961	37,835
Carmarthen	606,331	22,465	1,176	99	110,632	53,076	57,556
Carnarvon	370,273	18,005	590	132	87,870	42,978	44,892
Denbigh	386,052	19,124	812	136	92,583	46,708	45,875
Flint	184,905	14,041	798	80	68,156	34,452	33,704
Glamorgan	547,494	43,202	1,557	459	231,849	120,748	111,101
Merioneth	385,291	8,159	372	31	38,843	19,151	19,692
Montgomery	488,323	13,350	716	25	67,335	33,634	33,701
Pembroke	401,691	19,136	937	111	94,140	43,675	50,465
Radnor	272,128	4,614	217	28	24,716	12,693	12,023

Houses and Population enumerated (continued).

9	10	11	12	13	14	15	16	17
1861 (April 8th).						Increase or Decrease in the Number of Persons between 1851 and 1861.		Counties (Proper).
Houses.			Population.					
Inhabited.	Uninhabited.	Building.	Persons.	Males.	Females.	Increase.	Decrease.	
116,405	6,949	669	561,728	272,588	289,140	96,715	—	Warwick
11,809	603	76	60,809	30,665	30,144	2,522	—	Westmoreland
53,181	2,302	255	249,445	122,529	126,926	—	4,778	Wilts
62,893	3,791	355	307,601	150,989	156,612	30,675	—	Worcester
49,385	2,561	273	240,359	118,652	121,707	19,376	—	York (*East Riding*)
8,243	407	26	40,377	19,133	21,244	4,074	—	" (*City*)
50,306	2,942	381	244,804	121,845	122,959	29,590	—	" (*North Riding*)
316,061	18,555	1,998	1,507,511	740,696	766,815	182,016	—	" (*West Riding*)
								WALES.
12,361	477	63	54,546	26,191	28,355	—	2,781	Anglesey
12,929	916	80	61,627	31,004	30,623	153	—	Brecon
15,731	572	109	72,255	33,105	39,150	1,459	—	Cardigan
23,106	915	129	111,757	53,166	58,591	1,125	—	Carmarthen
20,261	660	251	95,668	46,752	48,916	7,798	—	Carnarvon
21,386	618	259	100,862	51,027	59,835	8,279	—	Denbigh
15,146	781	137	69,870	34,744	35,126	1,714	—	Flint
59,356	3,721	736	317,751	163,271	154,480	85,902	—	Glamorgan
8,499	357	45	38,888	19,085	19,803	45	—	Merioneth
13,518	413	115	67,075	33,843	33,232	—	260	Montgomery
19,416	837	202	96,093	45,683	50,410	1,953	—	Pembroke
4,706	162	22	25,403	13,144	12,259	687	—	Radnor

TABLE VI.—*England and Wales. Houses and Population in Superintendent Registrars' Districts on March* 31*st*, 1851, *and on April* 8*th*, 1861.

[Five only out of the eleven divisions are here inserted, viz., *London*, *South Midland*, *Northern*, *North Western*, and *York*. These five will present a fair average of the whole.—ED. *A. M.*]

No.	Superintendant Registrar's District.	Inhabited Houses. 1851.	Inhabited Houses. 1861.	Population Enumerated. 1851.	Population Enumerated. 1861.	Increase or Decrease in the Number of Persons between 1851 and 1861. Increase.	Increase or Decrease in the Number of Persons between 1851 and 1861. Decrease.	Excess of Registered Births over Registered Deaths in the Ten Years 1851-60.
	1. LONDON DIVISION.							
	MIDDLESEX (*part of*).							
1	Kensington (a)	17,151	25,854	120,004	186,463	66,459	—	14,209
2	Chelsea	7,591	8,318	56,538	63,423	6,885	—	4,302
3	St. George, Hanover Square	8,792	10,421	73,230	87,747	14,517	—	3,969
4	Westminster	6,642	6,880	65,609	67,676	2,067	—	4,235
5	St. Martin-in-the-Fields	2,307	2,283	24,640	22,636	—	2,004	457
6	St. James, Westminster	3,399	3,331	36,406	35,324	—	1,082	1,676
7	Marylebone	15,826	16,370	157,696	161,609	3,913	—	11,039
8	Hampstead	1,719	2,653	11,986	19,104	7,118	—	1,175
9	Pancras	18,584	21,928	166,956	198,882	31,296	—	19,649
10	Islington	13,528	20,676	95,329	155,291	59,962	—	15,881
11	Hackney	9,818	13,412	58,429	83,295	24,866	—	8,538
12	St. Giles	4,700	4,662	54,214	53,981	—	233	3,565
13	Strand (b)	3,949	3,815	44,417	42,956	—	1,461	2,516
14	Holborn (c)	4,311	4,125	46,621	44,861	—	1,760	2,022
15	Clerkenwell	7,224	7,086	64,778	65,632	854	—	8,236
16	St. Luke	6,349	6,368	54,055	56,997	2,942	—	11,619
17	East London	4,739	4,495	44,406	40,673	—	3,733	4,582
18	West London	2,670	2,616	28,833	27,144	—	1,689	—4,647
19	London, City (d)	7,297	6,367	55,932	45,550	—	10,382	2,861
20	Shoreditch	15,337	17,231	109,257	129,339	20,082	—	18,462
21	Bethnal Green	13,298	14,812	90,193	104,905	14,712	—	15,963
22	Whitechapel	8,812	8,667	79,759	78,963	—	796	3,676
23	St. George-in-the-East	6,146	6,187	48,376	48,878	502	—	5,291
24*a*	Stepney	7,392	7,465	54,173	56,567	2,394	—	5,083
24*b*	Mile End Old Town	8,867	10,768	56,602	73,064	16,462	—	9,473
25	Poplar	6,831	11,163	47,162	79,182	32,020	—	9,465

Note.—The Superintendent Registrars' Districts are *generally* co-extensive with Poor Law Unions or Single Parishes under the Poor Law Amendment Act. Where this is not the case, the Districts are thus distinguished:—* *two or more* Unions or Single Parishes with Boards of Guardians; X Poor Law Unions with Places, not under the Poor Law Amendment Act, annexed for Registration purposes; † Single Parishes or Incorporations of Parishes under *Local Acts;* § Gilbert's Incorporations, or Parishes still under the Act of 43rd Elizabeth. The population of Unions, where stated in the notes, refers to the Census of 1861.

(a) The Superintendent Registrar's District of Kensington is composed of Kensington Parish (pop. 70,614 in 1861), the Fulham Union (pop. 40,042 in 1861), and Paddington Parish (pop. 75,807 in 1861).

(b) Strand Union and the Middle Temple.

(c) The Holborn Union, Gray's Inn, and the Charterhouse.

(d) West London Union and the Inner Temple.

TABLE VI.—*England and Wales. Houses and Population* (continued).

Superintendent Registrar's District.		Inhabited Houses.		Population Enumerated.		Increase or Decrease in the Number of Persons between 1851 and 1861.		Excess of Registered Births over Registered Deaths in the Ten Years, 1851–60.
		1851.	1861.	1851.	1861.	Increase.	Decrease.	
No.								
	SURREY (*part of*).							
26	St. Saviour, Southwark	4,600	4,495	35,731	36,026	295	—	2,229
27	St. Olave, Southwark	2,360	2,214	19,375	19,053	—	322	— 5,447
28	Bermondsey	7,007	8,211	48,128	58,355	10,227	—	7,903
29	St. George, Southwark	6,992	7,234	51,824	55,509	3,685	—	5,337
30	Newington	10,458	12,815	64,816	82,157	17,341	—	10,015
31	Lambeth	20,447	23,001	139,325	162,008	22,683	—	19,884
32	Wandsworth	8,276	11,136	50,764	70,381	19,617	—	5,835
33	Camberwell	9,412	12,122	54,667	71,489	16,822	—	6,348
34	Rotherhithe	2,792	3,529	17,805	24,500	6,695	—	2,422
	KENT (*part of*).							
35	Greenwich	14,383	17,826	99,365	127,662	28,297	—	9,312
36	Lewisham	5,927	9,701	34,835	65,752	30,917	—	7,215
	3. SOUTH MIDLAND DIVISION.							
	6. MIDDLESEX (*extra metropolitan*).							
132	Staines	2,796	3,160	13,973	15,988	2,015	—	1,458
133	Uxbridge	3,524	4,018	19,475	23,139	3,664	—	2,121
134	Brentford	7,820	9,397	41,325	50,406	9,171	—	4,407
135	Hendon	2,811	3,384	15,916	19,207	3,291	—	1,802
136	Barnet	2,706	3,220	14,619	19,124	4,505	—	— 55
137	Edmonton	8,369	10,865	45,298	59,325	14,027	—	5,919
	7. HERTFORDSHIRE.							
138	Ware	3,259	3,341	16,482	16,516	34	—	2,029
139	Bishop Stortford	4,098	4,187	20,356	20,230	—	126	2,742
140	Royston * (a)	5,192	5,215	26,355	25,012	1,343	—	3,551
141	Hitchin	4,790	5,147	24,729	25,603	874	—	3,526
142	Hertford	2,762	2,912	15,090	15,301	211	—	1,549
143	Hatfield* (b)	1,569	1,676	8,499	8,400	—	99	1,062
144	St. Alban's	3,519	3,759	18,004	18,926	922	—	2,090
145	Watford	3,730	4,171	18,800	20,354	1,554	—	2,250
146	Hemel Hempstead	2,576	2,849	13,120	13,992	802	—	1,834
147	Berkhampstead	2,424	2,727	12,527	13,209	682	—	1,535
	8. BUCKINGHAMSHIRE.							
148	Amersham	3,774	3,820	18,637	18,238	—	399	2,112
149	Eton	3,942	4,362	21,490	22,356	866	—	2,180
150	Wycombe	6,630	7,198	33,562	35,139	1,576	—	4,663
151	Aylesbury	4,881	5,068	23,071	23,598	527	—	2,756
152	Winslow	1,969	2,016	9,376	9,207	—	169	1,169
153	Newport Pagnell	4,909	5,269	23,109	24,841	1,732	—	3,229
154	Buckingham	3,076	3,059	14,410	13,755	—	655	1,371

(a) Two Unions, viz., the Royston Union (pop. 18,623) and the Buntingford Union (pop. 6,389).
(b) Two Unions, viz., the Hatfield Union (pop. 6,389) and the Welwyn Union (pop. 2,211).

TABLE VI.—*England and Wales. Houses and Population* (continued).

No.	Superintendent Registrar's District.	Inhabited Houses. 1851.	Inhabited Houses. 1861.	Population Enumerated. 1851.	Population Enumerated. 1861.	Increase or Decrease in the Number of Persons between 1851 and 1861. Increase.	Increase or Decrease in the Number of Persons between 1851 and 1861. Decrease.	Excess of Registered Births over Registered Deaths in the Ten Years, 1851-60.
	9. OXFORDSHIRE.							
155	Henley	3,579	3,722	17,895	18,209	314	—	2,179
156	Thame	3,261	3,277	15,640	15,306	—	334	1,910
157	Headington	3,067	3,494	15,771	17,182	1,411	—	1,517
158	Oxford†	3,528	3,770	20,172	20,038	—	134	2,111
159	Bicester	3,277	3,879	15,562	15,553	—	9	2,097
160	Woodstock	2,960	3,074	14,453	14,236	—	217	1,564
161	Witney	4,897	5,117	23,558	23,231	—	327	2,527
162	Chipping Norton	3,506	3,633	17,427	17,307	—	120	2,121
163	Banbury	6,384	6,739	29,769	30,161	392	—	3,658
	10. NORTHAMPTONSHIRE.							
164	Brackley	2,961	3,021	13,747	13,464	—	283	1,588
165	Towcester	2,847	2,957	12,806	13,003	197	—	1,434
166	Potterspury	2,131	2,308	10,663	11,621	958	—	1,312
167	Hardingstone	1,883	2,146	9,157	9,928	771	—	1,464
168	Northampton	6,433	7,960	33,857	41,159	7,302	—	4,553
169	Daventry	4,527	4,679	21,926	20,600	—	1,326	2,493
170	Brixworth	3,229	3,328	14,771	15,358	587	—	1,632
171	Wellingborough	4,476	5,203	21,367	24,234	2,867	—	3,229
172	Kettering	3,770	4,081	18,097	19,086	989	—	2,887
173	Thrapston	2,711	3,071	12,841	14,064	1,223	—	2,063
174	Oundle	3,167	3,221	15,655	15,462	—	193	2,232
175	Peterborough	5,793	6,877	28,957	33,164	4,207	—	5,274
	11. HUNTINGDONSHIRE.							
176	Huntingdon	4,218	4,327	20,900	20,516	—	384	2,990
177	St. Ives	4,316	4,278	20,594	19,649	—	945	2,641
178	St. Neots	3,930	4,092	18,825	18,962	137	—	2,719
	12. BEDFORDSHIRE.							
179	Bedford	7,189	7,926	35,523	38,063	2,540	—	4,623
180	Biggleswade	4,609	5,022	23,436	25,389	1,953	—	3,763
181	Ampthill	3,221	3,521	16,542	16,970	428	—	2,177
182	Woburn	2,435	2,491	12,075	11,682	—	393	1,348
183	Leighton Buzzard	3,407	3,566	17,142	17,641	499	—	2,540
184	Luton	4,848	5,851	25,087	30,705	5,618	—	4,214
	13. CAMBRIDGESHIRE.							
185	Caxton	2,132	2,202	11,065	10,966	—	99	1,861
186	Chesterton	5,209	5,465	25,170	25,082	—	88	3,397
187	Cambridge	5,194	5,411	27,815	26,351	—	1,464	2,204
188	Linton	2,857	2,895	14,148	13,509	—	639	1,952
189	Newmarket	6,104	6,123	30,655	28,776	—	1,879	3,269
190	Ely	4,641	4,709	22,896	21,805	—	1,091	3,135
191	North Witchford	3,331	3,253	16,243	14,787	—	1,456	2,415
192	Whittlesey	1,678	1,594	7,687	6,966	—	721	1,115
193	Wisbeach	7,590	7,528	36,215	33,304	—	2,911	4,234

† Under Local Acts.

TABLE VI.—*England and Wales. Houses and Population* (continued).

No.	Superintendent Registrar's District.	Inhabited Houses. 1851.	Inhabited Houses. 1861.	Population Enumerated. 1851.	Population Enumerated. 1861.	Increase or Decrease in the Number of Persons between 1851 and 1861. Increase.	Increase or Decrease in the Number of Persons between 1851 and 1861. Decrease.	Excess of Registered Births over Registered Deaths in the Ten Years, 1851-60.
	4. EASTERN DIVISION.							
	14. ESSEX.							
194	West Ham	6,003	9,809	34,395	59,261	24,866	—	6,089
195	Epping	3,086	3,385	15,631	16,544	913	—	1,726
196	Ongar	2,281	2,262	11,855	11,314	—	541	1,466
197	Romford	4,758	5,419	24,607	26,920	2,313	—	3,722
198	Orsett	1,878	2,128	10,642	11,529	887	—	1,499
199	Billericay	2,526	2,763	13,787	15,013	1,226	—	1,218
200	Chelmsford	6,566	6,921	32,272	32,793	521	—	3,497
201	Rochford	3,069	3,382	15,838	18,270	2,432	—	1,910
202	Maldon	4,471	4,579	22,137	22,573	436	—	1,999
203	Tendring	5,339	5,784	27,710	27,094	—	616	2,926
204	Colchester	4,145	4,459	19,443	23,815	4,372	—	1,893
205	Lexden	4,628	4,966	21,666	22,922	1,256	—	3,593
206	Witham	3,306	3,452	16,099	16,324	225	—	1,931
207	Halstead	4,040	4,137	19,273	18,482	—	791	2,105
208	Braintree	3,770	3,841	17,561	17,169	—	392	1,988
209	Dunmow	4,249	4,373	20,498	19,759	—	739	2,625
210	Saffron Walden	4,260	4,294	20,716	19,721	—	995	2,604
	15. SUFFOLK.							
211	Risbridge	3,724	3,708	18,125	17,432	—	693	2,613
212	Sudbury	6,553	6,939	30,814	31,414	600	—	3,863
213	Cosford	3,790	3,884	18,107	17,374	—	733	2,154
214	Thingoe	3,827	3,864	19,014	18,221	—	793	2,540
215	Bury St. Edmund's†	2,752	2,847	13,900	13,316	—	584	922
216	Mildenhall	2,093	2,051	10,354	9,592	—	762	1,388
217	Stow	4,320	4,474	21,110	20,917	—	193	2,751
218	Hartismere	3,670	3,693	19,028	17,664	—	1,364	2,354
219	Hoxne	3,231	3,177	15,900	14,695	—	1,205	1,953
220	Bosmere	3,581	3,560	17,219	16,173	—	1,046	2,100
221	Samford	2,560	2,766	12,493	12,729	236	—	1,644
222	Ipswich	6,949	8,266	32,759	37,880	5,121	—	4,046
223	Woodbridge	5,044	5,068	23,776	22,748	—	1,028	2,339
224	Plomesgate	4,377	4,599	21,477	20,719	—	758	2,234
225	Blything	5,870	5,878	27,883	26,859	—	1,024	3,639
226	Wangford	2,922	2,977	14,014	13,620	—	394	1,408
227	Mutford†	4,061	5,193	20,163	24,069	3,906	—	3,020
	16. NORFOLK.							
228	Yarmouth	6,006	6,856	26,880	30,318	3,438	—	2,186
229	Flegg†	1,822	1,958	8,497	8,630	133	—	1,356
230	Tunstead†	3,402	3,346	15,614	14,515	—	1,099	1,480
231	Erpingham X (a)	4,698	4,895	21,722	20,875	—	847	2,100
232	Aylsham	4,323	4,300	20,007	19,050	—	957	2,142
233	St. Faith's	2,459	2,538	11,890	11,752	—	138	1,536

† Under Local Acts.
(a) The Erpingham Union (pop. 20,580 in 1861) and the Brinton Gilbert's Incorporation.

TABLE VI.—*England and Wales. Houses and Population* (continued).

No.	Superintendent Registrar's District.	Inhabited Houses. 1851.	Inhabited Houses. 1861.	Population Enumerated. 1851.	Population Enumerated. 1861.	Increase or Decrease in the Number of Persons between 1851 and 1861. Increase.	Increase or Decrease in the Number of Persons between 1851 and 1861. Decrease.	Excess of Registered Births over Registered Deaths in the Ten Years 1851-1860.
	16. NORFOLK—*contd.*							
234	Norwich†	14,988	17,012	68,195	74,414	6,219	—	5,731
235	Forehoe†	2,909	2,895	13,565	12,818	—	747	1,228
236	Henstead	2,375	2,432	11,545	11,290	—	255	1,291
237	Blofield	2,309	2,358	11,574	11,521	—	53	1,061
238	Loddon	3,138	3,137	15,095	14,242	—	853	1,678
239	Depwade	5,356	5,602	26,395	25,249	—	1,146	3,184
240	Guiltcross	2,626	2,580	12,744	11,541	—	1,203	1,298
241	Wayland	2,528	2,555	12,141	11,558	—	583	1,449
242	Mitford	6,147	6,149	29,389	28,018	—	1,371	3,181
243	Walsingham	4,610	4,735	21,883	21,115	—	768	2,257
244	Docking	3,716	3,794	18,148	17,579	—	569	1,968
245	Freebridge Lynn	2,689	2,833	13,557	13,474	—	83	2,014
246	King's Lynn	4,028	3,768	20,530	16,602	—	3,928	1,489
247	Downham	4,317	4,337	20,985	20,260	—	725	2,977
248	Swaffham	2,843	2,924	14,320	13,745	—	575	1,821
249	Thetford	3,855	4,048	19,040	18,711	—	329	2,215
	8. NORTH-WESTERN DIVISION.							
	33. CHESHIRE.							
452	Stockport	17,323	19,167	90,208	94,361	4,152	—	9,664
453	Macclesfield	12,845	13,386	63,327	61,517	—	1,810	5,5[illegible]
454	Altrincham	6,363	7,784	34,043	40,515	6,473	—	4,794
455	Runcorn	4,832	5,235	25,797	26,129	332	—	3,085
456	Northwich	6,221	6,938	31,202	33,331	2,129	—	5,015
457	Congleton	5,893	7,012	30,512	34,329	3,817	—	3,331
458	Nantwich	6,876	8,079	35,941	40,954	5,013	—	5,776
459	Great Boughton*† (a)	9,889	11,248	52,950	58,503	5,553	—	4,457
460	Wirrall	9,109	12,227	57,157	79,826	22,669	—	11,[illegible]
	34. LANCASHIRE.							
461	Liverpool†	35,293	37,045	258,236	269,733	11,497	—	[illegible]
462	West Derby* (b)	25,031	37,512	153,279	225,595	72,316	—	24,[illegible]
463	Prescot	9,323	13,120	56,074	73,112	17,038	—	11,[illegible]
464	Ormskirk	6,737	8,364	38,307	46,250	7,943	—	6,[illegible]
465	Wigan	13,965	17,422	77,539	94,559	17,020	—	13,[illegible]
466	Warrington	6,647	8,229	36,164	43,788	7,624	—	6,[illegible]
467	Leigh	6,015	7,618	32,734	37,697	4,963	—	4,[illegible]
468	Bolton	20,240	25,155	114,712	130,270	15,558	—	17,[illegible]

† Under Local Acts.

(a) The Superintendent Registrar's District of Great Boughton is composed of—

	Population, 1861.
Chester City (Local Act)	30,116
Great Boughton Union	18,795
Hardwarden Union	9,592

(b) The West Derby Union (pop. 156,327 in 1861) and Toxteth Park (pop. 69,268) a single parish under the Poor Law Amendment Act.

TABLE VI.—*England and Wales. Houses and Population* (continued).

Superintendent Registrar's District.		Inhabited Houses.		Population Enumerated.		Increase or Decrease in the Number of Persons between 1851 and 1861.		Excess of Registered Births over Registered Deaths in the Ten Years, 1851-60.
		1851.	1861.	1851.	1861.	Increase.	Decrease.	
No.	34. LANCASHIRE—*contd.*							
469	Bury	16,727	19,803	88,815	101,142	12,327	—	12,336
470	Barton-upon-Irwell	5,737	7,465	31,585	39,050	7,465	—	4,051
471	Chorlton	23,159	32,879	123,841	169,573	45,732	—	21,601
472	Salford	15,769	19,831	87,523	105,334	17,811	—	11,902
473	Manchester	36,701	43,003	228,433	243,615	15,182	—	16,929
474	Ashton	21,569	26,527	119,199	134,761	15,562	—	12,792
475	Oldham	16,485	21,951	86,788	111,267	24,479	—	13,046
476	Rochdale	14,200	18,407	72,515	91,758	19,243	—	9,763
477	Haslingden	9,489	13,402	50,424	69,782	19,358	—	8,615
478	Burnley	12,039	14,532	63,868	75,588	11,720	—	9,145
479	Clitheroe	4,238	4,146	22,368	20,476	—	1,892	2,029
480	Blackburn	15,916	21,888	90,738	119,937	29,199	—	14,861
481	Chorley	6,723	7,869	37,701	41,679	3,978	—	5,773
482	Preston	15,913	20,071	96,545	110,488	13,943	—	11,046
483	Fylde	3,930	4,581	22,002	25,681	3,679	—	2,977
484	Garstang	2,364	2,378	12,695	12,424	—	271	1,741
485	Lancaster X (a)	6,272	6,704	34,660	35,299	639	—	3,664
486	Ulverstone	5,676	6,844	30,556	35,734	5,178	—	5,168
	9. YORK DIVISION.							
	35. WEST RIDING.							
487	Sedbergh	917	915	4,574	4,396	—	178	536
488	Settle	2,774	2,608	13,762	12,529	—	1,233	1,408
489	Skipton X (b)	5,719	6,047	28,766	28,761	—	5	3,566
490	Pateley Bridge	1,928	1,992	9,334	9,534	200	—	1,105
491	Ripon	3,471	3,485	16,041	15,742	—	299	1,939
492a	Great Ouseburn	2,579	2,509	12,167	11,532	—	635	984
492b	Knaresborough	3,448	3,741	15,473	17,176	1,703	—	1,861
492c	Wetherby §	1,055	1,092	5,129	5,123	—	6	411
493	Otley §	5,588	6,069	28,541	29,508	967	—	3,304
494	Keighley X (c)	8,638	9,597	45,903	45,681	—	222	5,548
495	Todmorden	5,940	6,436	29,727	31,105	1,378	—	3,114
496	Saddleworth	3,367	3,808	17,799	18,630	831	—	2,037
497	Huddersfield	23,468	26,658	128,860	131,334	7,474	—	19,671
498	Halifax	23,626	27,016	120,958	128,667	7,709	—	15,917
499	Bradford X (d)	34,439	41,860	181,964	196,463	14,499	—	26,242
500	Hunslet §	18,776	23,895	88,679	109,949	21,270	—	15,331
501	Leeds	21,061	24,999	101,343	117,553	16,210	—	11,079

§ Parishes, &c. in Gilbert's Incorporations or under the Act of 43rd Elizabeth.

(a) The Lancaster Union (pop. 24,019 in 1861), Caton Gilbert's Incorporation, and Parishes under 43rd Elizabeth.

(b) The Skipton Union (pop. 28,398) and two townships not under the Poor Law Amendment Act.

(c) The Keighley Union (pop. 43,112) and a township under 43rd Elizabeth.

(d) The Bradford Union (pop. 106,218), the North Bierley Union (pop. 85,768), and a part of a Gilbert's Incorporation.

TABLE VI.—*England and Wales. Houses and Population* (continued.)

Superintendent Registrar's District.		Inhabited Houses.		Population Enumerated.		Increase or Decrease in the Number of Persons between 1851 and 1861.		Excess of Registered Births over Registered Deaths in the Ten Years, 1851-60.
		1851.	1861.	1851.	1861.	Increase.	Decrease.	
No.	**35. West Riding—** *contd.*							
502	Dewsbury	14,351	19,381	71,768	92,873	21,105	—	13,813
503	Wakefield X (a)	9,874	10,815	48,956	53,001	4,045	—	6,119
504a	Pontefract §	6,353	7,475	29,937	34,752	4,815	—	5,028
504b	Hemsworth	1,658	1,657	8,158	7,793	—	365	1,024
505	Barnsley	6,777	9,102	34,980	45,790	10,810	—	7,209
506	Wortley X (b)	6,255	7,577	32,012	38,509	6,497	—	6,598
507	Ecclesall Bierlow	7,587	13,009	37,914	63,618	25,704	—	8,651
508	Sheffield	20,785	26,658	103,626	128,929	25,303	—	16,281
509	Rotherham	6,686	9,055	33,082	44,330	11,248	—	6,826
510	Doncaster	7,302	8,625	34,675	39,341	4,666	—	5,020
511	Thorne	3,518	3,650	15,886	16,010	124	—	2,203
512	Goole	2,892	3,332	13,686	15,156	1,470	—	1,908
513	Selby X (c)	3,370	3,547	15,672	15,985	313	—	1,858
514	Tadcaster §	4,093	4,223	19,710	19,919	209	—	2,317
	36. East Riding *(with York).*							
515	York X (d)	10,469	11,941	54,324	59,967	5,643	—	5,306
516	Pocklington	3,250	3,420	16,098	16,710	612	—	2,128
517	Howden	3,060	3,159	14,436	15,076	640	—	2,419
518	Beverley	4,167	4,674	20,040	21,029	989	—	2,623
519	Sculcoates	9,187	11,136	44,719	51,942	7,223	—	6,236
520	Hull †	9,733	10,882	50,670	56,889	6,219	—	4,161
521	Patrington	1,872	2,010	9,407	9,680	273	—	1,316
522	Skirlaugh	1,868	1,952	9,279	9,653	374	—	1,297
523	Driffield	3,684	3,953	18,265	19,223	958	—	2,923
524	Bridlington	2,926	3,082	14,322	14,371	49	—	1,805
	37. North Riding.							
525	Scarborough	5,905	6,401	24,615	30,424	5,809	—	3,410
526	Malton	4,546	4,734	23,128	23,482	354	—	3,264
527	Easingwold	2,075	2,098	10,211	10,148	—	63	1,375
528	Thirsk	2,682	2,695	12,760	12,299	—	461	1,525
529	Helmsley X (e)	2,366	2,436	11,734	11,832	98	—	1,376
530	Pickering	2,083	2,197	9,978	10,547	569	—	1,373
531	Whitby	4,490	4,915	21,592	23,634	2,042	—	3,212
532	Guisbrough	2,544	4,367	12,202	22,125	9,923	—	3,173
533	Stokesley	2,984	2,226	9,387	10,381	994	—	1,309
534	Northallerton	2,650	2,661	12,460	12,174	—	286	1,399
535	Bedale	1,914	1,945	8,980	8,650	—	330	992
536	Leyburn	2,082	2,103	10,057	10,104	47	—	1,319
537	Askrigg ‖	1,248	1,229	5,635	5,649	14	—	757
538	Reeth	1,399	1,286	6,820	6,195	—	625	1,047
539	Richmond	2,894	2,842	13,846	13,456	—	390	1,464

† Under Local Act.
§ Parishes, &c., in Gilbert's Incorporations or under the Act of 43rd Elizabeth.
‖ Parishes, &c., not under the Poor Law Amendment Act.
(a) The Wakefield Union (pop. 51,092) and parts of Gilbert's Incorporations.
(b) Two Unions, viz., Wortley (pop. 24,091) and Penistone (pop. 14,418).
(c) The Selby Union (pop. 14,902) and parts of Gilbert's Incorporations.
(d) The York Union (pop. 59,157) and parishes not under Poor Law Amendment Act.
(e) Two Unions, viz., Helmsley (pop. 6,093) and Kirkby Moorside (pop. 5,739).

TABLE VII.—*England and Wales. Population in* 1851 *and* 1861, *ascertained Increase or Decrease in the interval, and Natural Increase or Excess of Registered Births over Deaths in the Ten Years,* 1851-60.

1 DIVISIONS.	2 Population Enumerated. 1851.	3 Population Enumerated. 1861.	4 *Ascertained Increase* in the interval between 1851 and 1861.	5 *Natural Increase* or Excess of Registered *Births* over Registered *Deaths*, in the Ten Years, 1851-60.
ENGLAND AND WALES	17,927,609	20,061,725	2,134,116	2,260,576
DIVISIONS.				
I. LONDON (within the limits of the Metropolis Local Government Act)	2,362,236	2,803,034	440,798	253,989
II. SOUTH EASTERN (Registration Districts chiefly in *Surrey* and *Kent* extra-metropolitan, *Sussex*, *Hunts*, *Berks*)	1,628,416	1,846,876	218,460	196,992
III. SOUTH MIDLAND (Registration Districts chiefly in *Middlesex* extra-metropolitan, *Herts*, *Bucks*, *Oxford*, *Northampton*, *Hunts*, *Beds*, *Cambridge*)	1,234,332	1,295,375	61,043	*155,742
IV. EASTERN (Registration Districts chiefly in *Essex*, *Suffolk*, *Norfolk*)	1,113,982	1,142,202	28,220	*129,726
V. SOUTH-WESTERN (Registration Districts chiefly in *Wilts*, *Dorset*, *Devon*, *Cornwall*, *Somerset*)	1,803,261	1,835,551	32,290	*200,673
VI. WEST MIDLAND (Registration Districts chiefly in *Gloucester*, *Hereford*, *Salop*, *Stafford*, *Worcester*, *Warwick*)	2,136,573	2,436,137	299,564	298,980
VII. NORTH MIDLAND (Registration Districts chiefly in *Leicester*, *Rutland*, *Lincoln*, *Nottingham*, *Derby*)	1,215,501	1,288,718	73,217	*161,763
VIII. NORTH-WESTERN (Registration Districts chiefly in *Cheshire*, *Lancashire*) .	2,488,438	2,934,722	446,284	308,022

TABLE VII.—*England and Wales. Population in* 1851 *and* 1861 (continued).

1 Divisions.	2 Population Enumerated. 1851.	3 Population Enumerated. 1861.	4 *Ascertained Increase* in the interval between 1851 and 1861.	5 *Natural Increase* or Excess of Registered *Births* over Registered *Deaths*, in the Ten Years, 1851-60.
IX. YORK (Registration Districts chiefly in *Yorkshire*) . . .	1,789,047	2,015,329	226,282	*256,117
X. NORTHERN (Registration Districts chiefly in *Durham*, *Northumberland*, *Cumberland*, *Westmoreland*)	969,126	1,151,281	182,155	152,694
XI. WELSH (Registration Districts chiefly in *Monmouthshire* and Wales) . .	1,186,697	1,312,500	125,803	*145,878

Note.—In the cases marked (*) the ascertained (actual) increase (col. 4) falls *short* of the natural (presumed) increase as shown by excess of Births over Deaths.

The following is a summary of *Occupations* as ascertained, 1851.

Registration Divisions and Union Counties (England and Wales).	Number of Persons Aged 20 Years and upwards, 1851.	Ratio per Cent. of Persons Aged 20 Years and upwards, occupied (in 1851) in: Mechanical Arts, Trade, and Domestic Service.	Agriculture.	Manufactures.	Mining and Mineral Works.
		Per Cent.	Per Cent.	Per Cent.	Per Cent.
I. The Metropolis	1,394,963	47·6	1·1	6·0	3·5
II. South-Eastern	887,134	30·7	20·8	2·5	2·4
III. South Midland	660,775	28·3	25·4	7·1	2·4
IV. Eastern	603,720	27·4	26·5	4·0	2·3
V. South-Western	978,024	28·6	23·3	4·6	5·6
VI. West Midland	1,160,387	29·1	15·5	5·2	12·6
VII. North Midland	654,679	31·8	21·7	6·4	5·3
VIII. North-Western	1,351,830	29·8	8·3	21·5	5·4
IX. York	961,945	25·2	14·3	17·5	7·3
X. Northern	521,460	27·7	16·1	4·2	12·4
XI. Welsh	641,680	21·8	25·7	2·5	12·4
England and Wales . .	9,816,597	31·0	16·1	8·4	6·3

TABLE VIII.—*Houses and Population of the Principal Cities and Boroughs,* 1851 *and* 1861.

*** "These Statements of the Number of Houses and Population in the principal Cities and Boroughs have been furnished generally by the respective Superintendent Registrars, and refer to the *Parliamentary or Municipal Limits*, or to both where co-extensive, as denoted by the letters placed after the names of places, viz., M., Municipal limits; P., Parliamentary limits; M. and P., Municipal and Parliamentary limits (co-extensive).

"The numbers of seamen and others on board vessels in the ports, as ascertained by the officers of Her Majesty's Customs, have been added to the general population. Several Boroughs, with respect to which the information could not be obtained, are omitted."

1	2	3	4	5	6
City or Borough.	Inhabited Houses.		Population.		
	1851.	1861.	1851.	1861.	
London, within the limits of the Metropolis Local Government Act*	305,933	362,890	2,362,236	2,803,034	—
Abingdon M. & P.	1,244	1,187	5,954	—	5,691
Andover M.	1,040	1,059	5,187	5,221	—
Andover P.	1,079	1,103	5,395	5,430	—
Arundel M. & P.	552	529	2,748	—	2,488
Ashburton P.	622	574	3,432	—	3,062
Ashton-under-Lyne M.	5,501	6,665	30,676	34,894	—
Ashton-under-Lyne P.	5,346	6,478	29,791	33,925	—
Banbury M.	769	790	4,026	4,055	—
Banbury P.	1,721	2,067	8,715	10,194	—
Bangor P.	1,228	1,336	6,338	6,795	—
Barnstaple M. & P.	2,116	2,187	11,371	—	10,738
Basingstoke M.	892	938	4,263	4,664	—
Bath M. & P.	7,744	8,021	54,240	—	52,528
Beccles M.	954	985	4,398	—	4,266
Bedford M. & P.	2,307	2,754	11,693	13,412	—
Berwick-on-Tweed M. & P.	2,028	1,872	15,094	—	13,254
Beverley M.	1,934	2,178	8,915	9,654	—
Beverley P.	2,183	2,423	10,058	10,901	—
Bewdley M.	718	691	3,124	—	2,905
Bewdley P.	1,582	1,516	7,318	—	6,786
Bideford M.	1,101	1,211	5,775	5,851	—
Birmingham M. & P.	45,844	59,090	232,841	295,955	—
Blackburn M. & P.	7,919	11,314	46,536	63,125	—
Bodmin M.	722	793	4,327	4,466	—
Bodmin P.	1,103	1,189	6,337	6,381	—

Note.—Col. 6 contains the cases in which in 1861 the population is *less* than in 1851.

* For the Metropolitan *Parliamentary* Boroughs, *see* Alphabetical List.

TABLE VIII.—*Houses and Population* (continued).

1		2	3	4	5	6
City or Borough.		Inhabited Houses.		Population.		
		1851.	1861.	1851.	1861.	
Bolton	M. & P.	10,394	13,348	61,171	70,396	—
Boston	M.	2,992	3,266	14,733	—	13,995
	P.	3,622	3,898	17,518	17,885	—
Bradford	M. & P.	19,002	22,537	103,778	106,218	—
Brecknock	M.	1,147	1,134	5,673	—	5,234
	P.	1,236	1,206	6,070	—	5,517
Brydgnorth	M.	1,227	1,629	6,172	6,569	—
	P.	1,516	1,891	7,610	7,892	—
Bridgwater	M. & P.	1,911	2,124	10,317	11,361	—
Bridport	M. & P.	1,468	1,570	7,566	7,672	—
Brighton	M.	*	12,708	*	77,693	—
	P.	10,843	13,946	69,673	87,311	—
Bristol	M. & P.	20,873	23,578	137,328	154,093	—
Buckingham	M.	809	827	4,020	—	3,847
	P.	1,717	1,716	8,069	—	7,625
Bury	P.	5,825	7,241	31,262	37,564	—
Bury St. Edmund's	M. & P.	2,752	2,847	13,900	—	13,316
Calne	M.	475	505	2,544	—	2,494
	P.	1,047	1,103	5,195	—	5,151
Cambridge	M. & P.	5,194	5,411	27,815	—	26,351
Canterbury	M. & P.	3,654	3,919	18,398	21,323	—
Cardiff	M.	2,565	4,666	18,351	32,421	—
Carlisle	M. & P.	3,956	4,878	26,310	29,436	—
Carmarthen	M. & P.	1,800	1,763	10,524	—	9,992
Carnarvon	M. & P.	1,723	1,820	8,674	—	8,530
Chatham	P.	4,337	5,227	28,424	36,177	—
Cheltenham	P.	6,356	7,016	35,051	39,590	—
Chester	M. & P.	5,173	5,980	27,766	31,101	—
Chesterfield	M.	1,455	1,928	7,101	9,835	—
Chichester	M. & P.	1,653	1,597	8,662	—	8,040
Chippenham	M.	309	300	1,707	—	1,603
	P.	1,139	1,345	6,283	7,075	—
Chipping Wycombe	M.	690	825	3,588	4,222	—
	P.	1,441	1,652	7,179	8,375	—
Christchurch	P.	1,543	1,837	7,475	9,386	—
Cirencester	P.	1,211	1,300	6,096	6,334	—
Clitheroe	M.	1,371	1,433	7,244	—	7,000
	P.	2,192	2,247	11,480	—	10,864
Cockermouth	P.	1,306	1,546	7,275	—	7,056
Colchester	M. & P.	4,145	4,459	19,443	23,815	—
Congleton	M.	2,146	2,631	10,520	12,338	—
Coventry	M.	7,657	8,994	36,208	40,937	—
	P.	7,783	9,158	36,812	41,647	—
Dartmouth	M. & P.	799	822	4,508	—	4,443
Daventry	M.	889	892	4,430	—	4,124
Deal	M.	1,465	1,590	7,067	7,531	—

* Brighton had not received a Charter of Incorporation in 1851.

TABLE VIII.—*Houses and Population* (continued).

1	2	3	4	5	6
City or Borough.	Inhabited Houses.		Population.		
	1851.	1861.	1851.	1861.	
Denbigh ..... M.	1,215	1,261	5,498	5,946	—
Derby ..... M. & P.	8,199	9,014	40,908	43,091	—
Devizes ..... M. & P.	1,292	1,389	6,554	6,639	—
Devonport ..... M.	3,789	4,193	38,180	50,504	—
Devonport ..... P.	4,961	5,435	50,159	64,798	—
Doncaster ..... M.	2,583	3,595	12,052	16,430	—
Dorchester ..... M. & P.	960	1,028	6,394	6,823	—
Dover ..... M. & P.	3,747	4,087	22,244	24,970	—
Droitwich ..... M.	582	672	3,125	—	3,123
Droitwich ..... P.	1,407	1,406	7,095	—	6,540
Dudley ..... P.	7,119	8,725	37,962	44,975	—
Durham ..... M. & P.	1,768	2,001	13,188	13,743	—
Evesham ..... M. & P.	918	991	4,605	4,680	—
Exeter ..... M.	5,109	5,404	32,818	33,737	—
Exeter ..... P.	6,499	6,885	40,688	41,791	—
Eye ..... M.	480	489	2,587	—	2,430
Eye ..... P.	1,374	1,406	7,531	—	7,039
Falmouth ..... M.	600	669	4,953	5,706	—
Faversham ..... M.	895	1,125	4,595	5,891	—
Finsbury ..... P.	37,427	44,363	323,772	386,844	—
Folkestone ..... M.	1,149	1,475	6,726	8,528	—
Frome ..... P.	2,122	2,069	10,148	—	9,523
Gateshead ..... M. & P.	3,520	4,394	25,568	33,589	—
Gloucester ..... M. & P.	2,843	2,769	17,572	—	16,320
Grantham ..... M.	904	940	5,375	—	4,954
Grantham ..... P.	1,968	2,256	10,873	11,116	—
Gravesend ..... M.	2,722	3,074	16,633	18,776	—
Great Grimsby ..... M.	1,634	2,302	8,860	11,067	—
Great Grimsby ..... P.	2,354	3,177	12,263	15,013	—
Great Marlow ..... P.	1,211	1,287	6,523	—	6,505
Great Yarmouth ..... M. & P.	6,886	7,836	30,879	34,803	—
Greenwich ..... P.	15,401	19,500	105,784	139,286	—
Guildford ..... M. & P.	1,176	1,464	6,740	8,032	—
Halifax ..... M. & P.	6,528	7,820	33,582	37,015	—
Hartlepool ..... M.	1,466	1,723	9,503	12,205	—
Harwich ..... M. & P.	751	812	4,451	5,062	—
Hastings ..... M.	2,471	3,327	16,966	23,098	—
Hastings ..... P.	2,477	3,328	17,011	23,103	—
Helston ..... M.	672	745	3,355	3,841	—
Helston ..... P.	1,459	1,714	7,328	8,657	—
Hereford ..... M. & P.	2,426	No ret.	12,108	15,625	—
Hertford ..... M. & P.	1,150	1,230	6,605	6,769	—
Holyhead ..... P.	1,040	1,256	5,622	6,190	—
Honiton ..... M. & P.	692	714	3,427	—	3,301
Horsham ..... P.	1,081	1,267	5,947	6,747	—
Huddersfield ..... P.	5,739	6,933	30,880	34,874	—
Hull ..... M. & P.	16,634	20,581	84,690	98,994	—
Huntingdon ..... M.	725	740	3,882	—	3,816
Huntingdon ..... P.	1,244	1,284	6,219	6,254	—

TABLE VIII.—*Houses and Population* (continued).

1		2	3	4	5	6
City or Borough.		Inhabited Houses.		Population.		
		1851.	1861.	1851.	1861.	
Hythe	M.	486	588	2,857	2,998	—
	P.	2,261	2,841	13,164	21,372	—
Ipswich	M. & P.	6,979	8,284	32,914	37,949	—
Kendal	M. & P.	2,457	2,582	11,829	12,028	—
Kidderminster	M. & P.	3,656	3,349	18,462	—	15,398
King's Lynn	M. & P.	3,845	3,641	19,355	—	16,071
Kingston-on-Thames	M.	1,119	1,519	6,279	9,114	—
Knaresborough	P.	1,326	1,320	5,536	—	5,404
Lambeth	P.	39,154	45,252	251,345	298,032	—
Lancaster	M.	2,583	2,680	14,604	—	14,478
	P.	2,891	No ret.	16,168	—	15,996
Launceston	M.	562	624	3,397	—	2,773
	P.	1,051	1,040	6,005	—	5,139
Leeds	M. & P.	36,165	44,646	172,270	207,153	—
Leicester	M. & P.	12,805	14,680	60,584	68,052	—
Leominster	M. & P.	1,118	1,158	5,214	5,660	—
Lewes	P.	1,747	No ret.	9,533	9,709	—
Lichfield	M. & P.	1,412	1,471	7,012	—	6,872
Lincoln	M. & P.	3,450	4,332	17,536	20,995	—
Liskeard	M.	623	781	4,386	4,689	—
	P.	965	1,170	6,204	6,704	—
Liverpool	M. & P.	54,310	65,999	375,955	443,874	—
London, City	M. & P.	14,580	13,373	127,869	—	112,247
Louth	M.	2,209	2,401	10,467	10,568	—
Ludlow	M.	1,003	1,076	4,691	5,178	—
	P.	1,133	1,264	5,376	6,034	—
Lyme Regis	M.	522	507	2,661	—	2,413
	P.	708	687	3,516	—	3,206
Lymington	M.	487	440	2,651	—	2,416
	P.	1,029	1,025	5,282	—	5,152
Macclesfield	M. & P.	8,312	8,345	39,048	—	36,095
Maidstone	M.	3,667	4,111	20,740	22,984	—
	P.	3,676	4,119	20,801	23,026	—
Maldon	M.	902	1,014	4,558	4,798	—
	P.	1,179	1,329	5,888	6,274	—
Malmesbury	P.	1,420	1,425	6,998	—	6,883
Malton	P.	1,545	1,696	7,661	8,072	—
Manchester	M.	50,731	61,662	303,382	338,346	—
	P.	53,204	65,553	316,213	357,604	—
Marlborough	M.	608	644	3,908	—	3,684
	P.	781	820	5,135	—	4,893
Marylebone	P.	40,513	48,027	370,957	436,298	—
Merthyr Tydfil	P.	11,684	16,147	63,080	83,844	—
Morpeth	P.	1,467	2,325	10,012	13,796	—
Newark	M. & P.	2,370	2,569	11,330	11,562	—
Newbury	M.	1,362	1,337	6,574	—	6,161
Newcastle-under-Lyme	M. & P.	2,153	2,657	10,569	12,938	—
Newcastle-on-Tyne	M. & P.	10,441	14,222	87,784	109,291	—
Newport (I. Wight)	M. & P.	1,550	1,591	8,047	—	7,934

TABLE VIII.—*Houses and Population* (continued).

1	2	3	4	5	6
City or Borough.	Inhabited Houses.		Population.		
	1851.	1861.	1851.	1861.	
Newport (Monmouth) ..M. & P.	2,908	3,695	19,323	23,248	—
Northallerton ..... P.	1,064	1,059	4,995	—	4,755
Northampton .....M. & P.	4,886	6,132	26,657	32,813	—
Norwich .....M. & P.	14,988	17,012	68,195	74,414	—
Nottingham .....M. & P.	11,549	15,307	57,407	74,531	—
Oldham ..... M.	9,900	13,936	52,820	72,334	—
Oldham ..... P.	13,658	18,463	72,357	94,337	—
Oswestry ..... M.	995	1,129	4,817	5,414	—
Oxford .....M. & P.	4,933	5,230	27,843	27,561	—
Pembroke .....M. & P.	1,792	2,144	10,107	15,051	—
Penzance ..... M.	1,878	1,968	9,214	9,414	—
Peterborough ..... P.	1,755	2,509	8,672	11,732	—
Petersfield ..... P.	1,072	1,135	5,550	5,655	—
Plymouth .....M. & P.	5,171	6,081	52,221	62,823	—
Pontefract ..... M.	1,069	1,120	5,106	5,340	—
Pontefract ..... P.	2,496	2,624	11,515	11,678	—
Poole .....M. & P.	1,903	2,036	9,255	9,745	—
Portsmouth .....M. & P.	12,825	15,803	72,096	94,546	—
Preston .....M. & P.	11,348	15,045	69,542	82,961	—
Reading .....M. & P.	4,098	4,851	21,456	24,965	—
Reigate .....M. & P.	792	1,581	4,927	9,975	—
Richmond ..... M.	843	863	4,106	4,290	—
Richmond ..... P.	1,032	1,046	4,969	5,134	—
Ripon .....M. & P.	1,345	1,382	6,080	6,172	—
Rochdale ..... P.	5,829	7,701	29,195	38,164	—
Rochester .....M. & P.	2,549	3,171	14,938	16,672	—
Rye ..... M.	726	784	4,071	—	3,794
Rye ..... P.	1,557	1,682	8,541	—	8,202
Saffron Walden ..... M.	1,173	1,246	5,911	—	5,474
St. Alban's ..... M.	1,361	1,500	7,000	7,675	—
St. Ives ..... M.	1,403	1,453	6,525	7,019	—
St. Ives ..... P.	2,003	2,116	9,872	10,354	—
Salford ..... M.	11,447	19,286	63,850	102,414*	—
Salford ..... P.	15,342	19,286	85,108	102,414	—
Salisbury .....M. & P.	2,311	2,347	11,657	11,833	—
Sandwich ..... M.	602	633	2,966	—	2,893
Sandwich and Deal .... P.	2,474	2,658	12,710	13,701	—
Scarborough .....M. & P.	2,838	3,987	12,915	18,380	—
Shaftesbury ..... P.	1,894	1,928	9,404	—	8,987
Sheffield .....M. & P.	27,099	38,083	135,310	185,157	—
Shrewsbury .....M. & P.	3,900	4,360	19,681	22,055	—
Southampton .....M. & P.	5,749	7,718	35,305	46,970	—
South Molton ..... M.	929	868	4,482	—	3,830
South Shields .....M. & P.	3,439	4,436	28,974	35,223	—
Southwark .... P.	23,751	25,683	172,863	193,443	—

* Since the Census of 1851, the Municipal Borough has been enlarged, and its boundaries are now the same as those of the Parliamentary Borough.

TABLE VIII.—*Houses and Population* (continued).

1	2	3	4	5	6
City or Borough.	Inhabited Houses.		Population.		
	1851.	1861.	1851.	1861.	
Stafford ..... M. & P.	1,977	2,241	11,829	12,487	—
Stamford ..... M. & P.	1,616	1,661	8,933	—	8,044
Stockport ..... M. & P.	10,568	11,286	53,835	54,681	—
Stoke-upon-Trent ..... P.	15,562	19,870	84,027	101,302	—
Stratford-upon-Avon .... M.	694	787	3,372	3,672	—
Stroud ..... P.	8,182	8,196	36,535	—	35,513
Sudbury ..... M.	1,280	1,476	6,043	6,878	—
Sunderland ..... M.	7,975	10,222	63,897	80,324	—
Sunderland ..... P.	8,519	11,216	67,394	85,748	—
Swansea ..... M.	6,001	7,931	31,461	42,581	—
Tamworth ..... M.	826	935	4,059	4,326	—
Tamworth ..... P.	1,760	2,118	8,655	10,202	—
Taunton ..... P.	2,645	2,905	14,176	14,660	—
Tavistock ..... P.	1,009	1,134	8,086	8,804	—
Tewkesbury ..... M. & P.	1,274	1,265	5,878	—	5,876
Thetford ..... M. & P.	844	898	4,075	4,208	—
Thirsk ..... P.	1,154	1,208	5,319	5,351	—
Tiverton ..... M. & P.	2,181	2,209	11,144	—	10,444
Totnes ..... M. & P.	728	790	4,419	—	3,993
Tower Hamlets ..... P.	75,710	88,664	539,111	647,585	—
Truro ..... M. & P.	2,194	2,389	10,733	11,336	—
Tynemouth ..... M. & P.	4,295	4,883	29,170	33,991	—
Wakefield ..... M.	4,391	4,779	22,065	23,181	—
Wakefield ..... P.	4,390	4,779	22,057	23,199	—
Wallingford ..... M.	522	553	2,819	—	2,786
Wallingford ..... P.	1,635	1,662	8,064	—	7,794
Walsall ..... M. & P.	4,521	7,452	25,680	37,762	—
Wareham ..... P.	1,351	1,394	7,218	—	6,977
Warrington ..... M.	4,285	4,985	22,894	25,953	—
Warrington ..... P.	4,380	5,159	23,363	26,852	—
Warwick ..... M. & P.	2,229	2,273	10,973	—	10,589
Wells ..... M. & P.	906	860	4,736	—	4,648
Westbury ..... P.	1,535	1,519	7,029	—	6,495
Westminster ..... P.	24,755	26,430	241,611	253,985	—
Weymouth and Melcombe Regis .... M. & P.	1,722	1,867	9,458	11,383	—
Whitby ..... P.	2,239	2,465	10,989	12,054	—
Whitehaven ..... P.	4,627	3,818	18,916	—	18,842
Wigan ..... M. & P.	5,686	6,698	31,941	37,657	—
Wilton ..... P.	1,721	1,799	8,607	8,674	—
Winchester ..... M. & P.	2,077	2,385	13,704	14,784	—
Windsor ..... M. & P.	1,417	1,570	9,596	9,827	—
Wisbeach ..... M.	2,141	2,086	10,594	—	9,275
Wolverhampton ..... M.	9,184	11,785	49,985	60,858	—
Wolverhampton ..... P.	22,284	28,458	119,748	147,646	—
Woodstock ..... P.	1,623	1,664	7,983	—	7,820
Worcester ..... M. & P.	5,695	6,317	27,528	31,123	—
York ..... M.	7,077	8,243	36,303	40,377	—
York ..... P.	7,778	9,162	40,359	45,326	—

TABLE IX.—*Islands in the British Seas. Houses and Population Enumerated in the Islands in the British Seas on April 8th,* 1861.

	Houses.			Population.		
	Inhabited.	Uninhabited.	Building.	Persons.	Males.	Females.
Islands in the British Seas	23,000	1,349	170	143,779	66,394	77,385
Isle of Man	8,948	502	90	52,339	24,544	27,795
Island of Jersey	8,327	381	46	56,078	25,304	30,774
Island of Guernsey and Adjacent Islands	5,725	466	34	35,362	16,546	18,816

TABLE X.—*Population in the Islands in the British Seas.*

	1821.	1831.	1841.	1851.	1861.
Islands in the British Seas	89,508	103,710	124,040	143,126	143,779
Isle of Man	40,081	41,000	47,975	52,387	52,339
Island of Jersey	28,600	36,582	47,544	57,020	56,078
" Guernsey (with Herm and Jethou)	20,339	24,540	26,698	29,806	29,846
" Alderney	No return	1,045	1,088	3,333	4,933
" Sark	488	543	785	580	583

Note.—Between 1831 and 1841 the population of the islands collectively increased about 18 per cent., and between 1841 and 1851 about 15 per cent. Between 1851 and 1861 a slight decrease took place in the Isle of Man and Jersey; while Guernsey would also have shown a decrease but for the circumstance of the military force in that island as well as in Alderney having been larger in 1861 than at the previous Census.

TABLE XI.—*Emigration (England and Wales). Return by the Government Emigration Board, showing the Number of Emigrants from the United Kingdom during the Ten Years* 1851 *to* 1861.

Year.	Total Number of Emigrants.	English.	Scotch.	Irish.	Foreigners.	Not distinguished.
1851.....	335,966		No	return.		
'52.....	368,764					
'53.....	329,937	62,915	22,605	192,609	31,459	20,349
'54.....	323,429	90,966	25,872	150,209	37,704	18,678
'55.....	176,807	57,132	14,037	78,854	10,554	16,230
'56.....	176,554	64,527	12,033	71,724	9,474	18,796
'57.....	212,875	78,560	16,253	86,238	12,624	19,200
'58.....	113,972	39,971	11,815	43,281	4,560	14,345
'59.....	120,432	33,930	10,182	52,981	4,442	18,897
'60.....	128,469	26,421	8,733	60,835	4,536	27,944
Total.....	2,287,205	454,422	121,530	736,731	115,353	154,439

Note.—It appears from the above Table that 2,287,205 emigrants sailed from the ports of the *United Kingdom* in the ten years 1851-60. Of that number, 1,582,475 were emigrants in the eight years 1853-60; and 454,422 were found to be *English*, 121,530 *Scotch*, 736,731 *Irish*, and 115,353 *foreigners*. The origin of 154,439 emigrants was not ascertained. If we assume that these unascertained numbers should be distributed proportionally over the rest, the numbers of the several classes will be shown in the annexed Table.

The emigrants in the first three months of 1851 were 56,584, consequently the number in the two years 1851-2, namely 704,730, have to be reduced to 648,146, to obtain the number of emigrants in the interval extending from 1st April, 1851, to 31st December, 1852. The nationality was not then distinguished, but a large proportion of the emigrants was of Irish origin, and the best estimate will be made by assuming that the proportions were the same as in 1853; the results appear in the table. 18,734 emigrants sailed from the United Kingdom in the portion of 1861 terminating on April 7th, the Census day, and they have been distributed in classes in the proportions ascertained to exist in 1860.

Estimated Number of Four Classes of Emigrants in the Interval between the Census of 1851 *and* 1861.

Years.	Totals.	English.	Scotch.	Irish.	Foreigners.
Total.....	2,249,355	640,210	183,627	1,230,986	194,532
1851 (April 1st)-1852	648,146	131,718	47,325	403,246	65,862
1858-60	1,582,475	503,568	134,674	816,408	127,825
1861 (to April 7th)	18,734	4,924	1,628	11,337	845

It may be inferred from the official returns that the emigration from the United Kingdom was not less than—

717,913 in the interval of the Censuses 1831 and 1841;
1,692,063 ditto ditto 1841 and 1851; and
2,249,355 ditto ditto 1851 and 1861.

On the Rule for finding the Value of an Annuity on Three Lives. By AUGUSTUS DE MORGAN, *Professor of Mathematics in University College.**

THE rule which was given by Thomas Simpson for finding the value of an annuity on three lives is as follows:—let A, B, and C be the three lives, and let K be a single life, the annuity on which is equal to an annuity on the joint lives of B and C; then the value of an annuity on the joint lives of A and K is the required annuity on the joint lives of A, B, and C. The life A should be the youngest of the three.

No reason has ever been given for this rule, simple as it is; and the only considerations of which I know, partaking of the nature of demonstration, and connected with it, are those by which Mr. Milne has shown that A should be the youngest of the three, and also that, instead of interpolating the age of K to a fraction of a year, the complete year of age next above the real age of K should be chosen.

If the rule be examined by the hypothesis of equal decrements, and if $1-\alpha$, $1-2\alpha$, &c., be the chances of A's living one, two, &c. years, and $1-\beta$, &c., $1-\gamma$, &c., the same for B and C, the error of the rule, or at least its order, will be represented by the fraction

$$\frac{2\alpha\beta\gamma}{r^4}, \text{ (r=interest of £1 for one year),}$$

whenever α, β, γ are so small that $\alpha\beta\gamma$ is small compared with r^4. As far as my investigations go, I doubt if, on the hypothesis of equal decrements, the rule would be a good approximation when $\alpha\beta\gamma$ is not a small fraction of r^4; and this may account for De Moivre not having noticed a rule which would, it might be imagined, present itself for trial at least to any one who was considering the subject for the first time.

A much more satisfactory account of this rule can be given from the expression for the law of human mortality which was given to the Royal Society, by Mr. Benjamin Gompertz, in 1825. That law is as follows:—the number living at x years from a given age (upwards of 10) may be represented by Ag^{q^x}, where Ag is the number living at the given age.

The constants do not retain their value during the whole of life—Mr. Gompertz, for instance, finds one set of values repre-

* This is the paper in the *Phil. Mag.* (Nov., 1839) quoted by Professor De Morgan at page 181, vol. viii., of this *Journal*, and referred to by Mr. Makeham in the Note at page 361, vol. ix.—ED. *A. M.*

senting the Carlisle tables very nearly from 10 to 60, and another from 60 to 100. The closeness with which this theory represents the Northampton, Sweden, Carlisle, and Deparcieux's tables may be seen in the paper cited; and it adds not a little to the speculative value of the formula, that its author has deduced it from so simple a principle as that *the power of the human constitution to oppose decay loses equal proportions in equal times.*

If Mr. Gompertz's theory were accurately true, with a uniform value of the constants, throughout the whole remainder of life, Thomas Simpson's rule would no longer be an approximation, but an exact method. Let Ag^{q^x} be the number living at the age of A, and let B be y years older than A, and C z years older than B. Consequently the numbers now alive in the table at the ages of B and C will be

$$Ag^{q^{x+y}} \text{ and } Ag^{q^{x+y+z}}, \text{ or } Ah^{q^x} \text{ and } Ak^{q^x},$$

where $h=g^{q^y}$, $k=g^{q^{y+z}}$.

Hence the chances of the parties living t years are

$$g^{q^x(q^t-1)},\ h^{q^x(q^t-1)},\ k^{q^x(q^t-1)},$$

and, v being the present value of £1 to be received in one year, the value of the annuity is

$$(ghk)^{q^x(q-1)}\,.\,v+(ghk)^{q^x(q^2-1)}\,.\,v^2+\ .\ .\ .\ .\ .\ .\quad (1)$$

Determine w and l from $hk=g^{q^w}=l$, then it will be evident that, if (1) be called $\phi(ghk)$, we have $\phi(ghk)=\phi(gl)$, the annuity on the joint life of A and another, whose single life is worth $\phi(l)$ or $\phi(hk)$, the values of an annuity on the joint lives of B and C.

The various causes of error in the preceding, arising from the change in the values of the constants, will readily present themselves to those who have studied the subject.

It is not, of course, necessary that the progression of powers should be precisely that of Mr. Gompertz, and the following theorem may easily be demonstrated. If a_n be the chance of a life living n years, and if

$$a_n=(a_1)^{p_n},$$

where p_n is not a function of a, then the annuity on any number of lives is not altered in value if, instead of any part of those lives, a single life of equivalent value be substituted.

CORRESPONDENCE.

MR. EDMONDS: COLLEGE LIFE.

To the Editor of the Assurance Magazine.

My dear Sir,—In your last Number (p. 334) Mr. Edmonds has made some statements which, he says, it is "of importance that the reader should know." Unless he intend to insinuate that my exposure of his unfair treatment of Mr. Gompertz is the result of an old grudge, I cannot imagine how the matters he brings forward can concern any reader. But be the importance what it may, it is just as necessary that the reader should have it correctly as that he should have it at all. I therefore rectify mis-statements, destroy insinuations, and fill up suppressions.

1. That we were "not strangers" to one another at college—that we were of the same college, class-room, &c. *Not strangers* may mean anything: the fact is that we had not the slightest acquaintance, and that my full conviction is that we never exchanged a word during our joint college life (1823–1826). I cannot swear that no single word ever passed, but I have not the slightest recollection of it, nor belief in it.

2. That, in the third year, the (now) Astronomer-Royal was our lecturer; and that, towards the end of the course, the class was reduced to our two selves. The first statement is correct; my memory did not agree with the second: but as my old friend the Astronomer-Royal is well known for a preserver of documents, I wrote to him, and, begging he would excuse my saying why I wanted to know, I asked him what he could tell me about the attendance on his class of 1824–25, towards the end of the academical year. He replied that his *side* of the college had 20 students in the third year—that the attendance at the end was about $4\pm$; and he gave me the names of *eight* students (myself and Mr. Edmonds being two of them) from whom this average of four was recruited. Among these were three old friends of my own, whose attendance my memory tells me was tolerably regular.

3. That at a personal interview (in 1832) I expressed general approbation of Mr. Edmonds's work, but objected to one sentence, which I recommended him to cancel. That I did express such general approbation is perfectly true; but it is of importance the reader should know that I then had no knowledge of what Mr. Gompertz had done, except from Mr. Edmonds's "suppressive mention." The matter to which I objected, which should have been produced, since it is of at least as much importance to the reader as the fact that the Astronomer-Royal was our lecturer, is as follows:—Mr. Edmonds, finding that Mr. Finlaison's observations did not suit his theory, explained the discrepancy by saying (p. xiv.) that Mr. Finlaison was "a person whose qualifications for the task undertaken are unknown to the public." I thought this sentence, from a young gentleman then first appearing in the subject, savoured not a little of presumption; and, out of kindness to Mr. Edmonds, I endeavoured to induce him to cancel it: this he declined, as he says. He adds that on a subsequent occasion of communication with me he found that my favourable disposition towards him had ceased. I do not remember this second communication: if it took place after I had seen Mr. Gompertz's paper, I have no doubt that I made Mr. Edmonds feel the change of disposition which he describes.

4. That I have been, for 28 years, "secretly writing and speaking" against Mr. Edmonds. Private conversation is not "secret speaking" unless express means be used to make it so. I have never spoken about Mr. Edmonds in any but the usual way. As to the writing, the article "Mortality" in the *Penny Cyclopædia*, and also its authorship, were neither secret nor secrets.

One or two sentences have reference to the subject-matter of my accusation, about which I hold it needless to say any more. Should Mr. Edmonds favour you with any more college life, or other "secret" matter, I have no doubt you will insist on his making it appear what the reader's concern with it is before you insert it. One thing, however, I must particularly beg of you, namely, that if Mr. Edmonds should dare to make any definite assertion of the existence of any private grudge on my part, supported by any pretence of proof, no matter how trivial, you will print it all at once. Such publication will be due to me and due to the subject. Both of us have a right to any exposure of this kind which Mr. Edmonds may choose to make of himself.

Yours very truly,

August 6th, 1861. A. DE MORGAN.

P.S.—If you read Cooper's novels, you may remember that once, when somebody's Indian assailants were dispersed by the arrival of assistance, the party assailed did not think it worth while to send a shot after his enemies until he happened to see two of them in a line, when he could not resist the temptation of finishing them both with one barrel, merely from love of practice. I make a sort of imitation of this proceeding: there are two similar fallacies in one page, if not in one line, which both go down by one bullet. The form of logic under which they both come is as follows:—A is B, or rather it would have been if it had not been C. The first is, 'We attended the same mathematical class for three years—or, rather, we should have done so if I had not omitted to read mathematics during the whole of the second year." The second is contained in the last sentence of the page: speaking of the explanation of 1860, Mr. Edmonds proceeds, "which explanation, as Mr. De Morgan himself admits, would have exonerated me from the charge of 'unfair suppression' if it had been given in the year 1832." That is, I have admitted that the suppression of 1832 would not have existed if the statement made in 1860 had been made in 1832. Mr. Edmonds is correct—I did admit it; certainly if the proceeding of 1832 had been *description*, it would not have been *suppression*. But even here Mr. Edmonds suppresses something, and alters the grammar of the rest. Once more I have recourse to parallel columns.

The original (Jan. 1861, p. 214).	*The misquotation* (July, 1861, p. 333).
"If Mr. Edmonds had given all the description he has now given, weak as it is, there would have been foolish and unfounded self-assertion, but at least there would not have been suppression."	"If Mr. Edmonds had given all the description which he has now given . . . . there would have been no suppression."

MR. FINLAISON'S REPORT AND THE ENGLISH LIFE TABLE.

To the Editor of the Assurance Magazine.

SIR,—You will, no doubt, accord me a small space in your *Magazine* to enable me to offer a few words of explanation on the subject of a letter in your last Number from Mr. Bailey.

That gentleman, I consider, has quite lost sight of the scope and object of the letter which has become the subject of his criticism. I did not pretend to undertake an exhaustive analysis of Mr. Finlaison's elaborate Report: on the contrary, my letter was written at the period when the decennial census was on the very point of being taken, with the expressed object only of referring to such portions of the Report as had a bearing on the census question—with the view of showing in how many different ways the public might be affected by false or incorrect census returns.

I propose to offer a few very short remarks upon each head of Mr. Bailey's letter.

1. I am not answerable for Mr. Finlaison's "omission of eight words," by which the meaning of a passage in the Registrar-General's Report is stated to be altered.

Surely a person writing *currente calamo* upon a subject of popular interest is not bound to verify every statement contained in the subject-matter upon which he is forming his remarks, as if he were engaged upon an analytical investigation of a mathematical process.

2. As regards the over-estimate of age in very advanced life, I beg to observe that I do not admit, because a person is stated, in the census returns, or in those of the Registrar-General, to have attained the age of 100 years, that he has actually done so.

3. Mr. Bailey states that I have borrowed from my own imagination the description I gave of the manner in which certificates of death are frequently given by medical men. He says such gentlemen exercise no judgment whatever as to the ages of their patients.

I distinctly differ with him on this point, and beg to refer again to my account of the method adopted in practice to determine the age at death, the truth of which I still maintain.

4 and 5. I did not attempt to decide the question as to whether the vitality of females was or was not superior to that of males.

I confined my attention to showing what *might* be the effect, as regards the grant of life annuities and assurances, according as either sex might be shown to possess longevity superior to the other.

6. Upon the question of the discredit likely to be thrown upon the English Life Table in consequence of incorrect returns to the Census Enumerators or District Registrars, I can only reiterate my opinion that its value *must* be deteriorated in proportion to the number and magnitude of the errors included in the returns upon which it is based.

I expressed no opinion as to whether or not—any errors to the contrary notwithstanding—it was a better or a worse table than the Carlisle or any other table of mortality.

Mr. Bailey has, in fact, altogether misunderstood the intention of my letter, which was to point out to how great an extent the public are interested in the attainment of correct statistical information.

I thank my friend for the advice contained in his classically-concluding letter. I see that I must in future more frequently turn the stilus if I wish to avoid the criticism of so severe a Metius Tarpa.

I am, Sir,
Your obedient servant,
H. W. PORTER.

Alliance Assurance Office,
2nd Sept., 1861.

ON THE RECENT IMPUTATIONS MADE AS TO MR. GOMPERTZ'S ACCURACY.

To the Editor of the Assurance Magazine.

Sir,—The lengthy paper by Mr. Edmonds, which appeared in the last Number of the *Assurance Magazine*, renders it necessary that I should recur to a subject, which I should not again have approached if I had consulted my own inclination. In that paper, Mr. Edmonds has taken particular pains to answer some of my criticisms; and it will probably be correct to assume, that but for those criticisms this last paper of his would never have been written. Nevertheless my name is not once mentioned in that paper, but I am designated "the new advocate of Mr. Gompertz." If my remarks had been made anonymously, this would have been a perfectly natural and appropriate course for Mr. Edmonds to adopt; but under existing circumstances, I am unable to perceive what advantage the course adopted by him, has over the more obvious one of naming the person whose arguments he undertakes to meet. As there may, however, perhaps be some hidden merit in this course, to be discovered only upon trial, it may be worth while to try the effect of adopting some similar circumlocution. Thus I hope that in speaking of the "plagiarist of Mr. Gompertz," I shall be as well understood by your readers, as if I mentioned a name with which recent controversy will have rendered them familiar.

I have carefully read the paper by the plagiarist of Mr. Gompertz, and examined thoroughly all the arguments made use of by him in his defence; and I find that in one particular only, to be mentioned directly, is there any necessity to alter what I had previously written. Thus I might have rested content with making the single necessary correction, and referring the reader back to my unanswered arguments, without opening any fresh ground; and I should certainly have done so, had not the plagiarist of Mr. Gompertz taken the opportunity of throwing several fresh imputations upon the accuracy of Mr. Gompertz, equally unfounded with the former ones. In consequence of his having pursued this course, it becomes desirable to examine in detail both these new imputations and the answers to my former criticisms.

First, as to the point on which my former remarks require modification. I stated that I had come to the conclusion that the plagiarist of Mr. Gompertz had been "under a misconception as to the real nature of the process of integration." From his recent explanation (pp. 339, 340) I gather that he uses the phrase "process of integration" as equivalent to "solution" of the differential equation, and not, as I had supposed, in the more limited application to the step in which both sides of the equation are *integrated*—this step being (3) in my demonstration on p. 289. I therefore withdraw the particular remark as as to the "misconception" (p. 291); but in doing

this, I wish it to be particularly understood that I do not modify in the least the other remarks contained in the same paragraph. On the contrary, I find the conclusions I had arrived at singularly confirmed, and the observed phenomena at the same time explained, by a statement made by the plagiarist of Mr. Gompertz on p. 334, that he had omitted to read mathematics during the whole of the second year of his academic course. Those who are acquainted with the nature of the studies pursued at Cambridge, will at once admit how impossible it is for a student who has neglected to "read" during the second year of his course, to take up with any benefit or prospect of success, the subjects belonging to the third year. Thus I can now readily account for what seemed a very curious circumstance previously, that the plagiarist of Mr. Gompertz, having formed the differential equation $dy = -yap^x dx$, should have been unable to integrate it, as he frankly confesses (p. 178), although the process was "simple and well known."

Before proceeding to examine the new ground taken by the plagiarist of Mr. Gompertz, it will be convenient to review the answers he has put forward to my former remarks.

First, with regard to the constant d. I remarked that d is "an essential part of the formula, *and as such is employed by Mr. Edmonds himself.*" To this he replies (p. 340), "There is not the remotest approach to truth in this statement." It would be difficult to invent a more precise form of contradiction than is here employed. Let us see how far it is justified by facts. If the reader will turn to p. 336, he will find this identical letter d employed by Mr. Edmonds in the equation

$$\log. y = \log. d - \frac{ap^x}{\log. p}.$$

Again, if he will turn to p. 88, he will find the same quantity introduced, but there denoted by the letter g. How can the plagiarist of Mr. Gompertz say after this that he has not employed the quantity d? If d were really "superfluous" and "useless," it should be used neither in the process of solution of the differential equation, nor yet in the formula for L_x. It should therefore be omitted, and we should have simply $L_x = g^{q^x}$. But the plagiarist of Mr. Gompertz must admit this is not accurate, and that it would be impossible to deduce therefrom his favourite formula $10^{\frac{k^x a}{\lambda p}(1-p^x)}$. It makes no difference to the argument that the plagiarist of Mr. Gompertz has assigned a particular value to d in his formula. If he had not employed d at all, his formula would have been $10^{-\frac{k^x a}{\lambda p}.p^x}$, instead of that which he has actually employed. It should be noticed that this formula, which must be acknowledged by the plagiarist of Mr. Gompertz to be "defective," is very little more so, if at all, than his own, $10^{\frac{k^x a}{\lambda p}(1-p^x)}$. Neither gives the actual number of persons living at the age x. The former requires to be multiplied by the constant factor $L_0 . 10^{\frac{k^x a}{\lambda p}}$; and the latter by the constant factor L_0. On the contrary, Mr. Gompertz's formula, dg^{q^x}, does give the actual number living at the age x. It may be allowed to a person entertaining the views expressed by the plagiarist of Mr. Gompertz, to call

this formula "defective," as not being put in the form which appears to him the most complete and convenient; but there can be no possible pretext for terming it "erroneous," as is done on p. 336. I have thus shown that the formula is incomplete without d, and that if the plagiarist of Mr. Gompertz had not made use of d, his formula would have had a very different form. Therefore, I repeat, *d is not superfluous, and is employed by Mr. Edmonds himself;* and I leave it to those who are competent, to decide which of us is correct. Before leaving this point, it is worth while to notice the amusing inconsistency of the plagiarist of Mr. Gompertz, who (p. 181) says that the process of integration is rendered obscure and ambiguous by the aid of the superfluous and useless indeterminate constant d, and yet himself introduces this very constant as I have shown.

Nor, secondly, is d "indeterminate." It would appear (*v.* p. 338) that the plagiarist of Mr. Gompertz has purposely employed this word in a sense different to that to which it is usually limited. To this course there is only one objection, viz., that a writer pursuing it is certain to be misunderstood. This may be obviated by a process which tradition affirms to have been employed in the "schools" at Cambridge. A student under examination made the error of writing 4 instead of 3 throughout a mathematical demonstration. Arrived at the conclusion, he discovered his error; but being pressed for time, instead of correcting the demonstration throughout, he appended a note—"N.B. Throughout the above demonstration the symbol (4) denotes three units." So, the plagiarist of Mr. Gompertz should say, "By the word *indeterminate*, I denote, not, as is commonly understood, that of which the value cannot be determined, but"—something quite different. Such a course may be thought by some persons to have its recommendations; and the meaning of the writer might be ascertained by the use of sufficient care. But the use of a word without comment in a sense different from that attached to it by all other persons, is wholly unjustifiable!

The plagiarist of Mr. Gompertz argues (p. 337) that d is rightly called *indeterminate* in the usual sense, because (he asserts) "Mr. Gompertz and his two advocates" have been unable to determine its value. If this were true, still he forgets that since *he* has succeeded in getting the value of d, it can no longer be called indeterminate. But as to the question of fact, he would have found, if he had read two lines further (p. 291), that Mr. Gompertz's "new advocate" had succeeded in determining the value of d! He would have found the equation $L_n = dg^{q^n}$, whence $d = \frac{L_n}{g^{q^n}}$, which reduces to the form given by the plagiarist of Mr. Gompertz $\left(\frac{L_0}{g}\right)$ on making $n=0$. This is a misstatement of fact, of which, whether intentional or accidental, I have just reason to complain. I regret to say that it is not the only instance of the kind which appears in the paper now under consideration.

I next say that the formula $10^{\frac{k^n a}{\lambda p}(1-p^x)}$ is defective, because it does not contain the factor L_0; and I point out, by a reference to a table of mortality calculated and published by the plagiarist of Mr. Gompertz, that he in practice supplies the defect of his formula. It will be seen by any person who is at the trouble to refer to the table A.1. of Mr. Edmonds's *Life Tables*, that $L_{12}=100{,}000$ and $L_{22}=92865{\cdot}8$; whereas if "the

superfluous quantity L_0'' (here L_{12}) is omitted from the formula, we should have $L_{22}=\cdot928658$. How far this can be properly described as reproducing "the identical figures given as obtained by the substitution of unity for L_0'' (*v.* p. 341), I again leave the reader to decide.

The next point to which I drew attention was that the plagiarist of Mr. Gompertz had committed the serious error of calling d a particular value of y. He confesses to this error (p. 338), saying that "in strictness" he should have said something quite different to what he did actually say. Having made an undeniable mistake, it is only proper to acknowledge it when pointed out. Why then does he, nine lines further down, retract that confession by speaking of "the *pretended* verbal error"? This appears to me at least a great want of candour; for the error is beyond doubt a real, not a pretended one, whether it is only "verbal" or something more. The plagiarist of Mr. Gompertz expresses surprise that while pointing out his errors, I have not noticed a "really serious error of Mr. De Morgan." The reason is easily given. I have no ambition to become a public censor. I have, as stated in my former letter, a definite object in view in exposing the errors of the plagiarist of Mr. Gompertz. I expect to demonstrate by doing this, that his imputations on Mr. Gompertz, besides being unfounded, are without weight, as coming from a person whose acquaintance with the subject is imperfect. If Professor De Morgan had committed a really serious error, no similar reason existed why I should point it out, and no good object was to be gained thereby.

Further on, I show that the plagiarist of Mr. Gompertz has himself changed the sign of the quantity c, which he had asserted that Mr. Gompertz changed. The truth of my statement is not denied; but the plagiarist of Mr. Gompertz appears to have had this remark in view while writing the second paragraph on p. 337. He there says that Mr. Gompertz has changed the sign of the exponent of g. This is a phrase which is literally unmeaning. We have $g=10^c=10^{-\frac{k^2a}{\lambda q}}$, where it is obvious that the "exponent of g" is unity;* and how its sign is changed by Mr. Gompertz, or can be changed at all, it is impossible to imagine. What the plagiarist of Mr. Gompertz apparently meant to state, was, that Mr. Gompertz committed an error in making the above supposition as to the values of g and c, instead of taking $g=10^c=10^{+\frac{k^2a}{\lambda q}}$. When stated in this plain way, and cleared of the mist of verbiage in which it has been diligently involved by the plagiarist of Mr. Gompertz, the whole objection is obviously futile. Mr. Gompertz was clearly at liberty, without committing an error or introducing a "defect," to make whichever supposition he pleased as to the values of g and c. The one supposition would have given him the formula, $\frac{d}{g^{q^x}}$, instead of that which he has actually employed, dy^{q^x}. The plagiarist of Mr. Gompertz considers the former of these formulæ to be preferable. Mr. Gompertz preferred the latter: and so, I believe, will all your readers who consider the point.

Here I would call attention to a curious inconsistency between the first and third paragraphs on p. 337. The first states that the equation

* It seems probable that the quantity c is intended by this term, but c is the *common logarithm* of g, not its exponent.

$y=dg^{x^n}$ is defective because it has not introduced the quantity L_0 by putting $\frac{L_0}{g}$ for d; the third asserts that L_0 ought *not* to have been introduced, and that the formula is defective because use *has* been made of L_0! It is, of course, obviously impossible that both these charges can be true, and the slightest examination of Mr. Gompertz's process will satisfy the reader that it is not chargeable with the "defect" of introducing L_0. I cannot leave this point without noticing that L_0 is incorrectly introduced into Mr. Gompertz's process by his plagiarist; for, as I have already pointed out (p. 291), Mr. Gompertz takes x throughout to denote the age *measured from birth*, so that the correct quantity to introduce is L_n, and not L_0.

The last of my remarks to which the plagiarist of Mr. Gompertz has attempted a reply, has reference to a misquotation by him of Mr. Gompertz's words. I will not occupy your space by again quoting the passage misquoted, but refer those who wish to verify what I state, to p. 293. The plagiarist of Mr. Gompertz has quoted a portion of a hypothetical sentence, which was not intended as a formal statement of the law of mortality, and argues on the strength of the misquoted passage, that Mr. Gompertz has committed an error, in not stating that his law of mortality does not prevail throughout the whole of life with the same constants. The truth being, that when Mr. Gompertz does state his law of mortality, he is very careful to limit it to a "long portion of life." I could have easily supposed that the misquotation was made through haste, but it is actually defended in the recent paper of the plagiarist of Mr. Gompertz! It is described as a simple omission of the "insignificant word" *if!* This is the first time I have ever heard the word *if* described as insignificant. The point will be made clearer by an example. Thus I may say, "*If* Mr. Edmonds has knowingly misrepresented facts in order to deprive Mr. Gompertz of the credit to which he is justly entitled, *then* he will not be correctly described by the term 'plagiarist,' but a far stronger term must be applied to him!" Suppose this passage to be quoted thus:—"Mr. Edmonds has knowingly misrepresented facts in order to deprive Mr. Gompertz of the credit to which he is justly entitled;" and we shall have an exact parallel to the misquotation with which I charge Mr. Edmonds. Perhaps this example may convince that gentleman that the little word "if" is not so "insignificant" as he has hitherto imagined. But it seems (p. 340) that "if" is used by Mr. Gompertz in the sense "let it be assumed." This I deny; but supposing it were, I cannot see that it would in the least affect the question.

The simple facts of the misquotation are these. Mr. Gompertz carefully limits the application of his hypothetical law of mortality to a "long portion of life." His plagiarist omits all reference to the passage in which this limitation is made—whether from design or oversight I will not pretend to say—but quotes part of a sentence in an altered form, as a proof that Mr. Gompertz has erroneously omitted to limit the application of his law. It is not true, as the plagiarist implies, that the only difference between the passage as written by Mr. Gompertz and quoted by him, consists in the presence or absence of the word "if". The reader will see, by comparing the passages (pp. 174, 293), that "if" is replaced by "that"; "were" is replaced by "are", and "lost" by "loses"; while in further illustration of the looseness with which the plagiarist of Mr. Gompertz has made the quotation, it may

be mentioned that he wholly omits the significant word "remaining", as applied to the "power of resisting destruction" by Mr. Gompertz.

A similar looseness exists throughout the recent papers of the plagiarist of Mr. Gompertz. In one instance, I have to complain that my words are misquoted. I am represented (p. 339) to have said that "the process of integration had not *commenced* when the erroneous (b) first appeared." On referring to p. 291, it will be seen that my actual words are, "b makes its appearance in step (2), while the integration is not *performed* till (3)." By the substitution, whether from design or carelessness, of the word *commenced* for the original one *performed*, a totally new complexion is given to the sentence, and the original meaning is obscured and perverted.

Again, it is stated (p. 338) that "Mr. Gompertz has *recently* presented to the Institute of Actuaries a corrected copy of his paper." The indefinite word *recently* is calculated to leave a false impression on the mind of the reader—whether designedly or not, I will not pretend to say. It would naturally be supposed, on reading the passage from which I have quoted, that the corrected copy of Mr. Gompertz's paper had been presented to the Institute of Actuaries, since attention has been drawn to that paper by the communication of Professor De Morgan, which appeared in the *Assurance Magazine* for July, 1860. So far from this being the case, I have ascertained that the copy was presented so long ago as February, 1855. It is true that this may be termed *recently* in comparison with the date (1825) of the original paper; but this very circumstance renders the use of the ambiguous term more objectionable.

As another example of the same kind, I would refer to the description given by the plagiarist of Mr. Gompertz, of the problem proposed for solution. He says (p. 335; see also p. 338) that the object of Mr. Gompertz was "to find an expression for the number living at any age (x) in terms of the annual mortality." This is incorrect. *This was Mr. Edmonds's object, but was not Mr. Gompertz's.* The object of the latter may be briefly expressed as follows:—The mortality (not the *annual* mortality) at any age x being proportional to q^x (*i.e.* $=aq^x$), required a formula for the number living at the age x. Mr. Gompertz's formula ($d_y q^x$) gives the correct solution of his problem, and Mr. Edmonds's formula, *corrected by the introduction of* L_0 (thus, $L_0 . 10^{\frac{k^2 a}{\lambda p}(1-p^x)}$), gives the correct solution of *his* problem.

The plagiarist of Mr. Gompertz expresses his belief (p. 333) that that gentleman has never complained that any wrong has been done him by a failure of due acknowledgment of the priority of his discovery. Here I would direct attention to the gradual change of tone in the paragraph relating to this point. First it is "I do not believe," then "I have never heard," and finally "Mr. Gompertz himself, during the period of 28 years, remained insensible to the supposed wrong." Truly a very positive statement to base upon so slender a foundation of fact, or rather belief! With reference to this point, I happen to have heard from a gentleman, himself a Fellow of the Royal Society, that Mr. Gompertz has, upon one occasion at least, in the course of conversation at the Royal Society, some twenty-five years ago, expressed in emphatic language his belief that his theory had been adopted without proper acknowledgment. It is easy enough to understand that gentlemen who have been "in free personal communication" with both

Mr. Gompertz and his plagiarist, and therefore, it is to be presumed, on friendly terms with both, would not make known a circumstance of this kind to the latter; and this consideration may suggest to that gentleman that it is not safe to assume that an event has not taken place because he has not heard of it.

The plagiarist of Mr. Gompertz states (p. 334) that his paper of Oct., 1860, "was written in self-defence (against an outrageous attack of Mr. De Morgan)." It appears to me that that paper contained an outrageous and unprovoked attack upon Mr. Gompertz, and that the more recent paper of last July contains another "outrageous attack" upon the same gentleman.

Thus a very laboured effort is made (p. 335) to show that Mr. Gompertz has fallen into a serious error with regard to the nature of the quantity a in his formula aq^x for the mortality at any age x. There is not the slightest foundation in fact for this charge, as I will now proceed to prove. It is by no means easy to perceive at first sight the precise meaning to be attached to the quantities a and α as employed by the plagiarist of Mr. Gompertz, in consequence of the obscurity of the language he has employed. I believe, however, that the following will be found to be a clear and faithful exposition of the distinction he means to draw. Adopting for the present his own letters, we have ap^x for the rate of mortality at the age x. The meaning of this quantity is best seen by the application that is made of it: thus the number dying in the small time dx, out of L_x alive, is stated to be $L_x ap^x dx$. Whence we see that, in mathematical language, $L_x ap^x$ is the limit of the ratio which the number dying in a small time bears to that time, when both are indefinitely diminished. If we represent the number alive at age x by unity, this ratio becomes ap^x, and at the age 0 it is simply α. Now in making a comparison with any actual table of mortality, we naturally take the *annual* mortality at the age considered. This will not be represented by α, *i.e.* if the rate of mortality as explained above is α at the commencement of the year, and is supposed to remain constant throughout the year, the number who die in the year out of L_0 persons living at the commencement will not be αL_0. In order to determine the annual mortality we must proceed as follows:—the number alive at the commencement of the year being L_0, suppose the year divided into ν equal portions, then the number dying in the first small interval $\frac{1}{\nu}$ will be $\frac{L_0\alpha}{\nu}$. Thus there remain alive $L_0\left(1-\frac{\alpha}{\nu}\right)$ persons, and of these $L_0\frac{\alpha}{\nu}\left(1-\frac{\alpha}{\nu}\right)$ will die in the next interval $\frac{1}{\nu}$, leaving $L_0\left(1-\frac{2\alpha}{\nu}+\frac{\alpha^2}{\nu^2}\right)$ $=L_0\left(1-\frac{\alpha}{\nu}\right)^2$ alive at the end of the second interval. Pursuing the same process, it is easily seen that the number surviving at the end of the year must be $L_0\left(1-\frac{\alpha}{\nu}\right)^\nu$. Now pass to the limit by supposing ν indefinitely large, and this quantity becomes $L_0 e^{-\alpha}$. Thus the number dying in the year will be $L_0(1-e^{-\alpha})$, whence the quantity $1-e^{-\alpha}$ represents the *annual mortality*. The plagiarist of Mr. Gompertz supposes the annual mortality to be such that out of $1+a$ persons alive at the beginning of the year, a die in the course of the year, so that the annual mortality is $\frac{a}{1+a}$. Thus

then he gets $\frac{a}{1+a}=1-e^{-\alpha}$, whence $e^{-\alpha}=1-\frac{a}{1+a}=\frac{1}{1+a}$, and $1+a=e^{\alpha}$; also $\alpha=\log_e(1+a)$.

The only argument put forward in order to support the charge against Mr. Gompertz that he was ignorant of the real nature of the quantity α, is that he has made no statement of "any algebraical or arithmetical value of a or α." It is difficult to attach any precise meaning whatever to these words; but the thing intended appears to be this:—Mr. Gompertz has not explained the precise meaning of his quantity a, nor given any numerical example from which that meaning might be inferred—*therefore* he was ignorant of its meaning! The truth is that for Mr. Gompertz's object of comparing the survivors at any age with his formula dg^{q^x}, it was not necessary to make any inquiry as to the nature of the quantity a. But this affords not the slightest ground for supposing he was ignorant of its nature.

It is true that there is no unqualified assertion made that Mr. Gompertz is in error on this point. We read however (p. 336), that "the differential equation of Mr. Gompertz is erroneous *if* he has used the finite decrement (a) to represent the mortality;" see also p. 340. Now in reading this sentence, it must be borne in mind that the plagiarist of Mr. Gompertz looks on the little word *if* as an insignificant word, so that his meaning must be that there is very little doubt indeed, in fact none at all, that Mr. Gompertz was in error. We have seen that there is no foundation whatever for this assumption, and that Mr. Gompertz in this, as in all the other points I have considered, is strictly accurate. Let us now turn and consider more closely the remarks of his plagiarist on this point, and there will remain no doubt that he has fallen into two very distinct and serious errors.

In the first place, he has taken to denote the annual mortality a quantity a such that the number living at the commencement of the year is to the number living at its end as $1+a$ is to 1. Instead of this, the correct assumption is to take a so that the number entering on the year is to the number surviving it as 1 to $1-a$. This is a fundamental error, and one that was not to be expected from the veriest tyro in actuarial science. To put the point in another way:—the annual mortality at any age x is found by comparing the finite decrement for one year with the number living at the *beginning* of the year; whereas the plagiarist of Mr. Gompertz has compared it with the number living at the *end* of the year! The practical result of this error will be, that supposing a to be the mortality at the beginning of a year, the plagiarist of Mr. Gompertz makes the actual mortality of the year greater than a, instead of less as it obviously should be. To take the figures given by him in p. 335, if $a=\cdot0063643$, then will the actual mortality of the year be $1-e^{-a}$ or $\cdot0063440$, instead of $\cdot0063845$ as stated.

Again, in calculating the actual mortality of the year, the plagiarist of Mr. Gompertz supposes the rate of mortality constant throughout the year. This is contrary to the fundamental hypothesis, that the mortality at any age x, integral or fractional, is αp^x. The mortality varies throughout the year, and the actual annual mortality will be neither $e^{\alpha}-1$ as the plagiarist of Mr. Gompertz has it, nor $1-e^{-\alpha}$ as found by me above. The real annual mortality is found by a very simple process, for it is equal to

$$\frac{L_x-L_{x+1}}{L_x}=1-\frac{L_{x+1}}{L_x}.$$

But $$L_x = dg^{p^x},\ L_{x+1} = dg^{p^{x+1}};$$

whence $$\frac{L_{x+1}}{L_x} = \frac{g^{p^{x+1}}}{g^{p^x}} = g^{p^{x+1}-p^x} = g^{(p-1)p^x}.$$

Also $g = 10^a = 10^{-\frac{ak^x}{\lambda p}}$; so that the correct value of the annual mortality in terms of known quantities is

$$1 - g^{(p-1)p^x} = 1 - 10^{-\frac{ak^x(p-1)}{\lambda p}\cdot p^x};$$

which becomes when $x=0$, $1-10^{-\frac{ak^x(p-1)}{\lambda p}}$.

Before leaving this point, I would call the attention of your readers to the circumstance that the plagiarist of Mr. Gompertz has termed a (p. 335) the most important quantity in the formula dq^x. What! Is it more important than q, which is "part of the foundations of the universe"? (*V.* p. 177.) We have here a remarkable instance of a complete revolution in opinion in the course of nine months. In October, 1860, it was stated that q is "distinguished from other but *less important* constants used"; in July, 1861, it appears that a and not q is the most important constant!

On the same page we read:—"Previous to forming the differential equation, it is essential that the nature of the quantity (a), given to represent the proportional mortality for one year when the mortality is constant, should be determined." With regard to this, I simply remark that the success with which Mr. Gompertz has applied his formula to the examination of various tables of mortality, without previously examining into the nature of a, sufficiently shows that it is *not* essential that the nature of a should be determined. Secondly, I would particularly draw attention to the fact that a is *not* "given" by Mr. Gompertz "to represent the proportional mortality for one year when the mortality is constant." He nowhere throughout his paper speaks of the mortality being *constant for one year;* but on the contrary consistently supposes the mortality to vary at every instant of life; and he nowhere speaks of a being the mortality *for one year*. His words are that "the intensity of his mortality at the age x may be denoted by aq^x." In this instance, as in several others, Mr. Gompertz's words are wholly misrepresented. Before leaving this point, I may remark that it is quite impossible the annual mortality should be expressed by two different quantities (*v.* p. 335). If the one quantity correctly expresses the annual mortality, it is obvious the other cannot do so too. I have already shown that *neither* of the two quantities described by the plagiarist of Mr. Gompertz correctly expresses the annual mortality.

Not only is Mr. Gompertz's accuracy called in question, but his truthfulness is also impeached. We are told (p. 330) that Mr. Gompertz has lent "his approbation to papers containing *statements which, from his superior knowledge of the facts, he could not, with any regard to truth, have uttered himself.*" Unfortunately, we are left to conjecture what those untrue statements are. For myself, I can find no statement, either in Professor De Morgan's paper or my own, to which that description will apply. It is therefore impossible to disprove this serious charge; but I feel not the slightest doubt, that all your readers will agree with me in considering that the plagiarist of Mr. Gompertz has pursued an unwarrantable course, in recklessly making this grave charge without the least

attempt to substantiate it. Similarly, we are told that "his advocates advance claims on his behalf, which Mr. Gompertz has never advanced himself." Here again, no attempt is made to point out these claims, and I am therefore only able to give a general denial to the assertion. I can discover no new claims put forward on behalf of Mr. Gompertz by Professor De Morgan and myself, and I do not believe that any such have been put forward. So, again, we are told there is only one instance of "concurrence of views" between Mr. Gompertz and his two advocates, "in the recently published papers of the three parties." It would have been more to the point if the plagiarist of Mr. Gompertz had given any example of discordance of views between "the three parties." Probably he has not done so, because he was unable to find any. With regard to the particular point which he has noticed as one of agreement, viz., the statement that he has given no proof of the truth of his supposed law of mortality, I have several times carefully read Mr. Gompertz's letter which is contained in the *Assurance Magazine* for April, 1861, and have failed to discover that he makes any statement of the kind at all. In fact, after carefully reviewing the whole of the remarks on p. 330, the conclusion appears unavoidable that they were written with far greater regard to effect than to truth.

A great deal has been said about the errors which appear in Mr. Gompertz's process, as printed in the *Philosophical Transactions*. I had hoped that I had satisfactorily shown that those errors are simply errors of the press, and that the process itself is free from any intrinsic error. In this view I am supported by the manuscript corrections in the copy of Mr. Gompertz's paper belonging to the Institute of Actuaries, and by Mr. Gompertz's own remarks in the *Assurance Magazine* (p. 297). Even without such support, I submit that when a gentleman of undoubted mathematical ability starts with a correct equation $aL_x q^x \dot{x} = -(\dot{L_x})$, and ends with a correct solution $L_x = dq^{q^x}$, it is only reasonable to suppose that he is aware of the correct intermediate steps, and that any errors which appear in the published process arise—not from ignorance or carelessness on the part of the author—but from a want of proper supervision of the press. On one point I venture to differ with Mr. Gompertz. He wishes for "a sufficiently clever mathematical superintendent of the press." On the contrary, I am of opinion that it is preferable to have a superintendent of the press who is ignorant of mathematics, but familiar with mathematical symbols. For it is clearly impossible to obtain one with sufficient mathematical knowledge to "restore" a demonstration such as Mr. Gompertz's published one; and a person of imperfect knowledge, who attempts to make conjectural emendations, is likely to introduce more new errors than he corrects. On such grounds I have been given to understand that women are often found better compositors of Latin and Greek works than men, because being quite ignorant of the language, they simply set up what is put in their hands, and do not introduce errors by attempting to correct fancied mistakes. Thus the whole responsibility of correcting the press is left to the author. It appears from the remarks on pp. 338–9, that the plagiarist of Mr. Gompertz does not yet allow that the second b is a printer's error. He has put forward a conjecture (p. 339) to account for the introduction of b, which supposes a degree of ignorance or carelessness on the part of Mr. Gompertz, which is quite inconceivable on the part of a gentleman of such proved mathematical attainments. Mr. Gompertz is represented as having erro-

neously supposed that because a constant was introduced by integration on one side of the equation, therefore another was required on the opposite side! This is wholly opposed to the fundamental notion of integration, by which *one* constant only is introduced; and it is clearly impossible to entertain the supposition for an instant. This theory has been put forward in apparent forgetfulness of a different explanation that was volunteered (pp. 181, 184) to account for the appearance of b. Both of these explanations cannot be correct, and I believe neither is.

I would ask the plagiarist of Mr. Gompertz how he accounts for the corrections made by Mr. Gompertz's authority in the copy of his paper belonging to the Institute of Actuaries—made, it should be remembered, before February, 1855? This at least shows that Mr. Gompertz at that date repudiated the letter b as a misprint, and as forming no part of the correct process. When does his plagiarist suppose he became aware of the error? This error is exactly such a one as I should expect to see introduced by a compositor who knew something of algebra. The letter b having been introduced in the previous step erroneously, and the equation $a\dot{x}q^x = -\frac{\dot{L}_x}{L_x}$ having been written $abq^x = -\frac{\dot{L}_x}{L_x}$, such a compositor would naturally argue—"the letter b must have been forgotten by the author in "the next step. Instead of aq^x, which the MS. shows in the next step, we "should clearly have abq^x, as in the previous one." It must be borne in mind that the plagiarist of Mr. Gompertz allows the introduction of b in the former of these cases to be a genuine misprint.

In the same passage (p. 339) occurs a statement for which I am wholly unable to account:—"The equation, as corrected, is more defective, in form, at least, than was the equation intended to be corrected. . . . . The correction now made would leave no new constant on either side of the equation." The corrected equation in question will be found on p. 289 indicated by (3), *and contains the new constant* (d). I shall feel obliged if the reader will refer to this passage and verify my statement, and then endeavour to reconcile it with the one I have quoted above.

On the same page it is stated that I do "not call in question the truth of the substantial part of the statement" that "b is introduced through an incorrect process of integration, and is superfluous, useless, and erroneous." I certainly intended to call in question the truth of this. I have stated and, I believe, proved, that Mr. Gompertz's process is not erroneous, and that b is not a superfluous, useless, and erroneous constant, but is simply a misprint. It appears to me that I have thus, in a sufficiently plain manner, called in question the truth of the statement, not only of its form but of its substance.

Besides the imputations on Mr. Gompertz's accuracy, we have others on his language and style (p. 329) to which it is only desirable to refer in order to show the spirit which animates the remarks now under review. I should be glad to learn how it can affect the questions at issue, whether Mr. Gompertz's language is "homely" or "transcendental." Surely it must be a very weak case which needs the support of such an argument as is here implied!

In my former letter I showed the necessity of detailed evidence before admitting the truth of the supposed "true law of mortality." In consequence of this challenge apparently, we have now set forth (p. 331) a short

table comparing the mortality among the male population of England during 10 years according to observation and according to theory. On this I would remark that the agreement of the two does not proceed in any part of the table beyond the first decimal place, so that we are still left in ignorance as to the materials from which the values of p were computed to *seven* decimal places, and the logarithms to *four* decimal places. In addition to this, a large number of instances must be produced before it could be allowed that the law is commonly, much less universally, true. I would also ask whether the plagiarist of Mr. Gompertz will assert that the example he has produced is not equally in favour of the new law of mortality as recently stated by Mr. Gompertz?

Besides the errors which I have already shown are chargeable to the plagiarist of Mr. Gompertz, his language in various passages renders it extremely probable that he has fallen into another serious error. He says (p. 328) that his law of mortality is such that if the annual mortality at age 25 were 1 per cent, and at 45 were 1·80 per cent, then the annual mortality at the intermediate ages would be found by interpolating the terms of a geometric progression. Again, on p. 329, he says that "the annual rate of mortality increases in a geometric progression, of which the common ratio is (p)." Now, bearing in mind the distinction already pointed out, viz., that if αp^x be the rate of mortality at the age x, then the *annual mortality* at that age will be $1-g^{(p-1)p^x}$, it is obvious that the annual rate of mortality does not increase in geometric progression; but on the contrary, the chance of living a year ($=g^{(p-1)p^x}$) decreases in a geometric progression.

After all that has been already said, it is needless to comment at any length on the remarks contained in p. 330 and elsewhere, to the effect that Mr. Gompertz's method of procedure, in comparing the numbers living at various ages with those given by his formula dg^{q^x}, is not so good as the method of comparing the mortality in successive decennial periods of age. This simply amounts to saying that Mr. Gompertz's method is inferior to that of his plagiarist. This is a point upon which difference of opinion may fairly exist. The plagiarist of Mr. Gompertz should therefore have given it as his opinion, not asserted it as a fact, that the one method is preferable to the other. We must not omit to notice however the curious statement on p. 329, that the formula $10^{\frac{k^x a}{\lambda p}(1-p^x)}$ is "dg^{p^x} corrected and reduced to its *simplest* terms." It might be thought that it would be allowed on all hands that the latter formula is far simpler than the former; but it seems that long continued admiration of the former has led its author to claim for it merits which it certainly does not possess.

In conclusion, I beg to state that I regret extremely the personal character which this controversy has taken, but it has been forced upon me. When general and unsupported charges of inaccuracy are preferred, such as those which I have already pointed out, and such as that preferred against myself on p. 340, it becomes necessary to examine into the accuracy and knowledge of the person who makes them. If he is found to be a person free from error and accurate in his writings, his charges, although unsupported, will have weight with the world; but if on the other hand he should be found singularly inaccurate, then his charges will be disregarded and fall harmless to the ground, or recoil upon himself. Bearing these facts in mind, I have endeavoured to show the credit which I think should

be attached to the charges preferred by Mr. Edmonds against Mr. Gompertz, and I must leave it now to the scientific public to pronounce their verdict.

I have the honour to be, Sir,
Your obedient servant,

25, *Pall Mall,*
September, 1861.

T. B. SPRAGUE.

ON THE TABLES OF DEFERRED ANNUITIES PUBLISHED BY THE NATIONAL DEBT OFFICE.

To the Editor of the Assurance Magazine.

Dear Sir,—Having had occasion to refer to the above-mentioned Tables, I have been surprised to find that they are not computed in the usual way, but apparently on some principle wholly different from it. I append instances in respect of single premiums (but the same remark applies to the annual ones), in which your readers will observe that the premiums charged by the Government are, for the most part, greatly in excess of those resulting from calculations made on the true principles. The publication of these discrepancies may possibly bring about some explanation of the reason of them; and I therefore have to beg the favour of your inserting this communication in your *Journal;* remaining—

Dear Sir,
Yours truly,

London, August, 1861.

J. W. STEPHENSON.

Deferred Annuities of Thirty Pounds—Males.

Single Premiums, returnable without Interest at any time prior to commencement of Annuity, pursuant to 16 & 17 Vict., cap. 45.

Age at Entry.	Term.	Government Premiums.			Premiums computed on Government Data at 3 per Cent.			Difference.			Difference per Cent.
		£	s.	d.	£	s.	d.	£	s.	d.	
21	After 10 years	403	12	5	403	5	0	0	7	5	
"	" 20 years	261	15	0	252	11	10	9	3	2	3·63 per cent.
"	" 30 years	156	17	6	140	18	10	15	18	8	11·3 "
"	" 40 years	89	12	6	66	14	5	22	18	1	34·33 "
"	" 50 years	44	7	6	23	9	7	20	17	11	89 "
31	After 10 years	360	5	0	360	14	6				
"	" 20 years	216	0	0	207	13	3	8	6	9	4·01 per cent.
"	" 30 years	121	7	6	101	17	6	19	10	0	19·14 "
"	" 40 years	63	12	6	37	6	0	26	6	6	70·57 "
"	" 50 years	24	5	0	7	7	6	16	17	6	228·8 "
41	After 10 years	297	7	6	297	5	11	0	1	7	
"	" 20 years	167	2	6	153	5	10	13	16	8	9·02 per cent.
"	" 30 years	84	2	6	59	8	3	24	14	3	41·59 "
"	" 40 years	34	15	0	12	8	5	22	6	7	179·79 "
51	After 10 years	230	0	0	225	14	8	4	5	4	1·88 per cent.
"	" 20 years	115	17	6	95	7	6	20	10	0	21·49 "
"	" 30 years	47	17	6	21	19	1	25	18	5	118·33 "
56	After 25 years	56	2	6	30	4	10	25	17	8	85·69 per cent.

INSTITUTE OF ACTUARIES.

SOLUTIONS OF THE SECOND YEAR'S EXAMINATION QUESTIONS.

Some years ago, Mr. Porter, with a view to assist candidates for the Institute's certificates, gave solutions of the questions proposed to them, which, it may be remembered, were published in the first and second volumes of this *Journal*. Since these solutions appeared, modifications have, on more than one occasion, been made in the syllabus put forth by the examiners, and corresponding changes have consequently arisen in the order and character of the questions proposed. Under these circumstances, we willingly adopt the suggestion made by Mr. Sprague, who was one of the examiners of the last year, as to the desirableness of again publishing some solutions of the questions submitted of late to the candidates, so that future ones may have the best information as to the kind of answers expected from them; and having been favoured by Mr. Sprague with the following solutions of some of the questions proposed in the second year's examination of 1860, and with the references he has made, as regards others, to such text-books as are likely to be most useful to the student in his preparation, we now lay them before the reader. The questions themselves appeared in the last volume, see page 301.

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1. "Assuming the formula for $\log_e(1+x)$, prove that

$$\log_e 3 = 1 + \frac{1}{3}\cdot\frac{1}{4} + \frac{1}{5}\cdot\frac{1}{4^2} + \frac{1}{7}\cdot\frac{1}{4^3} + \ldots.$$

$$\tfrac{1}{2}\log_e 10 = \log_e 3 + \frac{1}{19} + \frac{1}{3}\cdot\frac{1}{19^3} + \frac{1}{5}\cdot\frac{1}{19^5} + \ldots."$$

We have $$\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \&c.$$

Changing the sign of x in this equation,

$$\log_e(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \&c.$$

Subtracting this equation from the former,

$$\log_e(1+x) - \log_e(1-x), \text{ or } \log_e\frac{1+x}{1-x} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5}, \&c.\right) \quad . \; . \; (1)$$

Now let $$\frac{1+x}{1-x} = m, \text{ whence } x = \frac{m-1}{m+1},$$

then $$\log_e m = 2\left\{\frac{m-1}{m+1} + \frac{1}{3}\left(\frac{m-1}{m+1}\right)^3 + \frac{1}{5}\left(\frac{m-1}{m+1}\right)^5 + \ldots\right\}.$$

Here make $m=3$, whence $\frac{m-1}{m+1} = \frac{2}{4} = \frac{1}{2}$,

then we get $$\log_e 3 = 2\left\{\frac{1}{2}+\frac{1}{3}\cdot\frac{1}{2^3}+\frac{1}{5}\cdot\frac{1}{2^5}+\ldots\right\}$$

$$=1+\frac{1}{3}\cdot\frac{1}{2^2}+\frac{1}{5}\cdot\frac{1}{2^4}+\frac{1}{7}\cdot\frac{1}{2^6}\ldots.$$

$$=1+\frac{1}{3}\cdot\frac{1}{4}+\frac{1}{5}\cdot\frac{1}{4^2}+\frac{1}{7}\cdot\frac{1}{4^3}+\ldots.$$

which is the first of the proposed formulæ.

Again, in formula (1) put $\frac{1+x}{1-x}=\frac{n+1}{n}$, whence $x=\frac{1}{2n+1}$; then we have

$$\log_e\frac{n+1}{n}=2\left\{\frac{1}{2n+1}+\frac{1}{3}\cdot\frac{1}{(2n+1)^3}+\frac{1}{5}\cdot\frac{1}{(2n+1)^5}+\ldots\right\}.$$

Here make $n=9$, and we have

$$\log_e\frac{10}{9}=2\left\{\frac{1}{19}+\frac{1}{3}\cdot\frac{1}{19^3}+\frac{1}{5}\cdot\frac{1}{19^5}+\ldots\right\}.$$

But $$\log\frac{10}{9}=\log_e 10-\log_e 9=\log_e 10-2\log_e 3,$$

$$\therefore \log_e 10-2\log_e 3=2\left\{\frac{1}{19}+\frac{1}{3}\cdot\frac{1}{19^3}+\frac{1}{5}\cdot\frac{1}{19^5}+\ldots\right\},$$

and $$\frac{1}{2}\log_e 10=\log_e 3+\frac{1}{19}+\frac{1}{3}\cdot\frac{1}{19^3}+\frac{1}{5}\cdot\frac{1}{19^5}+\ldots$$

—the second of the proposed formulæ.

2. "Explain what is meant by the modulus of a system of logarithms. "By means of the formulæ in question (1), show that

$$\log_e 3 = 1{\cdot}098612;$$
$$\log_e 10 = 2{\cdot}302585.$$

"Also show that the modulus of the common system of logarithms is "·434294."

If N be any number, it can be proved that

$$\log_a N=\log_a e\,.\,\log_e N=\frac{\log_e N}{\log_e a},$$

whence we see that the logarithm of any number to the base a can be obtained by multiplying its Napierian (or hyperbolic) logarithm by the constant quantity $\log_a e$ or $\frac{1}{\log a}$. This quantity is called the *modulus* of the system of logarithms to the base a.

For the second part of the question, we have

$\frac{1}{4} = \cdot 250,000,0$	$\frac{1}{3}\cdot\frac{1}{4} = \cdot 083,333,3$
$\frac{1}{4^2} = \cdot 062,500,0$	$\frac{1}{5}\cdot\frac{1}{4^2} = \cdot 012,500,0$
$\frac{1}{4^3} = \cdot 015,625,0$	$\frac{1}{7}\cdot\frac{1}{4^3} = \cdot 002,232,1$
$\frac{1}{4^5} = \cdot 003,906,3$	$\frac{1}{9}\cdot\frac{1}{4^5} = \cdot 000,434,0$
$\frac{1}{4^6} = \cdot 000,976,6$	$\frac{1}{11}\cdot\frac{1}{4^6} = \cdot 000,088,8$
$\frac{1}{4^7} = \cdot 000,244,2$	$\frac{1}{13}\cdot\frac{1}{4^7} = \cdot 000,018,8$
$\frac{1}{4^8} = \cdot 000,061,1$	$\frac{1}{15}\cdot\frac{1}{4^8} = \cdot 000,004,1$
$\frac{1}{4^9} = \cdot 000,015,3$	$\frac{1}{17}\cdot\frac{1}{4^9} = \cdot 000,000,9$
$\frac{1}{4^{10}} = \cdot 000,003,8$	$\frac{1}{19}\cdot\frac{1}{4^{10}} = \cdot 000,000,2$
$\frac{1}{4^{11}} = \cdot 000,001,0$	$\cdot 098,612,2$

Hence, to six places of decimals, $\log_e 3 = 1\cdot 098,612$.

For $\log_e 10$, we have

$\frac{1}{19} = \cdot 052,631,6$	$\frac{1}{19} = \cdot 052,631,6$
$\frac{1}{19^3} = \cdot 000,145,8$	$\frac{1}{3}\cdot\frac{1}{19^3} = \cdot 000,048,6$
$\frac{1}{19^5} = \cdot 000,000,4$	$\frac{1}{5}\cdot\frac{1}{19^5} = \cdot 000,000,1$
	$\cdot 052,680,3$
Adding the value of $\log_e 3$ found above,	$\log_e 3 = 1\cdot 098,612,2$
	$1\cdot 151,292,5$
and multiplying by 2,	2
the second equation in question (1) gives the value of $\log_e 10$ required	$2\cdot 302,585$

Here it may be noticed that in order to determine $\frac{1}{19^n}$, it is convenient to employ the binomial theorem.

Thus, $\frac{1}{19^n} = \frac{1}{(20-1)^n} = \frac{1}{20^n}\left(1-\frac{1}{20}\right)^{-n}$

$$= \frac{1}{20^n}\left\{1+\frac{n}{20}+\frac{n(n+1)}{1.2}\cdot\frac{1}{20^2}+\frac{n(n+1)(n+2)}{1.2.3}\cdot\frac{1}{20^3}+\&c.\right\}.$$

For example—

$$\frac{1}{19^3}=\frac{1}{20^3}\left\{1+\frac{3}{20}+\frac{3.4}{1.2}\cdot\frac{1}{20^2}+\frac{3.4.5}{1.2.3}\cdot\frac{1}{20^3}+\cdots\right\}$$

$$=\frac{1}{20^3}\left\{1+\frac{3}{20}+\frac{3}{200}+\frac{1}{800}+\cdots\right\}$$

$$=\frac{1}{20^3}\left\{\begin{array}{l}1+\cdot15\\ \quad+\cdot015\\ \quad+\cdot00125\end{array}\right\}$$

$$=\frac{1}{20^3}\times 1\cdot\dot{1}6625$$

$$=\quad\cdot000,145,8$$

The modulus of the common system of logarithms being $\frac{1}{\log_e 10}$, common division gives its value, $\frac{1}{2\cdot302,585}=\cdot434,294$.

Thus—

$$\begin{array}{r}
2\cdot30\dot{2},5\dot{8}\dot{5}\,)\,1\cdot000\,000\,00\,(\,\cdot434,294\\
78\,966\,00\\
9\,888\,45\\
678\,11\\
217\,59\\
10\,36\\
1\,15
\end{array}$$

3. "When an event can happen in more ways than one, show that the "probability of its happening is equal to the sum of the probabilities in "respect of the different ways."

Vide Gray's Tables and Formulæ, p. 34.

4. "Determine (1) the probability that two persons now aged x and y "respectively will both die in the nth year from the present time; (2) the "probability that one or both of them will die in that year."

Using Jones's notation, the probability that x will live $n-1$ years is $p_{x.n-1}$, and the chance that he will live n years is $p_{x.n}$; therefore the chance that he will die in the nth year is $p_{x.n-1}-p_{x.n}$.

So the chance that y will die in the nth year is $p_{y.n-1}-p_{y.n}$, and the chance that *both* will die in that year is therefore

$$(p_{x.n-1}-p_{x.n})(p_{y.n-1}-p_{y.n}),$$

Again, the chance that x will *not* die in the nth year is $1-(p_{x.n-1}-p_{x.n})$, and the chance that y will not die in the nth year is $1-(p_{y.n-1}-p_{y.n})$. Hence the chance that neither x nor y will die in that year is

$$(1-p_{x.n-1}+p_{x.n})(1-p_{y.n-1}+p_{y.n});$$

and, consequently, the chance that one or both will die in that year is

$$1-(1-p_{x.n-1}+p_{x.n})(1-p_{y.n-1}+p_{y.n}).$$

5. "Find the present value of a sum of money due at the end of any "time, assuming the operation of compound interest."

Vide Jones on Annuities, p. 13, art. 21; *Colenso's Algebra*, p. 155, art. 198.

"Prove that the value of £1 due at the end of $p+q$ years=value of "£1 due in p years × value of £1 due in q years."

If i be the interest of £1 for 1 year—

then the value of £1 due in p years is $(1+i)^{-p}$,

" " q " $(1+i)^{-q}$,

and " " $p+q$ " $(1+i)^{-(p+q)}$.

Now, $(1+i)^{-p}\times(1+i)^{-q}=(1+i)^{-(p+q)}$,

which proves the property stated.

6. "Describe a practical method of calculating a table of the value ot "£1 due at the end of any number of years from 1 to 100; and show how "the results may be employed to calculate a table of the values of annuities "certain."

See *Gray*, p. 22.

"*Ex.*—The value of £1 due at the end of 10 years, at 4 per cent., "is ·67556417: deduce the first ten terms of the value of an annuity "for any number of years."

Pursuing the method explained by Mr. Gray in the passage last referred to, the process will be as follows:—

$$
\begin{array}{lll}
(1+i)^{-10} = & \cdot 67556417 & \\
 & 2702257 & = \cdot 04\times(1+i)^{-10} \\
\hline
(1+i)^{-9} = & \cdot 70258674 & = (1+\cdot 04)(1+i)^{-10} \\
 & 2810347 & \\
\hline
(1+i)^{-8} = & \cdot 73069021 & \\
 & 2922761 & \\
\hline
(1+i)^{-7} = & \cdot 75991782 & \\
 & 3039671 & \\
\hline
(1+i)^{-6} = & \cdot 79031453 & \\
 & 3161258 & \\
\hline
(1+i)^{-5} = & \cdot 82192711 & \\
 & 3287708 & \\
\hline
(1+i)^{-4} = & \cdot 85480419 & \\
 & 3419217 & \\
\hline
(1+i)^{-3} = & \cdot 88899636 & \\
 & 3555985 & \\
\hline
(1+i)^{-2} = & \cdot 92455621 & \\
 & 3698225 & \\
\hline
(1+i)^{-1} = & \cdot 96153846 & \\
 & 3846154 & \\
\hline
(1+i)^{-0} = & 1\cdot 00000000 &
\end{array}
$$

Again, for the values of the annuities—

$$
\begin{array}{rcrl}
(1+i)^{-1} & = & \cdot 96153846 & = a_{\overline{1}|} \\
(1+i)^{-2} & = & 92455621 & \\
\hline
 & & 1\cdot 88609467 & = a_{\overline{2}|} \\
(1+i)^{-3} & = & 88899636 & \\
\hline
 & & 2\cdot 77509103 & = a_{\overline{3}|} \\
(1+i)^{-4} & = & 85480419 & \\
\hline
 & & 3\cdot 62989522 & = a_{\overline{4}|} \\
(1+i)^{-5} & = & 82192711 & \\
\hline
 & & 4\cdot 45182233 & = a_{\overline{5}|} \\
(1+i)^{-6} & = & 79031453 & \\
\hline
 & & 5\cdot 24213686 & = a_{\overline{6}|} \\
(1+i)^{-7} & = & 75991782 & \\
\hline
 & & 6\cdot 00205468 & = a_{\overline{7}|} \\
(1+i)^{-8} & = & 73069021 & \\
\hline
 & & 6\cdot 73274489 & = a_{\overline{8}|} \\
(1+i)^{-9} & = & 70258674 & \\
\hline
 & & 7\cdot 43533163 & = a_{\overline{9}|} \\
(1+i)^{-10} & = & 67556417 & \\
\hline
 & & 8\cdot 11089580 & = a_{\overline{10}|}
\end{array}
$$

The results in the former part of the process may be compared with those given in *Jones*, p. 80; and those in the latter part, with *Jones*, p. 92.

7. "Explain what is meant by a table of mortality, and state the "different methods in which the facts embodied in it may be exhibited."

For information on the subject of this question, reference may be made to De Morgan's *Essay on Probabilities*, pp. 161, 2.

8. "Give some account of the origin and relative merits of the tables "of mortality known as the Carlisle, Equitable, Experience, and English "Life Tables."

Vide Registrar-General's 5th, 6th, and 12th Reports, Dr. Farr's Appendices; *De Morgan on Probabilities*, p. 165; *Milne on Annuities*, p. 404; *Encyclopædia Britannica*, art. "Mortality"; *Morgan's Equitable Society Tables*; *Davies on Annuities*, chap. 3; *Jenkin Jones, New Rate of Mortality*, Introduction.

9. "State accurately what is meant by the expectation (or mean dura-"tion) of life at any age according to a given table of mortality."

On this point see *Gray*, art. 135 and 162–6.

"Prove the formula for calculating the chance of living a year at any "age from the expectations—

$$p_x = \frac{e_x - \frac{1}{2}}{e_{x+1} + \frac{1}{2}}."$$

It is proved (*Jones*, art. 122) that

$$e_x = \tfrac{1}{2} + \frac{l_{x+1} + l_{x+2} + \ldots\ldots + l_{x+s}}{l_x}.$$

Thus then,

$$e_x = \tfrac{1}{2} + \frac{l_{x+1}}{l_x} \cdot \frac{l_{x+1} + l_{x+2} + \ldots\ldots + l_{x+s}}{l_{x+1}}$$

$$= \tfrac{1}{2} + \frac{l_{x+1}}{l_x}\left(1 + \frac{l_{x+2} + l_{x+3} + \ldots\ldots + l_{x+s}}{l_{x+1}}\right).$$

Now, $\frac{l_{x+1}}{l_x} = p_x$, and $e_{x+1} = \frac{1}{2} + \frac{l_{x+2} + l_{x+3} + \ldots\ldots + l_{x+s}}{l_{x+1}}$.

Hence by substitution $\quad e_x = \frac{1}{2} + p_x(\frac{1}{2} + e_{x+1})$,

whence we get $\quad p_x = \frac{e_x - \frac{1}{2}}{e_{x+1} + \frac{1}{2}}$.

See also *Gray*, art. 152, 3.

10. "Prove that if $e_{x-1} = e_x = e_{x+1}$, then will $p_{x-1} = p_x$; but that if "$p_{x-1} = p_x = p_{x+1}$, e_{x-1} will not be equal to e_x unless $e_{x+2} = \frac{1}{2} \cdot \frac{1+p_x}{1-p_x}$."

By the formula in the last question,

$$p_x = \frac{e_x - \frac{1}{2}}{e_{x+1} + \frac{1}{2}}, \quad p_{x-1} = \frac{e_{x-1} - \frac{1}{2}}{e_x + \frac{1}{2}}, \quad p_{x+1} = \frac{e_{x+1} - \frac{1}{2}}{e_{x+2} + \frac{1}{2}}.$$

If, then, $e_{x-1} = e_x = e_{x+1}$, it is clear that $p_x = p_{x-1} = \frac{e_x - \frac{1}{2}}{e_x + \frac{1}{2}}$.

Again, if $\quad p_{x-1} = p_x = p_{x+1}$,

we have $\quad p_x = \frac{e_{x-1} - \frac{1}{2}}{e_x + \frac{1}{2}} = \frac{e_x - \frac{1}{2}}{e_{x+1} + \frac{1}{2}} = \frac{e_{x+1} - \frac{1}{2}}{e_{x+2} + \frac{1}{2}}$;

whence

$$e_{x+1} = \tfrac{1}{2} + p_x(e_{x+2} + \tfrac{1}{2}),$$
$$e_x = \tfrac{1}{2} + p_x(e_{x+1} + \tfrac{1}{2}),$$
$$e_{x-1} = \tfrac{1}{2} + p_x(e_x + \tfrac{1}{2}).$$

From these equations, it appears that e_{x-1} will not be equal to e_x unless $e_x = e_{x+1} = e_{x+2}$, and then the first of them gives

$$e_{x+2} = \tfrac{1}{2} + p_x(e_{x+2} + \tfrac{1}{2});$$

from which it follows, that

$$e_{x+2} = \tfrac{1}{2} \cdot \frac{1+p_x}{1-p_x}.$$

11. "The value of an annuity on the life of a person of a given age is "frequently supposed to be equal to that of an annuity certain for a number "of years equal to the mean duration of life at that age. Explain why this "is not the case, and state upon what hypothesis it would be true."

See *Jones*, art. 127, p. 122; *De Morgan on Probabilities*, p. 188.

The following mathematical proof will probably be considered by some persons more satisfactory than the reasonings in the passages above referred to.

The value of an annuity on the life x is

$$\frac{l_{x+1}}{l_x}.r+\frac{l_{x+2}}{l_x}.r^2+\ldots\ldots+\frac{l_{x+s}}{l_x}.r^s,$$

or,
$$p_1r+p_2r^2+\ldots\ldots+p_sr^s, \text{ suppose;}$$

where it must be noticed that all the quantities $p_1, p_2 \ldots\ldots p_s$, are from the nature of the case, less than unity. Now let n be the number of complete years in the mean duration of life at the age x, then the value of an annuity certain for n years is

$$r+r^2+r^3+\ldots\ldots+r^n,$$

and we have

$$n+\text{a fraction}=\tfrac{1}{2}+\frac{l_{x+1}+l_{x+2}+\ldots\ldots+l_{x+s}}{l_x},$$

or
$$n+\delta=\tfrac{1}{2}+p_1+p_2+\ldots\ldots+p_s.$$

Then the value of the annuity on the life x will be greater or less than the annuity certain for n years, according as

$$p_1r+p_2r^2+\ldots\ldots+p_sr^s > \text{or} < r+r^2+\ldots\ldots+r^n,$$

and therefore according as

$$p_{n+1}r^{n+1}+\ldots\ldots+p_sr^s > < (1-p_1)r+(1-p_2)r^2+\ldots\ldots+(1-p_n)r^n.$$

Now, since $r<1$, the first member of the inequality must be

$$< p_{n+1}r^{n+1}+p_{n+2}.r^{n+1}+\ldots\ldots+p_s.r^{n+1},$$
$$< (p_{n+1}+p_{n+2}+\ldots\ldots+p^s)r^{n+1};$$

and the second member of the inequality must be

$$> (1-p_1+1-p_2+\ldots\ldots+1-p_n)r^n,$$
$$> (n-p_1-p_2-\ldots\ldots-p_n)r^n,$$
$$> (p_{n+1}+p_{n+2}+\ldots\ldots+p_s+\tfrac{1}{2}-\delta)r^n.$$

If, then, we suppose δ to be not greater than $\frac{1}{2}$, or if we take n to be the *nearest* integer to the number expressing the mean duration of life, instead of the integer next less than it, we shall have

$$(p_{n+1}+p_{n+2}+\ldots\ldots+p_s)r^{n+1} < (p_{n+1}+p_{n+2}+\ldots\ldots+p_s+\tfrac{1}{2}-\delta)r^n,$$
$$\therefore p_{n+1}r^{n+1}+p_{n+2}r^{n+2}+\ldots\ldots+p_sr^s$$
$$< (1-p_1)r+(1-p_2)r^2+\ldots\ldots+(1-p_n)r^n,$$

whence the value of the annuity on the life x is always less than that of the annuity certain for the number of years indicated by the nearest integer to the number expressing the mean duration of life.

12. "If B represents the present value of a benefit of £1 upon a given "life (x), B_1 the same upon a life one year older ($x+1$), p the probability "of a payment of B being received in the first year, and Π the probability "of (x) surviving a year, prove that

$$\log B = \log v\,\Pi + \log\left(\frac{p}{\Pi} + B_1\right)."$$

For demonstration of this formula, see *Gray*, art. 193, 4.

13. "Describe the process of calculating a table of annuities by means "of the formula $a_{m-1} = (1+a_m)p_{m-1,1}.r$; and show that this formula is a "particular case of the one in the last question."

For the first part of this question, see *Jones*, art. 118, 9; for the second part, see *Gray*, art. 202.

14. "Explain fully the construction and use of columns D, N, and M, "in the columnar method of calculating the values of annuities and assur-"ances; also state the superior advantages this method possesses over the "old one."

See *Jones*, arts. 112–6; *Gray*, arts. 296–311; *De Morgan*, *Companion to the Almanac* for 1840.

15. "Prove that the value of an annuity of £1 during the joint lives of "x and y, and for t years afterwards, should x survive so long, is

$$a_x - \frac{D_{x+t}}{D_x}(a_{x+t} - a_{x+t.y})."$$

This problem has been investigated by Jones (art. 161), who has given, for the annuity in question, the formula—

$$a_{x_t\rceil} + \frac{a_{x.y-t\rceil t}}{p_{y-t,t}}.$$

This may be shown, as follows, to be equivalent to the formula given above.

We have
$$a_{x_t\rceil} = a_x - p_{x.t}r^t.\,a_{x+t}$$
$$= a_x - \frac{D_{x+t}}{D_x}.\,a_{x+t}.$$

Again,
$$a_{x.y-t\rceil t} = p_{(x.y-t)t}.\,r^t a_{x+t.y},$$

and, consequently,

$$\frac{a_{x.y-t\rceil t}}{p_{y-t.t}} = \frac{p_{(x.y-t)t}}{p_{y-t.t}}.\,r^t.\,a_{x+t.y}$$
$$= p_{x.t}r^t.\,a_{x+t.y}$$
$$= \frac{D_{x+t}}{D_x}.\,a_{x+t.y}.$$

Substituting these values in (1), we get the formula stated in the question.

On this point reference may be made to the *Assurance Magazine*, vol. vii., p. 239; or the formula may be proved by a process of general reasoning similar to that employed in the *Assurance Magazine*, vol. vii., p. 174.

16. "Give an expression for the annual premium for a contingent "annuity to commence at the death of A and to continue as long as either "B or C is living."

It will be noticed that it is only required here that the formula should be stated, not proved.

If x, y, z, are the ages of the lives at the present time, the annual premium will be

$$\frac{a_y+a_z-a_{y.z}-a_{x.y}-a_{x.z}+a_{x.y.z}}{1+a_{x.y}+a_{x.z}-a_{x.y.z}}.$$

See *Jones*, art. 165.

17. "Find, in a convenient form for computation, the single and annual "premiums for an annuity to commence at the death of y and continue pay-"able during the remainder of x's life, but to be payable only if y dies within "t years."

If, to the annuity described in the question, we add a reversionary annuity to x after y, payable only in case both x and y survive t years, the sum will be an ordinary reversionary annuity to x after y, the formula for which is a_x-a_{xy}. But the formula for the deferred reversionary annuity contingent on x and y both surviving t years is easily seen to be

$$p_{(x.y)t}\cdot v^t.\{a_{x+t}-a_{x+t.y+t}\},$$

$$\text{or } \frac{D_{x+t.y+t}}{D_{x.y}}\{a_{x+t}-a_{x+t.y+t}\}.$$

Hence the single premium for the proposed annuity will be

$$a_x-a_{xy}-\frac{D_{x+t.y+t}}{D_{x.y}}\{a_{x+t}-a_{x+t.y+t}\}.$$

Also, the divisor for the annual premium will be $1+$an annuity for $t-1$ years on the joint lives of x and y,

$$=1+a_{x.y\overline{t-1}|}=1+a_{x.y}-\frac{D_{x+t.y+t}}{D_{x.y}}\{1+a_{x+t.y+t}\}.$$

18. "Prove that the single premium for an assurance payable on a life "now aged n years attaining the age of $n+t$, or dying previously, may be "represented by the formula (Jones's notation), $\frac{1-ia_{n\overline{t-1}|}}{1+i}$; and show that "this formula is equivalent to $\frac{M_n-M_{n+t}+D_{n+t}}{D_n}$."

The annual interest on £1 being i, it is obvious that £1 is equal in value to an annuity of i for any term and a capital sum of $1+i$ at the end

of that term,—the sum $1+i$ being payable a year after the last payment of the annuity. Now let that term be $t-1$ years contingent on the existence of the life n, then

1 = an annuity of i for $t-1$ years on the life n
$+(1+i)$ receivable at the end of the year in which n dies, or at the end of n years, whichever first happens;

or,
$$1 = i a_{n_{\overline{t-1}|}} + (1+i)\mathrm{R}$$

putting R for the value of the assurance in question.

Hence $\mathrm{R} = \dfrac{1 - i a_{n_{\overline{t-1}|}}}{1+i}$, as stated in the question.

Next to prove that this formula is equivalent to $\dfrac{\mathrm{M}_n - \mathrm{M}_{n+t} + \mathrm{D}_{n+t}}{\mathrm{D}_n}$, which is easily seen to be another expression for the same assurance. (*V. Jones*, p. 212.)

We have
$$a_{n_{\overline{t-1}|}} = \frac{\mathrm{N}_n - \mathrm{N}_{n+t-1}}{\mathrm{D}_n}.$$

Hence
$$\frac{1 - i a_{n_{\overline{t-1}|}}}{1+i} = r - \frac{d\mathrm{N}_n - d\mathrm{N}_{n+t-1}}{\mathrm{D}_n}$$
$$= \frac{r\mathrm{D}_n - d\mathrm{N}_n + d\mathrm{N}_{n+t-1}}{\mathrm{D}_n}.$$

But, by the construction of the column M, we have the relations (see *Gray*, art. 305, p. 118),
$$\mathrm{M}_n = \mathrm{D}_n - d\mathrm{N}_{n-1}$$
$$= \mathrm{D}_n - d(\mathrm{D}_n + \mathrm{N}_n)$$
$$= r\mathrm{D}_n - d\mathrm{N}_n;$$

and
$$\mathrm{M}_{n+t} = \mathrm{D}_{n+t} - d\mathrm{N}_{n+t-1},$$

whence
$$d\mathrm{N}_{n+t-1} = \mathrm{D}_{n+t} - \mathrm{M}_{n+t}.$$

We thus have
$$r\mathrm{D}_n - d\mathrm{N}_n + d\mathrm{N}_{n+t-1} = \mathrm{M}_n + \mathrm{D}_{n+t} - \mathrm{M}_{n+t},$$

which proves the equivalence of the two expressions.

19. "Give an expression for the single premium for an assurance on "the life of A provided he die after B."

The expression required is
$$\mathrm{A}_x - \mathrm{A}_{\underset{1}{x.y}},$$

x and y being the ages of A and B. See *Jones*, art. 216.

20. "Prove that $\mathrm{A}_{(\underset{1}{x.y})\urcorner t} = r^t . p_{(x.y)t} . \mathrm{A}_{\underset{1}{x+t.y+t}}$."

Adopting Jones's notation, let $q_{(\underset{1}{x.y})n}$ denote the chance of x dying before y in the nth year from the present time; then
$$\mathrm{A}_{(\underset{1}{x.y})\urcorner t} = r^{t+1} q_{(\underset{1}{x.y})_{t+1}} + r^{t+2} q_{(\underset{1}{x.y})_{t+2}} + \&c.$$

But a little consideration shows that $q_{\underset{1}{(x.y)}t+n}$ is compounded of two chances—first, the chance that x and y will both live t years; and, secondly, the chance that, having survived t years, x will die before y in the nth year from that time, which is the same thing as the chance that a life $x+t$ will die before $y+t$ in the nth year. Hence—

$$q_{\underset{1}{(x.y)}t+n}=p_{(x.y)t}\cdot q_{\underset{1}{(x+t.y+t)n}}.$$

Making n in this equation successively equal to 1, 2 . . . . , and substituting above, we have

$$\begin{aligned}A_{\underset{1}{(x.y)}}\urcorner_t &= r^{t+1}p_{(x.y)t}\cdot q_{\underset{1}{(x+t.y+t)1}}+r^{t+2}p_{(x.y)t}\cdot q_{\underset{1}{(x+t.y+t)2}}+\&c.\\ &= r^t p_{(x.y)t}\{rq_{\underset{1}{(x+t.y+t)1}}+r^2q_{\underset{1}{(x+t.y+t)2}}+\&c.\}\\ &= r^t p_{(x.y)t}\cdot A_{\underset{1}{x+t.y+t}}.\end{aligned}$$

On this subject see *Gray*, art. 259, 60.

21. "Determine the value of an assurance on a life of 30, payable at "the age of 60 or previous death, commencing at £a, increased by £p at "the end of 5 and 10 years respectively, and thereafter increasing by £q "*per annum.*"

The value of the assurance of £a, payable at 60 or death, is

$$a\,.\,\frac{M_{30}-M_{60}+D_{60}}{D_{30}}.$$

The values of the deferred assurances of £p, payable at 60 or death, are

$$p\,.\,\frac{M_{35}-M_{60}+D_{60}}{D_{30}},\ p\,.\,\frac{M_{40}-M_{60}+D_{60}}{D_{30}}.$$

The value of the increasing assurance of £q *per annum* after 10 years will be

$$\begin{aligned}&q\frac{M_{41}-M_{60}+D_{60}}{D_{30}}+q\frac{M_{42}-M_{60}+D_{60}}{D_{30}}+\ldots\\ &\qquad+q\frac{M_{59}-M_{60}+D_{60}}{D_{30}}+q\frac{M_{60}-M_{60}+D_{60}}{D_{30}}\\ &=q\frac{M_{41}+M_{42}+\ldots+M_{60}}{D_{30}}+20q\frac{D_{60}-M_{60}}{D_{30}}\\ &=q\frac{R_{41}-R_{61}}{D_{30}}+20q\frac{D_{60}-M_{60}}{D_{30}}.\end{aligned}$$

Adding the different quantities together, the value of the assurance proposed is

$$\frac{aM_{30}+p(M_{35}+M_{40})+q(R_{41}-R_{61})+(a+2p+20q)(D_{60}-M_{60})}{D_{30}}.$$

22. "Prove the formulæ for the value of a policy of £1—

$$(1)\ 1-\frac{1+a_{m+n}}{1+a_m},\quad (2)\ (P_{m+n}-P_m)(1+a_{m+n}),$$

$$(3)\ 1-(d+P_m)(1+a_{m+n}).\text{"}$$

For (1) and (2), see *Jones*, art. 254.

The formula (3) is deduced from (1) by means of the equation (*Jones*, art. 191)—

$$P_m=\frac{1}{1+a_m}-(1-r)=\frac{1}{1+a_m}-d,$$

whence
$$P_m+d=\frac{1}{1+a_m}.$$

"Show that the same forms apply when the policy is on the joint "duration of two lives."

In order to prove this, it is only necessary to notice that all the formulæ employed in the proof of the above expressions are equally true whether the policy is on one life or on the first of two lives; and, therefore, those expressions must also be equally applicable in both cases.

23. "Find the value of an annuity on two successive lives, x and y, "of which the second is to be nominated at the death of the first, and is "supposed to be then y years of age."

See *Jones*, art. 238; or *Assurance Magazine*, vol. ii., p. 5.

"Also show that if I represent the value of a perpetuity of £1, the "value of the annuity will be equal to $I-(1+I)A_xA_y$."

If to the annuity on the successive lives be added a perpetuity, to commence on the death of y, and the first payment to be made at the end of the year in which y dies, the sum will be equal to a perpetuity from the present time=I. Now the value of the deferred perpetuity-due will be that of an assurance of $1+I$ on death of y, the value of which will be $(1+I)A_y$ at the time y is nominated, *i.e.* at the end of the year in which x dies, and therefore, equal to $(1+I)A_xA_y$ at the present time. Hence the value of the annuity on the succession is, as stated in the question, $I-(1+I)A_xA_y$.

The formula given by Jones is $a_x+\frac{1-ia_x}{1+i}(1+a_y)$, which may be shown to be equivalent to the above, as follows:—

We have
$$\frac{1-ia_x}{1+i}=A_x.$$

Hence $1-ia_x=(1+i)A_x$; or, since $\frac{1}{i}=I$, $I-a_x=(1+I)A_x$, and

$$a_x=I-(1+I)A_x.$$

So $a_y=I-(1+I)A_y$, and

$$1+a_y=1+I-(1+I)A_y=(1+I)(1-A_y).$$

Substituting these values in Jones's formula, it becomes

$$I-(1+I)A_x+A_x(1+I)(1-A_y)=I-(1+I)A_xA_y.$$

24. "Find the single premium for an annuity to x after the death of y, "with the condition that the premium is to be returned if x die before y."

Let P be the premium required, then the benefit to be insured is an annuity to x after y, or a_x-a_{xy}, and an insurance of P if x die before y, or $P.A_{\substack{x.y\\1}}$; and these together must be equal to P.

Thus, then, $$a_x-a_{xy}+PA_{\substack{x.y\\1}}=P,$$

whence $$P(1-A_{\substack{x.y\\1}})=a_x-a_{xy},$$

and $$P=\frac{a_x-a_{xy}}{1-A_{\substack{x.y\\1}}}.$$

25. "A, aged x years, is entitled to the interest of £1 for life. If A "die within t years, the interest is payable to B or his representatives "till the expiration of the t years; when C, if living, is entitled to the "capital. If, however, C, now aged y years, die either before A or within "the t years, the capital reverts to B or his representatives. Determine "the values of B's and of C's interests."

Let P and Q denote the values of B's and C's interests, then B's interest during t years, or $P_{t\rceil}$, consists of two parts, first his interest in the income (£i per annum) which he will receive after the death of A and to the end of the t years if C live so long; secondly, his chance of receiving the capital by the death of both x and y within the t years. In this case it must be remembered that he will be entitled to a year's income in addition to the capital, at the end of the year in which the last survivor of x and y dies, so that this second part is equal to an insurance of $1+i$ on the death of the last survivor of x and y provided that death take place within t years.

Hence $$P_{t\rceil}=i(a_{y_{t\rceil}}-a_{x.y_{t\rceil}})+(1+i)(A_{x_{t\rceil}}+A_{y_{t\rceil}}-A_{x.y_{t\rceil}}).$$

Again, B's interest after the expiration of the t years consists of the chance of his becoming entitled to the capital and one year's income, on the death of A after the expiration of t years, C having died before A.

Thus $$P_{\rceil t}=(1+i)A_{\substack{(x.y)\rceil t\\2}}=(1+i)\{A_{x\rceil t}-A_{\substack{(x.y)\rceil t\\1}}\}.$$

Adding the two parts of B's interest together, we have

$$\begin{aligned}P&=P_{t\rceil}+P_{\rceil t}\\&=i(a_{y_{t\rceil}}-a_{x.y_{t\rceil}})+(1+i)\{A_{x_{t\rceil}}+A_{y_{t\rceil}}-A_{x.y_{t\rceil}}+A_{x\rceil t}-A_{\substack{x.y\rceil t\\1}}\}\\&=i(a_{y_{t\rceil}}-a_{x.y_{t\rceil}})+(1+i)\{A_x+A_{y_{t\rceil}}-A_{x.y_{t\rceil}}-A_{\substack{x.y\rceil t\\1}}\}.\quad\ldots\ (1)\end{aligned}$$

Now it is proved (*Jones*, art. 197) that

$$(1+i)A_{y_{t\rceil}}=1-r^t p_{y.t}-i\,a_{y_{t\rceil}},$$

whence $$i\,a_{y_{t\rceil}}+(1+i)A_{y_{t\rceil}}=1-r^t p_{y.t}.^*$$

So also $$i\,a_{x.y_{t\rceil}}+(1+i)A_{x.y_{t\rceil}}=1-r^t p_{(x.y)t}.$$

Substituting now in equation (1) we get

$$P=(1+i)(A_x-A_{x.y_{\underset{1}{\rceil}t}})-r^t p_{y.t}+r^t p_{(x.y)t}$$

$$=(1+i)(A_x-A_{x.y_{\underset{1}{\rceil}t}})-r^t p_{y.t}(1-p_{x.t}).$$

For the value of C's interest, we notice that during the t years it is zero, and after t years it is equal to an assurance of $1+i$ at the death of x, provided that take place after the t years and y be then surviving, and to an endowment of £1 at the end of t years contingent on y being then alive and x previously dead. It is thus equal to

$$(1+i)A_{x.y_{\underset{1}{\rceil}t}}+r^t p_{y.t}(1-p_{x.t}).$$

Adding together the two values, we get

$$P+Q=(1+i)A_x,$$

as we evidently ought, since the sum of the interests of B and C is equal to a reversion to the capital and one year's income on the death of A.

DR. FARR'S HEALTHY LIFE TABLE.

To the Editor of the Assurance Magazine.

SIR,—I forward to you D, N, and S, computed from Dr. Farr's Healthy Life Table, to enable Actuaries to compare it with other tables.

I have not added any numerical comparisons; but my own opinion, at present, is, that Dr. Farr's Table is about as much an advance on the Carlisle Table at that was on the Northampton. But whether the Assurance Offices will adopt it, is another question. As regards the law of mortality, I may mention that there is a remarkable table by Brune, derived from the Prussian returns made during a long course of years, which, on account of the well-known care exercised, well deserves attention.

I am, Sir, &c., &c.,

WM. DAVIS.

22, *Grove Place, St. John's Wood.*
May 13*th*, 1861.

* This equation may be established as follows:—The yearly interest of £1 being £i, it is obvious that £1 is equal in value to an annuity of £i for any term and an assurance of $1+i$ at the end of that term: so £1 is equal in value to a temporary annuity of £i on the life of y, together with a temporary assurance of $1+i$ on y, together with an endowment of £1 at end of t years if y is then alive; which is exactly what is expressed by the equation in question.

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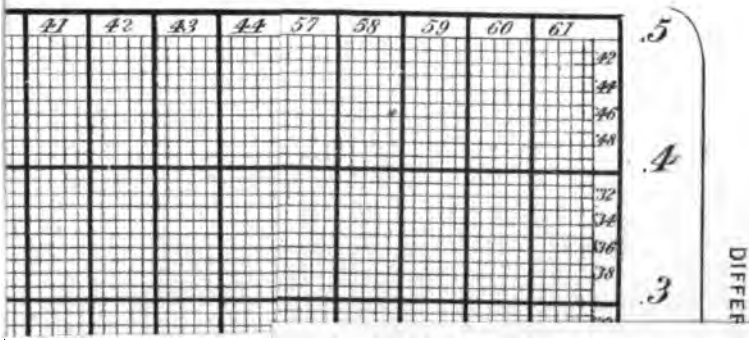


Values of Annuities at 4%

THE

ASSURANCE MAGAZINE,

AND

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

Observations submitted by WILLIAM SPENS, *Manager of the Scottish Amicable Life Assurance Society, in presenting to the Institute Tables of the Mortality Experience of that Society from* 1826 *to* 1860.

[Read before the Institute 25th November, 1861, and printed by order of the Council.]

AS stated in the introduction to the Tables, they relate to 10,255 persons, as under—

	Entered.	Discontinued.	Died.	Existing.
1. Non-hazardous—Males . .	8,596	2,550	632	5,414
2. Non-hazardous—Females . .	890	247	63	580
3. Hazardous—Both sexes (exclusive of West Indies)	431	132	47	252
4. Hazardous—Both sexes (West Indies)	338	129	31	178
Total . .	10,255	3,058	773	6,424

And they embrace the experience of the first thirty-four years of the Society—reckoning, as was found most suitable, the first year's experience as the experience of the persons entering in the first year for one year from their respective entries, and so on. In this way it will be found that the experience does not include any of the persons entering in 1860, and none of the deaths in 1860 of persons who had been assured for more than a year from the time of payment of their annual premium in 1859. These deaths,

and the entries of 1860, fall, according to the principle adopted, to be included in the experience of the thirty-fifth year of the Society.

The sums of the numbers of lives exposed to risk during each year of age amount to—

For Non-hazardous—Males	48,348·0
„ Non-hazardous—Females	4,518·0
„ Hazardous (both sexes), exclusive of West Indies	1,964·0
„ Hazardous (both sexes), West Indies . .	1,471·5
Total . . .	56,301·5

The average duration of the lives being—

For Non-hazardous—Males	5·62
„ Non-hazardous—Females	5·07
„ Hazardous (both sexes), exclusive of West Indies .	4·55
„ Hazardous (both sexes), West Indies . . .	4·35
„ Total	5·49

It will be thus found that a large proportion of the entries has been recent. While the experience has this disadvantage, it is conceived that its value—more particularly as bearing on the question of the value of selection of lives by Assurance Offices—is, on the other hand, much enhanced by the non-hazardous lives being stated separately—by the non-hazardous male lives being kept separate from the female lives—and by the lives at risk and the deaths being stated both for each year of the assurances and for each year of the Society.

It is unnecessary to allude to the well-known very great labour which is involved preparatory to the exhibition of tables of this nature. It is proper, however, that I should notice—

1st. That the policies were first arranged according to the years of birth of the lives assured, which gave facility for deleting policies in cases where more than one was held on the same life, so that not more than one of these should be reckoned at risk at the same time.

2nd. That the ages at entry, as for the first year's experience of a life, were taken as the difference between the year of birth and the year of entry—a mode which evidently (on the assumption, at least, of parties being born equally at any time of a year) states the ages on an average correctly.

3rd. That in the case of policies cancelled otherwise than by death, when the broken period was less than half a year, 0 was reckoned; when half a year, ·5; when more than a half, 1.

The work was ably executed by, or under the immediate personal superintendence of, the Secretary of the Society, with the assistance of a careful clerk. Besides that I think it due to Mr. Stott to make this statement, I consider that it is desirable that parties who may refer to, or make use of, the experience, should be aware that the details have this voucher for their correctness.

The tables show—

For each year of the Society, and of the Assurances, the lives at risk and the deaths in the above four classes—abstracts, in quinquennial groups of ages, following each table.

For each year of the Society, and of the Assurances, the deaths, actual and expected by the English Life Table No. 2, for non-hazardous males and females separately—and by the Northampton Table, for non-hazardous males and females jointly—in quinquennial groups of ages.

Totals at risk and deaths, actual and expected by various tables of mortality, arranged according to the classes in quinquennial groups of ages.

And there are added—

Nosological Tables, classified, in accordance with 20th and previous Reports of the Registrar-General, by Dr. Fleming, the Medical Officer of the Society, in decennial groups of ages, showing the special causes of death at the different ages; and, as regards the general classes of death, the duration of the assurances at the different ages; also, in the case of the non-hazardous males, proportioning the expected deaths by the English Life Life Table according to the deaths, 1848 to 1854, in Registrar-General's 18th Report—showing the shortcoming or excess of the actual deaths.

It was thought better to present the Tables without comment, and that I should submit to the Institute my observations upon some points which I think these and other Tables illustrate. It may be thought that in some respects the Tables are too minute and too detailed—and, if our own experience were to be solely

looked to, I dare say they might be considered too elaborate—but it is undoubtedly by such subdivisions and details that advances in our knowledge of the subject are to be made; and I feel confident that, as regards after combinations and comparisons which may be made with the like experience of other Offices, the labour will be found to have been well bestowed.

It appears from Table TD, that, during the whole period of the experience, the actual deaths in the Scottish Amicable Society have been the following percentages of calculated deaths by various tables of mortality:—

TABLE I.

	Percentages which Actual Deaths are of Deaths by various Tables.			
	Northampton.	E. L., No. 2.	Carlisle.	Experience, Table G.
Males—Non-hazardous	53·95	81·34	88·09	91·17
Females—Non-hazardous	54·08	82·63	84·96	85·56
Hazardous—Both sexes (exclusive of West Indies)	96·91	145·44	157·46	162·86
West Indies	95·34	153·61	163·09	177·68
Total	56·48	85·34	91·98	95·06

The above does not show any extra mortality among the female lives. An examination, however, of the details of the experience exhibits the following result:—

TABLE II.

	Percentage which Actual Deaths are of Deaths by English Life Table, No. 2, Males and Females respectively.	
	First Six Years.	After Six Years.
Males—Non-hazardous	69·7	98·2
Females—Non-hazardous	79·2	89·3

The observations on female life are too limited to draw a conclusion with any confidence, but it is believed that during the currency of the experience a greater proportion of ineligible lives has been proposed among females, and there are probably greater difficulties in the medical examination. Such causes would undoubtedly operate in the way of producing the result above indicated—viz., a greater percentage of deaths among the females at

first, followed, after the worst lives may have died out, by confirmation of the superior vitality of the females.

The experience of the West India risks cannot fail to be interesting to the Institute. It will be seen that the mortality is not far from double that of the non-hazardous males, and is about equal to that according to the Northampton Table. On the one hand, I am willing to believe that the percentage of deaths experienced is beyond what may be expected in favoured spots; and, on the other hand, it must be kept in view that the experience of these risks is only for an average of 4·35 years, and the prevalence of some severe tropical disease may yet materially add to the rate of mortality.

It is to be observed, in reference to the deaths among the hazardous lives, exclusive of the West Indies, that four of them occurred in a small Friendly Society for assurances which the Scottish Amicable took up—viz., two in the first year of the assurances and two in the second—of lives which would not otherwise have been assured.

The total number exposed to risk, of the whole assurances, during the first year, has been 10,109·5; and the total deaths during the first year, including the two noticed above and at least two deaths by suicide, 73—showing a percentage of ·72 or ·68, exclusive of above four deaths. This percentage does not appear to be below what might have been expected in a Society where no large proportion of the risks has been accepted before the period when careful medical examinations were made. It is above the percentage (·55) of sums assured becoming claims in the first year, deduced from an amount of about fifty millions assured. (See *Assurance Magazine*, vol. iv., p. 4.) It is right that I should mention that it is considerably under the experience which has been furnished me of an old Office, the deaths being 90 out of 7,419, and the percentage thus 1·213. From Mr. Neison's valuable analysis of the Gotha experience, in his *Contributions to Vital Statistics*, it would appear, from Tables VIII. and XV., pp. 165 and 191, that the percentage of deaths in the first year, 1840 to 1849, is about ·85. The Manager of an extensive American Assurance Office has been so good as to send me a note of the number of policies which have been exposed to risk during the different years of the assurances and the policies becoming claims; and I think it may be well that I exhibit this, along with similar experience of the Scottish Amicable, for the first six years:—

TABLE III.

American Office Policies.			Year of Assurances.	Scottish Amicable Lives.		
No. Exposed.	Deaths.	Per Cent.		No. Exposed.	Deaths.	Per Cent.
14,869	102	·68	1	10,109½	73	·72
11,902	112	·94	2	8,393	83	·98
9,873	91	·92	3	7,082½	82	1·15
8,164	102	1·25	4	5,875½	62	1·05
6,780	81	1·19	5	4,728	58	1·22
5,650	60	1·06	6	3,886½	58	1·49

It will be observed that the percentage in the American Society in the first year is ·68; that of the Scottish Amicable, ·72. In the second year the percentages are ·94 in the American Society and ·98 in the Scottish Amicable. In the experience of the London Equitable, involving as it does 21,398 lives, the deaths during, not the first year of assurance, but the first calendar year after entry, seems to be represented as 105—showing, after giving effect to discontinuances, a percentage of ·491. While, I believe, this is very near the percentage on non-hazardous lives as presently selected during the first year of the assurances, and the percentage in such Offices as deal little, or not at all, with hazardous cases, the percentage thus stated for the Equitable for the first calendar year after entry, contrasted with that of the second year, 1·167, and 245 the number of deaths, is very startling. I find that, in the Scottish Amicable, the deaths in the *first calendar year after entry* are 83; and, in the *second calendar year after entry*, 77—being, in the first year, a percentage of about ·9; and, in the second, about 1 per cent. It seems to me to be of very great importance that it should be ascertained if there is any explanation of the extraordinary result thus exhibited by the London Equitable experience. Is it not possible that some anomaly may have been produced in noting down the deaths for the first and second years, connected with the exclusion which is seemingly stated to have been made of the lives at risk during the first calendar year of entry? Mr. Morgan, in a Note on p. iv. of the experience, says:—

"One hundred and eighty-five new members were admitted into the Society in the course of the whole year 1829, at various ages; these are not included in the Table A, *although the mortality of that year is included in that table.* The reason for this omission will be apparent from a consideration of the principle upon which tables of this kind are formed and of the information they are intended to convey—namely, the number of deaths

which have happened, or may be expected to happen, out of a certain number of individuals, at any given age before they shall attain to the next year of age. In conformity with this principle, the duration of the lives of those who are or have been members of the Society, is reckoned according to the number of entire years of age which they have completed since the time of their admission.

"A person, therefore, making assurance in the course of the year 1829, in the 31st year of his age, cannot be reckoned as having lived a year in the Society on the 31st day of December, 1829; and is not to be added until 1830, when he must be classed under the age of 31, as he would complete his 32nd year in 1830, and have entered his 33rd year on the 1st of January, 1831, should his life have so long endured."

The following Note is appended in the article "Mortality, Human," by the late Mr. Milne, in the *Encyclopædia Britannica*, page 639 of vol. xv. of last edition:—

"None of the lives insured during any year commencing with the 1st of January are entered in these statements as having come into the Society until the 1st of January next following the day of their actual admission, and their age is stated to be a year older than it was when insured; *although whatever deaths may happen amongst them before the 1st of January next following the commencement of the insurances, are entered in the statements as having taken place amongst the lives previously insured.*"

Mr. Edmonds, in his paper in the *Lancet*, 1842, says:—

"He [Mr. Morgan] assumes that all who have been admitted during any year of the observation, in any specific year of age, will have completed that year of age at the end of such year of the observation. Having made this assumption, he disregards any mortality occurring in the fractional part of the first year of admission, and commences the observation from the beginning of the ensuing year, when the age is expressible without fractions of a year. For example: if a member had been admitted in the 45th year of age in any part of the year 1821, he will have been regarded as having completed his 45th year, or as being exactly 45 years old, on the 31st of December, 1821; and, for the purposes of the observation, no account will have been taken of this member before the 1st January, 1822. Although, as I have learned from inquiries, this is the mode of proceeding adopted by Mr. Morgan, yet there is no statement to this effect published by him."

The above is the whole which I can discover printed on this subject. Apparently, the mortality for the year 1829, noted separately, page 29 of the Equitable experience, is not included in the mortality in Table A, as it would seem it ought to be, and as it is stated by Mr. Morgan to be; and it would confer a great additional boon if the two points here noticed were carefully looked into again, and an authoritative statement made on the subject.

The observations which have preceded have had more especial reference to the general mortality; I propose now to consider, somewhat more in detail, the mortality among assured male lives non-hazardous—say select assured male lives.

And first, in regard to the mortality in the first year in the Scottish Amicable, the number exposed to risk has been 8,482, and the deaths 47—the percentage thus being ·554. Seeing, as we have stated above, that the percentage of sums assured in the Offices above referred to, including any hazardous risks, is ·55; there seems no reason to doubt that this ·554 is not below the probable general mortality for select assured male lives during the first year. The percentage which should have fallen by the English Life Table, No. 2, is 1·245—the Scottish Amicable (select male) deaths being 44·4 per cent. of this. That the 1·245 per cent., or 1,245 out of 100,000, includes a large proportion of lives not assurable within the year, appears also clearly from the following important observation which, some years ago, I was enabled to make by the kindness of medical gentlemen connected with the Scottish Amicable Society or personal friends of my own. I furnished these gentlemen with schedules as to deaths of parties above the age of 13, and asked them to fill in such deaths as had occurred in their practice, giving the cause, about the age, and stating "Yes" or "No" to the question whether the life was assurable as a good life within a year of death.

I had these returns filled up with 513 cases, of which

200 were stated to have been assurable as good lives within a year of death, and
313 were stated as not assurable as good lives within a year of death.
513

The following Table IV. shows, according to the Registrar-General's Reports referred to, the number of deaths, in the different classes of diseases among the lives assurable and not assurable as good lives within the year, at ages under 48, from 48 to 62 inclusive, and above 62.

It will be seen how much the proportion of deaths of lives non-assurable within the year, as good lives, increases with each period of age; and the marked feature of the great preponderance of deaths in the zymotic, &c., class (I.) in the assurable division, and of tubercular (III.) diseases in the non-assurable division, at ages under 48, will at once attract attention.

TABLE IV., *showing the Division, according to the Registrar-General's Classification* (20*th and previous Reports*), *of* 513 *Deaths reported by Medical Gentlemen, of which Deaths* 200 *were reported as assurable within a Year of Death, and* 313 *not so assurable.*

Class of Disease.	278 Lives under Age 48.		113 Lives 48 to 62, inclusive.		122 Lives above Age 62.		513 All Ages.	
	Assurable.	Not Assurable.	Assurable.	Not Assurable.	Assurable.	Not Assurable.	Assurable.	Not Assurable.
I.	55	5	15	7	7	8	77	20
II.	6	8	2	14	1	10	9	32
III.	16	60	1	7	1	..	18	67
IV.	14	9	1	11	5	17	20	37
V.	5	11	1	14	1	11	7	36
VI.	19	11	9	10	3	20	31	41
VII.	10	8	4	8	3	9	17	25
VIII.	2	7	..	4	.	9	2	20
IX.	12	4	..	3	..	..	12	7
X.	1	3	..	..	..	..	1	3
XI.	..	..	..	..	..	..	..	..
XII.	..	..	..	.	..	..	..	..
XIII.	..	1	..	..	..	5	..	6
XIV.	2	1	..	..	..	1	2	2
XV.	..	..	..	..	..	9	..	9
XVI.	..	..	..	..	..	..	..	..
XVII.	3	5	..	2	1	1	4	8
Totals..	145	133	33	80	22	100	200	313

I do not at present propose to analyse the details further, nor to argue for any precise accordance of the results with the experience of select lives for the first year, but it is obvious that they distinctly confirm the view that a very large proportion of the deaths among the population, as usually shown in mortality tables, are of lives not assurable as good lives within the year.

In reference to the apportionment of a general rate of mortality for the first year, similar to what is stated above, I have been unable, on a careful consideration—or, I may say, reconsideration—to discover whether, up to the age of about 45, the mortality is higher or lower at younger ages; and, in the meantime, I have stated it, up to age 42, at ·45. I do not say that the recent valuable paper on the mortality of the peerage, *Assurance Magazine*, vol. ix., p. 311, added to the paper on the vital statistics of the Society of Friends, read before the Statistical Society, 21st Dec., 1858, and the mortality exhibited among the Government annuitants, lead undoubtedly to the inference that such a result as the above is more likely to give undue advantages regarding vitality to the younger than the older lives, because it is quite possible that

such results as the peerage mortality shows may be consistent with an increasing mortality among select lives; but I think they may tend to promote opinions that the views here suggested for consideration are, at least, not unworthy of serious and careful attention.

Having made these observations in reference to the rate of mortality for the first year, I now proceed to consider the mortality for after years; and first, in regard to mortality in later years of the assurances. Following a similar process of argument to that adopted by Mr. Edmonds in his papers on the London Equitable Society in the *Lancet*, I believe that after six years it may be assumed that the mortality is in accordance with that of the English Life Table, No. 2; and it will be seen from the following table, that the total deaths in the London Equitable, "Town Males," and Scottish Amicable male lives non-hazardous, after the first six years, agree almost exactly with the English Life Table.

TABLE V.—*Deaths, Actual and Expected, by English Life Table, No. 2, for first Six Years, and thereafter, in the case of the London Equitable, "Town Males," and the Scottish Amicable Males, non-hazardous.*

	Deaths during first Six Years of Assurance.				Deaths after first Six Years of Assurance.			
	Actual.	Expected.	Excess of Expected Deaths.	Percentage of Excess on Actual.	Actual.	Expected.	Difference of Expected Deaths.	Percentage of Diff. on Actual.
Equitable	1,208	1,546·025	338·025	27·98	3,936	4,067·343	+ 131·343	+ 3·33
Do. if 176, the deaths for 1829, as mentioned above, be added (say 16 within six years and 160 after)	1,224	1,546·025	322·025	26·30	4,096	4,067·343	− 28·657	− ·68
"Town Males"—(doubling, for first year, deaths in first calendar year)	710	901·685	191·685	27·00	480	*467·315	− 12·685	− 2·63
Scottish Amicable Male Lives, non-hazardous	322	461·334	139·334	43·27	310	315·659	+ 5·659	+ 1·82

This is a result in itself accordant with what might be expected; for while, in six years, lives that may have been bad at first should have disappeared, still there will be a tendency always for bad

* Approximated.

lives to be continued rather than good ones; and a few bad lives continued will add to the rate of mortality more than the diminution from some hundreds of good lives discontinued. These causes will tend to assimilate different experiences in after years; and it will be seen from this table how, notwithstanding the superiority of the Scottish Amicable select male lives in the first six years, they come to agree very nearly in after years. It is unnecessary for me to say how desirable it is for parties anxious to study the value of selection to refer to Mr. Higham's valuable papers.

To come to a conclusion I found it necessary to adopt some such assumption as that above stated, in reference to the mortality after the first six years; and while, doubtless, future investigations will lead to more precision, yet I am persuaded that any correction which may be made can but little affect the values of annuities calculated on this assumption.

Having thus so far considered the rate of mortality for the first year, and the rate after six years, I submit the following table, made out with the view of assisting me to the concluding column of percentages, for the first six years, at ages 22, 27, 32, 37, 42, 47, 52 and 57, for select assured male lives—the observations being founded on lives of five ages where these were the medium ages. I have also represented this table by the Diagram No. 1 appended to this paper.

TABLE VI.—*Percentage of Deaths according to the Experience of the Equitable Office* (E.), "*Town Males*" (T. M.), *and the Scottish Amicable Non-hazardous Male Lives* (S. A.), *and Percentage deduced during the first Six Years of the Assurances.*

Age at Entry.	Age.	1. E.	2. T. M.	3. S. A.	Average of 1, 2, 3.	Percentage Deduced.
22	22	·288	·173	·457	·306	·450
	23	1·174	1·247	·943	1·121	·600
	24	·718	1·222	·953	·964	·725
	25	1·336	·673	·297	·769	·800
	26	·726	·206	·762	·564	·850
	27	·818	·755	1·478	1·017	·895
		5·060	4·276	4·890	4·741	4·320
27	27	·348	·480	·447	·425	·45
	28	·720	·505	·705	·643	·65
	29	·666	·937	1·641	1·081	·80
	30	·747	1·109	·358	·738	·88
	31	·765	·454	1·356	·858	·94
	32	·917	·862	1·278	1·019	·99
		4·163	4·347	5·785	4·764	4·71

TABLE VI. (continued).

Age at Entry.	Age.	1. E.	2. T. M.	3. S. A.	Average of 1, 2, 3.	Percentage Deduced.
32	32	·165	·540	·336	·354	·45
	33	·754	·724	·605	·694	·70
	34	·840	·993	·548	·793	·86
	35	·848	·746	·746	·780	·96
	36	·990	·928	·915	·944	1·04
	37	·952	1·351	·840	1·048	1·10
		4·569	5·282	3·990	4·613	5·11
37	37	·453	·797	·756	·668	·45
	38	1·300	·998	·528	·942	·73
	39	1·589	1·151	·176	·972	·90
	40	1·158	·536	1·244	·979	1·05
	41	1·170	1·024	1·167	1·120	1·18
	42	1·254	·851	1·896	1·334	1·30
		6·924	5·357	5·767	6·015	5·61
42	42	·366	·364	·423	·384	·45
	43	1·086	1·138	·996	1·073	·76
	44	1·048	·467	·919	·811	·96
	45	1·131	1·570	1·492	1·398	1·15
	46	1·163	·835	1·138	1·045	1·33
	47	1·186	2·112	1·165	1·487	1·50
		5·980	6·486	6·133	6·198	·615
47	47	·765	·658	·824	·749	·67
	48	1·428	·827	1·150	1·135	1·
	49	1·610	1·249	1·918	1·592	1·24
	50	1·917	1·270	2·005	1·731	1·46
	51	1·762	1·969	·278	1·336	1·66
	52	1·890	3·136	1·278	2·102	1·85
		9·372	9·109	7·453	8·645	7·88
52	52	1·343	·627	1·389	1·120	·80
	53	1·615	1·552	1·070	1·412	1·15
	54	2·230	3·811	1·235	2·425	1·49
	55	1·980	2·221	·728	1·643	1·82
	56	1·936	1·827	2·511	2·091	2·14
	57	2·185	3·968	1·990	2·714	2·45
		11·289	14·006	8·923	11·405	9·85
57	57	1·037	1·721	·791	1·183	·90
	58	2·244	2·505	1·380	2·043	1·44
	59	2·174	1·657	2·117	1·983	1·97
	60	2·220	4·514	3·077	3·270	2·49
	61	2·830	1·690	2·256	2·259	3·00
	62	3·379	2·372	1·770	2·507	3·50
		13·884	14·459	11·391	13·245	13·30

In reference to the Equitable experience, I have indicated that I have doubts of the first year's mortality; and as regards the

"town males," the first year's mortality being represented by double the deaths during the first calendar year, it may be thus understated; but both these experiences include, I understand, any hazardous lives which may have been accepted—though probably these will be few—and they are the experience of a considerable period during which medical examination was not made at all, or in a much less strict mode than more recently; and, indeed, while the Equitable experience for the first year shows a percentage of only 38·1 of the calculated deaths by the English Life Table, and the Scottish Amicable 44·4 per cent.; in the remaining five years, on the other hand, the Equitable shows a percentage of 86·9, and the Scottish Amicable 77·3. The "town males" again, by doubling the deaths as above stated, show a percentage of 53·9 during the first year, and 85·8 during the next five years, all as shown more in detail in Table VII., which has been made out at considerable trouble, and will, it is thought, be worthy of a careful perusal. (See page 74.)

Before passing from Table VII., I may notice that it pretty clearly indicates that the percentage on assured lives, according to the English Life Table, No. 2, will be for 10,000—

For 1st year . . .	from 125 to 130
For 2nd year . . .	„ 130 to 135
For 3rd year . . .	„ 135 to 140

Returning to Table VI., I may state, that while, in the results deduced in it, I have desired rather to follow the Scottish Amicable, it will be seen that they are very much in accordance with the average of the three tables; and I think it will be found difficult, on a careful consideration of the whole table, to believe that there could be a material alteration on the *sums* of the percentages deduced for the six years; and, if so, whatever doubts may be entertained of the divisions of the totals, it is certain that no alteration which could be made would, at all events, affect to any considerable extent the values of the annuities to be deduced. In stating the percentages for the first six years, I should add that I have rather leaned to the side of what may be called safety; and, more particularly as regards the advanced ages, and 57 in particular, I think the percentages might have been less, and probably the falling into the English Life decrement retarded; but without more clear grounds, I thought it better, for simplicity, to adhere to the merging in the English Life Table at all ages after six years.

In regard to the values of annuities for select assured male lives, I think it may be useful, before submitting a new table of

TABLE VII., *showing, in reference to London Equitable Experience, "Town Males," and Scottish Amicable Males—non-hazardous, the Deaths, Actual and Expected, by English Life Table, No. 2, during first Six Years and thereafter; also the Percentages which the Deaths, Actual and Expected, bear to the Lives exposed to Risk.*

Years of Assurance.	London Equitable.						Town Males.						Scottish Amicable.					
	Expected Deaths by E. L., No. 2.	Percentage on Lives at Risk.	Lives at Risk.	Actual Deaths.	Percentage on Lives at Risk.	Percentage of Expected Deaths, E. L., No. 2.	Expected Deaths, E. L., No. 2.	Percentage on Lives at Risk.	Lives at Risk.	Actual Deaths.	Percentage on Lives at Risk.	Percentage of Expected Deaths, E. L., No. 2.	Expected Deaths, E. L., No. 2.	Percentage on Lives at Risk.	Lives at Risk.	Actual Deaths.	Percentage on Lives at Risk.	Percentage of Expected Deaths, E. L., No. 2.
1	275·238	1·287	21,382	105	·491	38·148	200·264	1·299	15,406	*108	·701	53·929	105·641	1·245	8,482	47	·554	44·490
2	278·680	1·344	20,739½	245	1·181	87·914	181·616	1·353	13,424½	142	1·058	78·185	91·998	1·300	7,079	61	·862	66·305
3	266·971	1·408	18,961½	239	1·260	89·523	155·883	1·400	11,135	145	1·302	93·020	80·979	1·352	5,991	63	1·051	77·797
4	250·911	1·465	17,129½	216	1·261	86·086	134·669	1·406	9,361	117	1·250	86·879	70·614	1·409	5,013	52	1·037	73·639
5	241·246	1·532	15,748½	200	1·270	82·902	120·495	1·503	8,015½	87	1·085	72·205	59·956	1·473	4,069½	50	1·229	83·393
6	232·979	1·598	14,578½	203	1·392	87·132	108·758	1·566	6,946	111	1·598	102·050	52·146	1·550	3,365	49	1·456	93·965
First 6 years	1546·025	1·424	108,539½	1,208	1·113	78·135	901·685	1·400	64,388	710	1·102	78·742	461·334	1·357	33,999½	322	·947	69·798
Five years after 1st	1270·787	1·458	87,157½	1,103	1·265	86·796	701·421	1·432	48,982	602	1·229	85·827	355·693	1·394	25,517½	275	1·078	77·314
Total after six	4067·343	2·647	153,670½	3,936 † or 4,096	2·561 or 2·665	96·771 or 100·70	467·315	..	..	480	..	102·71	315·659	·928	14,348½	310	·912	98·207

* Double the deaths in first calendar year.

† In this table the deaths for the first six years are stated without reference to the 16 deaths noticed in Table V.

these, to illustrate, by a diagram (see Diagram No. 2), values at 4 per cent., which I deduced, in 1844, from the Equitable tables. I think that the table then formed must be, as near as possible, the true value of annuities for select lives, upon the assumption that these may be deduced from the Equitable experience as published. One table of the values of annuities was calculated according to the actual probabilities of living by Morgan's Table A. They were also calculated for the separate lives from 25 to 35, 35 to 45, 45 to 55, and 55 to 65—the calculations being made by the probabilities of living one year given for these lives in the Tables of the Experience of Seventeen Offices. It was clear that in each of these separate tables the value at the commencing age should be too great and those at the end too small. It was also clear that the real values at 25, 35, and 45, should be between the values stated in the different separate tables. It was further explained, in reference to the table alluded to, how the difference in the value of the annuities for select lives at one age and the next should be less than the differences between one age and the next in the Table of Annuities calculated from the probabilities by Morgan's Table. The Table of Annuities was made out having all these points in view—the principal difficulty being in reference to the values from 39 to 45, which it was found necessary to state nearly in accordance with the separate lives entered between 35 and 45. I have shown in the diagram these values by their differences from the values deduced in 1844, also the values as calculated by Mr. Davies, and also by Mr. Higham, at intervals of age, the values at 4 per cent. being kindly furnished to me by the latter gentleman by approximation from the values he deduced at 3 per cent. (See Mr. Higham's paper, read before the Institute 25th March, 1850.)

I have also shown in this diagram the values now deduced (Tables VIII. and XI.), and at intervals, values of Mr. Higham approximated at 4 per cent. from the values given in the second most able paper which he read on selection. (See *Assurance Magazine*, vol. i., p. 179.)

The values of annuities have again been calculated (see Table VIII.) for ages 22, 27, &c., up to 57, on the assumption of the mortality in the London Equitable, after six years, being in accordance with the English Life Table, and taking the actual mortality prior to this. The differences of the two values, as seen in this table, are not very great—being greatest at ages 37 and 47, where they are ·28 and ·21; and I think it may be held, so far as regards the Equitable Society, that no material error in calculating

the values of annuities at separate ages can arise by proceeding on the general assumption of agreement with the English Life Table after six years. It will be seen that the last calculated values are generally the lower, but if the deaths of 1829 be not included in Table A, as previously alluded to, page 67, the values of 1844 should have been stated less. The table shows, at ages 22, 27, &c., to 57, the values of the annuities in 1844 as calculated from Equitable experience, the values deduced of new from Equitable experience as above stated, the values of "town males" and Scottish Amicable on the same principle, and the values of annuities according to the rate of mortality deduced for the first six years and followed by that of the English Life. These last values will be found to differ in no material degree from those deduced from the Equitable in 1844, and, as seen in Diagram No. 2 and Table XI. below, they are almost identical with Mr. Higham's of 1851, deduced by a considerably different process; and, upon the whole, I think that it will be evident that they cannot be far from the truth—that is, they must be very near the real values of annuities for assured male lives non-hazardous. The principal feature of the values of annuities in 1844 was the high value it gave to advanced lives towards 60, and the comparatively modified value it assigned to young lives. The further investigations made do not contradict this. I need not add, that one great advantage of a true table of such annuities is the measure it enables us to take of the fairness of modes of allocations of additions.

TABLE VIII.—*Values of Annuities at 4 per Cent. on Observation of Select Assured Lives.*

Ages.	Values deduced from Equitable Experience, 1844.	Values calculated by Actual Mortality for Six Years, and after by English Life Table, No. 2.			Values per Mortality deduced in Table II. for first six years, and after by E. L., No. 2.
		Equitable.	"Town Males."	Scottish Amicable Males, Non-hazardous.	
22	17·90	17·81	17·92	17·84	17·93
27	17·29	17·32	17·29	17·08	17·24
32	16·55	16·52	16·44	16·63	16·47
37	15·69	15·41	15·60	15·59	15·60
42	14·65	14·64	14·59	14·67	14·61
47	13·45	13·24	13·31	13·42	13·41
52	12·12	11·94	11·71	12·17	12·10
57	10·74	10·65	10·55	10·85	10·72

If what we have given above be the values of select assured lives, there is no doubt, in reference to physiological inquiry and the values of annuities on select lives continued under observation during the whole period of these lives, the values will be repre-

sented too low and the percentages of mortality too high. In the first place, there will certainly be some bad lives accepted by Assurance Offices as good lives. The great damage from this may probably come forth principally in the first year; but of the 47 deaths which took place among the Scottish Amicable male lives, "non-hazardous," there were, undoubtedly, at least 8, and perhaps 12, more or less, bad—2 or 4 of these being suicides in the first year of the assurance, out of 7 or 9, the whole number of suicides in the Society's experience. It would seem that, taking hazardous and non-hazardous together, the deaths to be expected for the first year may be from 6 to 7 per 1,000; and, for select assured lives, about 5 per 1,000, one of which would likely be a life really ineligible as a good one. In the second place, there is, as noticed, the tendency of the bad lives to be less discontinued than the good ones. I have little doubt that, if the materials at present available were made use of, both values could, with very little difficulty compared with the importance of the subject, be ascertained very nearly; and I cannot think the value of annuities on select assured male lives will be found to differ in any considerable degree from those stated here. As regards the values of annuities on select lives continued under observation during the whole of the lives, it is very satisfactory to be able to refer to that most valuable addition made by Dr. Farr to his contributions to the science in the Life Table for Healthy Districts. Mr. William Davis, in the last number of the *Assurance Magazine*, has made out a D and N Table from this at 3 per cent., by which I have been enabled to contrast the values of the annuities at 3 per cent. with values at the same rate which I have deduced as for select assured male lives. The following are the results:—

TABLE IX.—*Values of Annuities at 3 per Cent.*

Ages.	Select Assured Male Lives.	Healthy Districts. Farr.
22	21·047	21·909
27	20·057	21·032
32	18·975	20·039
37	17·786	18·856
42	16·484	17·553
47	14·964	16·029
52	13·342	14·311
57	11·688	12·389

It is thus seen that the values of annuities by Dr. Farr's "Healthy Districts" Table considerably exceed the values for select assured male lives. There is every reason to suppose that lives

selected in these districts, and continued under observation for the whole period of life, would give still higher values; but I doubt if any practical selection of lives which has been made will be found to give higher values. I now subjoin—

TABLE X.—*Rates per Cent. of Mortality for Select Assured Male Lives, for the first Six Years of the Assurances—the Rate thereafter being assumed to be according to the English Life Table, No. 2.*

Age.	1st Year.	2nd.	3rd.	4th.	5th.	6th.	
22	·45	·60	·725	·80	·85	·895	Thereafter English Life Table, No. 2.
23	·45	·62	·74	·82	·85	·91	
24	·45	·63	·75	·84	·89	·93	
25	·45	·64	·78	·85	·91	·96	
26	·45	·65	·79	·87	·92	·98	
27	·45	·65	·80	·88	·94	·99	
28	·45	·66	·81	·90	·97	1·01	
29	·45	·66	·82	·93	·98	1·03	
30	·45	·68	·84	·93	1·00	1·06	
31	·45	·69	·84	·94	1·02	1·08	
32	·45	·70	·86	·96	1·04	1·10	
33	·45	·70	·86	·98	1·08	1·14	
34	·45	·70	·87	1·00	1·10	1·18	
35	·45	·70	·88	1·01	1·13	1·22	
36	·45	·72	·90	1·03	1·16	1·26	
37	·45	·73	·90	1·05	1·18	1·30	
38	·45	·74	·91	1·08	1·21	1·34	
39	·45	·75	·93	1·10	1·24	1·38	
40	·45	·75	·94	1·10	1·28	1·42	
41	·45	·76	·95	1·13	1·30	1·46	
42	·45	·76	·96	1·15	1·33	1·50	
43	·49	·81	1·01	1·21	1·39	1·58	
44	·53	·86	1·06	1·26	1·46	1·64	
45	·58	·91	1·12	1·34	1·52	1·70	
46	·62	·96	1·18	1·39	1·59	1·78	
47	·67	1·00	1·24	1·46	1·66	1·85	
48	·70	1·02	1·29	1·52	1·76	1·96	
49	·73	1·06	1·34	1·60	1·86	2·04	
50	·76	1·05	1·39	1·68	1·95	2·15	
51	·78	1·12	1·41	1·75	2·04	2·28	
52	·80	1·15	1·49	1·82	2·14	2·45	
53	·82	1·21	1·58	1·95	2·32	2·66	
54	·84	1·26	1·66	2·08	2·49	2·88	
55	·85	1·33	1·78	2·22	2·68	3·08	
56	·88	1·38	1·86	2·36	2·84	3·28	
57	·90	1·44	1·97	2·49	3·00	3·50	

This contains the adjusted column in Table VI., and the intermediate rates are interpolated.

The following Table XI. is calculated at the ages for which the rate of mortality is adjusted in Table VI. by those rates of mortality for the first six years, and English Life Table thereafter. The values for the intermediate ages are interpolated, but I have no doubt these would be very little different if the annuities had been calculated in detail from the preceding table; and it will be seen that the values fall in accordance with what has been

said to be stated as almost the same as those which I made out in 1844. They are both represented in Diagram No. 2, as above mentioned; and to impress their agreement also with Mr. Higham's, of 1851, I add here his approximate values, as shown in the diagram.

TABLE XI.—*Values of Annuities for Select Assured Male Lives at 4 per Cent.*—1861.

Ages.	Value of Annuity.	Ages.	Value of Annuity.	Ages.	Value of Annuity.	Ages.	Higham, 1851.
20	18·16	34	16·14	48	13·16	25	17·42
21	18·04	35	15·96	49	12·90	30	16·75
22	17·93	36	15·78	50	12·64	35	15·95
23	17·80	37	15·60	51	12·37	40	15·02
24	17·67	38	15·41	52	12·10	45	13·87
25	17·53	39	15·22	53	11·83	50	12·60
26	17·38	40	15·03	54	11·56	55	11·30
27	17·24	41	14·83	55	11·28	60	9·83
28	17·09	42	14·62	56	11·00		
29	16·95	43	14·39	57	10·72		
30	16·79	44	14·16	58	10·44		
31	16·64	45	13·92	59	10·16		
32	16·47	46	13·67	60	9·89		
33	16·31	47	13·42				

NOTE.—It will be seen that the values of annuities have been given for ages 20 and 21, and 58, 59, and 60. This has been done by observation of the differences in Diagram No. 2. The values now given by me are intended to exclude, more entirely than Mr. Higham, hazardous lives; and his values being generally a little lower, is rather in accordance with this than otherwise.

I may notice, in reference to the 632 deaths which took place among the Scottish Amicable males, non-hazardous, that, by referring to Table RP of the Scottish Amicable experience, it will be seen that, as compared with the deaths to be "expected" by the English Life Table No. 2, the difference 144·99 is thus accounted for.

TABLE XII.—*Shortcoming or Excess of Deaths in the following Classes of Diseases.*

	Shortcoming.		
II.	Diseases of uncertain seat	20·95	
III.	Tubercular disease	89·24	
VI.	Diseases of the respiratory organs	35·84	
XIV.	Atrophy	10·35	
XV.	Gradual decay and senile debility	7·45	
XVI.	Cause of death not ascertained	5·66	
XVII.	Accidental injuries, suicides, &c.	17·39	
	Other classes	13·14	
			200·02
	Excess in the following classes of disease.		
IV.	Disease of the brain and nerves	25·80	
V.	Disease of the heart and blood-vessels	10·02	
VII.	Disease of the digestive organs	17·24	
IX.	Disease of integumentary tissues	1·97	
			55·03
	Difference, as stated	..	144·99

TABLE XIII.—*Approximate Calculation of the Deaths of the Scottish Widows' Fund, Expected, per E. L. No. 2, for Seven Years, to 31st December, 1859, and Apportionment of them (as if Males) according to the Deaths in Registrar-General's 18th Report, 1848 to 1854; and also the Shortcoming or Excess of the Actual Deaths from the different Diseases.*

Ages at commencement	13 to 30.		31 to 40.		41 to 50.		51 to 60.		61 to 70.		71 to 80.		81 to 90.		All Ages.		Shortcoming of Actual Deaths.	Excess of Actual Deaths.
Say Ages at Death	25 to 35.		35 to 45.		45 to 55.		55 to 65.		65 to 75.		75 to 85.		85 to 95.					
CLASS.	Expected.	Actual.	Expected.	Actual.	Expected.	Actual.	Expected.	Actual.	Expected.	Actual.	Expected.	Actual.	Expected.	Actual.	Expected.	Actual.		
I.	14·85	10	36·99	26	43·59	18	34·77	27	23·39	17	7·43	9	·37	0	161·39	107	54·39	..
II.	2·47	0	10·72	6	21·24	8	26·37	21	23·19	13	7·72	1	·36	1	92·07	50	42·07	..
III.	33·72	15	70·76	26	63·08	15	32·36	9	9·67	2	·82	0	·02	0	210·43	67	143·43	..
IV.	5·65	8	22·48	32	31·89	61	35·45	51	31·88	54	10·74	28	·44	0	138·53	234	..	95·47
V.	2·93	2	12·06	6	19·98	27	21·60	51	15·45	33	3·43	16	·08	0	75·53	135	..	59·47
VI.	5·75	2	24·02	20	44·20	25	51·70	36	39·82	32	11·60	15	·51	0	177·60	130	47·60	..
VII.	3·85	6	15·63	21	26·73	29	26·59	33	15·88	13	3·28	13	·11	0	92·07	115	..	22·93
VIII.	1·29	3	4·74	5	6·99	12	7·95	11	8·25	16	2·83	5	·11	0	32·16	52	..	19·84
IX.	·01	1	·03	0	·04	0	·04	0	·03	1	·01	0	·00	0	·16	2	..	1·84
X.	·79	1	2·27	5	3·07	3	2·76	2	1·69	1	·36	2	·00	0	10·94	14	.	3·06
XI.	·08	0	·50	0	·93	0	1·05	0	·72	0	·21	0	·00	0	3·49	0	3·49	..
XII.	·01	0	·01	0	·02	0	·01	0	·00	0	·00	0	·00	0	·05	0	·05	..
XIII.	·16	0	·75	0	1·90	0	2·11	0	·22	0	·00	0	·00	0	5·14	0	5·14	..
XIV.	·14	2	·59	0	1·34	0	8·87	0	11·26	1	·21	0	·01	0	22·42	3	19·42	..
XV.	·00	0	·00	0	·00	0	·01	2	26·96	8	44·00	14	5·22	5	76·19	29	47·19	..
XVI. and not specified	1·36	0	4·89	2	8·43	1	9·87	3	8·10	1	2·61	1	·14	0	35·40	8	27·40	..
XVII.	7·97	4	20·44	10	22·37	8	13·88	6	6·09	1	1·47	0	·08	0	72·30	29	43·30	..
Totals..	81·03	54	226·88	159	295·80	207	275·39	252	222·60	193	96·72	104	7·45	6	1205·87	975	433·48	202·61

NOTE.—It is to be observed, that the actual deaths in this table occurred only in seven years recent experience, and include the deaths among females as well as males, and hazardous as well as non-hazardous lives.

Referring to the statement appended to Dr. Begbie's analysis of the deaths in the Scottish Widows' Fund Office for the seven years ending 31st December, 1859, I have made an approximate calculation of the number of deaths by the English Life Table, No. 2, and make them— . . . . 1,205·87

The actual deaths being . . .	975·
Difference . . .	230·87

The materials for precisely proportioning these deaths, according to the deaths 1848 to 1854 in the Registrar-General's Report, are not complete, but the preceding table, I think, must be pretty nearly correct; and it will be observed that the excess of deaths from diseases of the brain and nerves (IV.), and heart and blood-vessels (V.), and digestive organs (VII.), also appears in the case of this Society for the seven years referred to, and it is much more strongly marked in regard to the two former classes. These two tables suggest important questions for consideration, but I shall not enter upon them at present—more especially as Dr. Fleming may, probably, allude to them in some remarks on the subject of the causes of death, which, I hope, will be published by him shortly.*

I may, before concluding, refer to Abstract C, p. 160 of Mr. Neison's *Contributions to Vital Statistics.* He shows that, as a general rule, in the case of the Gotha Society (to which and its manager the profession are so much indebted), the period which elapsed from the date of the policies to date of death is less, on an average, in the case of younger than in the case of older entrants; and the same rule appears to prevail in the case of the Scottish Amicable Society; at least, the following is the average duration of policies in the case of deaths, among the male lives, non-hazardous:—

Under 30	5·5 years.
30 and under 40	7·0 „
40 and under 50	8·3 „
50 and under 60	9·2 „

But, in reference to the Scottish Amicable, this, no doubt, principally arises from there being an increasing proportion of lives at risk in the later years of the assurances in the case of the older than in the case of the younger lives. Thus, of the lives

* I may so far anticipate Dr. Fleming's remarks, as to mention that, to a great extent, he attributes the excess in Class VII. to the probable insertion in the Registrar-General's Reports, under Class II., of cases of *dropsy*, which, in the returns of Assurance Offices, would be entered among diseases of *liver*—the producing cause.

under 30, as contrasted with those at from 50 to 60 (when the deaths happen to amount to 114 among both sets of lives), the older lives are, in the

1st year of assurance,		29	per cent. of the younger.
5th	"	37	"
10th	"	43	"
20th	"	53	"

In concluding my remarks, I trust that, while I have not hesitated to express myself decidedly on some points to which I have given much attention and which appear to me clear, I shall not be supposed not to have felt how much more ably the valuable materials I have had to deal with might have been handled by others. They might have thrown light on points which some may suppose my remarks have rendered more obscure than they have hitherto appeared; but still, if the light has been deceiving, it is better that it should be darkened; and whether it be by pointing to the right or from the wrong direction, we must be so far satisfied when we are able to raise some landmarks, however rude, on the road to truth.

Heppel's Logarithms.

IT is now, we believe, nearly four years ago since Professor De Morgan made known to some of the members of the Institute that a Mr. Heppel had constructed some tables of logarithms to a large number of places, and that the manuscripts and some stereotyped plates were about to be sold by his executors. Through the intervention of the Professor, the tables and plates were submitted to the inspection of the gentlemen referred to, and were ultimately purchased by them—a small subscription being made for the purpose—and eventually they were placed in the hands of the writer of this notice, with a view to the publication of them. As some introduction was necessary to such a work, it was his intention to have prefaced the tables with a brief but general discussion of the subject, and to have given some account of the principal tables of the kind now in existence. This, however, he has found it impracticable hitherto to effect, and is now the less disposed to attempt, seeing that so good and so modern an account of these tables has just been published in Mr. Charles Knight's *English Cyclopædia**. At all events, it is likely to be very long before even

* See vol. vii., under article "Table."

the materials already collected could be used by him; and as some curiosity naturally exists as to the extent of the tables constructed by Mr. Heppel, he has thought it desirable to state in a few words of what they consist, and to append a short account of the author himself, written by a lady nearly related to him, and, as it would seem, at Professor De Morgan's request.

The principal table, then, consists of the logarithms to twelve places of the numbers 100,000 to 118,600, and it is this table which has been stereotyped in quarto. The next consists of the logarithms to the like number of places of the numbers 131,000 to 136,636, and the last comprehends the logarithms of the numbers 200,000 to 210,000; the logarithms in this being to fifteen places as far as the number 202,024, and to twelve only thereafter. The tables not stereotyped are in manuscript merely. They are comprised in two volumes, which they nearly fill, and are executed with sufficient neatness; but there is little to show how far dependence is to be placed on the accuracy of any of them. Mr. Heppel, it seems, was, at one time, the actuary of an Office called the "Standard of England," which had but a brief existence before it merged into the "Britannia." The account given of him is as follows:—

"George Hastings Heppel, who calculated these tables of logarithms—more for the occupation of his leisure time, and the pure love of science, than from any idea of a profitable undertaking—was born in the year 1793, and evinced, at a very early age, a decided mathematical talent. When at school, whilst other boys were enjoying the pastimes of the playground, he would be found engaged quietly at his desk with his friends Euclid and Hutton. Being the only son of a man of independent property, he was not bred to any profession; but, at the age of 22, his father established him in a paper-mill, in which, unfortunately, he was not successful. It was given up, and, after seven or eight years, he obtained an appointment under the Commissioners for the adjustment of the Spanish claims, for which employment he was far better fitted than for the ordinary details of business. During that period he gave Mr. Michael Quin, the chief Commissioner, valuable assistance in his work upon "Banking," which he acknowledges very handsomely in his preface to that work. Subsequently to the termination of the Commission, he followed the pursuits of an accountant, and was for some time occupied as an actuary of a Life Assurance Company. He was well known to most of the influential men of high standing in the city of London, and esteemed and appreciated by them for his rare talents and integrity; but, unhappily, in the spring of 1844, the seeds of consumption took so fatal a root, and worked their way so insidiously, that, till within three months of his decease, neither himself nor his family were aware of his dangerous state. To his own great satisfaction, however, he had completed such a portion of his table of logarithms as he deemed sufficient for publication, and made terms with Messrs.

Clowes and Sons for printing and stereotyping them; but, before half the plates were composed, his disease made suddenly such rapid progress as to terminate his existence on the 4th February, 1845."

The following memorandum also appears in the volume containing the figures which have been stereotyped :—

"These logarithms were constructed by my father on the basis of Hutton's logarithms of certain numbers to twenty places. He has not left any account of the method he adopted, but the following particulars appear to be clearly deducible from the books containing the calculations.

"Such as could be directly deduced from Hutton's were calculated from them, and those intervening were filled in from the method of differences. Two orders of differences were taken, and these, as well as the logarithms, were calculated to the thirteenth place of decimals—that is, to one more place than was intended for publication. The third order of differences for 100,000 commences with the figure 8 in the sixteenth place; and thus, in the case where they are largest, those in the second order would only alter by one in the course of 120 numbers. As many as possible, in the earlier part, were calculated by their factors, independently of those adjacent to them, so as to check the accuracy of the work; and afterwards, by division by 9 or 8, or by multiplication by 11 or 12, the later logarithms might be found from the earlier, and thus both would mutually check one another."

This memorandum is signed "G. Heppel," and has the date of Carlisle, Nov. 17th, 1855.—ED. *A. M.*

On the Construction and Use of Commutation Tables for Calculating the Values of Benefits depending on Life Contingencies. By PETER GRAY, *Esq.*, F.R.A.S.*

IT is to a Mr. George Barrett, of whom nothing besides is publicly known,† that we are indebted for the principle of the Commutation Tables, and for the method of computing, by means of them, the values of benefits depending on the contingencies of human life. The method was first introduced to public notice, after it had been refused a place in the *Transactions of the Royal Society*, by Mr. Baily, in an Appendix to the second edition of his *Doctrine of Life Annuities*, published in 1813. Mr. Griffith Davies, in a work on life contingencies, published in 1825, by certain additions to the tables, and alterations in their structure, according to Professor De Morgan, "increased the utility and extended the power of the method to an extent of which the inventor had not the least idea." Mr. Barrett's method was also

* Extracted from the *Mechanics' Magazine*, for 1842.

† For some account of Mr. Barrett, see vol. iv., p. 185, of this *Journal*.—ED. *A. M.*

briefly noticed in the Appendix to Mr. Babbage's Treatise on Life Assurance. The method, as improved by Mr. Davies, has since been treated, and a very large collection of tables adapted to it, for both one and two lives, has been given, by Mr. Jones, in his work on annuities, in the *Library of Useful Knowledge*. But by far the most valuable papers on the subject are two in the *Companion to the Almanack*, for 1840 and 1842, by Professor De Morgan, which contain the materials of many thousand formulæ, applicable to almost every case that can occur. There is also some notice of the method in the article "Reversions," in the *Penny Cyclopædia*, which article likewise is the production, we believe, of Professor De Morgan.

The above-named are understood to be the only works in which the new method is, in any sort, treated. They certainly are not numerous; but they are sufficiently so, and well enough known, to have induced a general adoption of this method, to the exclusion of that previously in use, but for two reasons. The first is, the want of tables adapted to this method; and the second, the want of an *elementary* and *systematic* treatise on the subject. The first of these wants is now amply supplied by the valuable collection of tables published by Mr. Jones, above referred to; but the second still exists. It is no disparagement to the able authors of the works above named, to say that this is the case. Generally speaking, it has not been their object to furnish a treatise of this kind; and they have accordingly taken for granted, on the part of their readers, the possession of a degree of acquaintance with the subject, which very many, to whom the power of using the Commutation Tables would be of the greatest service, certainly do not possess. Professor De Morgan, indeed, expressly says, that all that will be found of demonstration in his articles "is intended for those who are familiar with the subject." Now, although the formulæ, according to this method, are extremely simple, and easily intelligible to any one who is acquainted with the merest rudiments of algebra, and who will take the small degree of trouble necessary to enable him to comprehend the notation employed; yet it is a result of our own experience, which we have no doubt can be amply confirmed by the observations of others, that most people view with mistrust and will not willingly have recourse to formulæ, the *principles* of which they do not understand. And while Professor De Morgan's papers, in particular, have, doubtless, well served the end which, from the remark quoted above, the distinguished author appears to have had principally in

view, we do not see that this forms any good reason why others, to whom a knowledge of this method of computation would be of service, should be prevented from availing themselves of the vast fund of information, regarding this method, which those papers contain.

It is, therefore, as an humble contribution towards the supplying of the deficiency which has been shown to exist, that the present papers are intended; and if they shall serve to render more available than heretofore, to any of the numerous readers of the *Journal*, the valuable papers to which reference has so often been made, the writer will consider himself amply rewarded for his labour. While he thinks that the task of rendering more generally intelligible the principles on which the new method is grounded will not be a difficult one, he trusts that, should his success not be commensurate with his wishes, he may at least be found to have aided in clearing the way for some one better qualified to do justice to a subject which is daily growing in interest and importance.

The peculiarities of the old and the new methods may be here briefly stated. In the old method we are presented with a table of the values of annuities at all ages, which of themselves are rarely wanted, but from which, by operations more or less complex, the values of benefits of all other kinds may be computed. In the new method, on the other hand, we are presented with a table which, by mere inspection, tells us nothing; but from which, while the values of the ordinary benefits can be found by a simple division, those of benefits of the most complex description are found by operations consisting usually of nothing more than one or two subtractions and one division. In point of simplicity, moreover, in the deduction of the various formulæ, the methods admit of no comparison. For the establishment of what, according to the old method, required chapters, a few pages will suffice according to the new.

The opinion we have expressed as to the superiority of the new method, however, will probably be regarded as of little value; and that of Professor De Morgan, which is most unequivocally given, may, perhaps, be objected to, as the testimony of an interested witness, seeing he has bestowed such pains on the elucidation of this method. But Mr. Milne's testimony will certainly not be objected to on any such ground. He is the author of the best treatise that has appeared, or is now likely to appear, on the old method; and therefore his prejudices, if he has any, must be sup-

posed to be all on the side of that method which he has done so much to illustrate. Speaking of Mr. Baily's work, he says (*Encyclop. Brit.*, seventh edition, article "Annuities," vol. iii., p. 200):—"In an Appendix to it . . . . formulæ were given for calculating from tables of that kind [Commutation Tables] the values of temporary and deferred life annuities and assurances, when the annuity, instead of remaining always the same, increases or decreases from year to year by equal differences, *with considerably greater facility and expedition than the same things could have been done with by the tables and methods of calculation in previous use.*" And his testimony, be it observed, refers to the tables as devised by Mr. Barrett, and gives but a faint idea of their capabilities in their improved form.

The construction of the Commutation Tables is effected by combining in a particular manner (which will be explained hereafter) the rates of mortality and interest; and, as in the tables adapted to the old method, any rates that are most approved of, as regards these elements, may be employed. But whatever may be the rates of interest and mortality made use of, the demonstrations and formulæ which will be hereafter given, being generalized by the employment of symbols, will be equally applicable to all tables of the same form.

The rate of interest according to which our table (which has not been heretofore published) is constructed is 4 per cent.; and the rate of mortality is that given by Mr. Finlaison in his "Twentieth Observation on the Mortality of the Government Annuitants" (Parliamentary Paper, No. 122, 1829, p. 58). Tables of mortality, in their usual form, exhibit the numbers who, out of a number supposed to be alive at birth, or some other early age, attain each successive year of age, and consequently also the numbers who die in each year; and this form is the most convenient for the construction of Commutation Tables. But this is not the form in which Mr. Finlaison's data are arranged. What he gives are the *probabilities*, or rather the *logarithms of the probabilities*, that a life at each age will survive a year. A preliminary step, therefore, was, by means of these probabilities, to construct a mortality table of the more usual form; and in doing so, as well as in the subsequent construction of our Commutation Table, Mr. Finlaison's data have been made use of to *their full extent*. It is thought proper to mention this, because there is an "*abridged*" mortality table of the usual form, deduced from Mr. Finlaison's data, published in the *Report of the Select Committee on Friendly Societies*,

1827, p. 82; and some might be embarrassed by finding that the results of our Commutation Table do not exactly correspond with those which may be deduced from the abridged table referred to. This table states the survivors of 1,000 births to be, at the ages of 6 and 7, for instance, 919 and 912 respectively; while the use we have made of the data enables us to say, that the survivors at these ages, out of 100,000,000 of births, will be 91,912,811 and 91,239,410. As regards the correctness of our Commutation Table, it has been subjected to the severest tests in the way of verification, and we are confident that no error of any consequence will be found in it. Mr. Finlaison, in his Report, gives the values of annuities at all ages, at 4 per cent., as deduced from his data by the ordinary method, and carried to seven places, which are two more than are usually given. The values derived from our table will be found to correspond with those to the last place. It has been thought proper to make these remarks as to the degree of confidence that may be reposed in our table, in case any one should feel disposed to apply it to practical purposes.

We shall now explain the notation, as regards the rates of interest and mortality, which we shall employ in our demonstrations.

It is shown by writers on interest, and, indeed, in most elementary works on algebra, that if r represent the interest of £1 for a year, then will

$1+r$	be the	*amount*	of £1 in	1 year,
$(1+r)^2$	„	„	£1 „	2 years,
$(1+r)^3$	„	„	£1 „	3 „
$(1+r)^4$	„	„	£1 „	4 „

and, generally, $(1+r)^x$ will be the amount of £1 in x years, where x may represent any number whatsoever, all at compound interest. It is also shown that unity divided by the amount of £1 in a given time is equal to the *present value* of £1 to be received at the end of that time—that is, to the sum which would, in the given time, just amount, also at compound interest, to £1. Hence—

$\frac{1}{1+r}$	is the present value of	£1	due at the end of	1 year,
$\frac{1}{(1+r)^2}$	„	„	£1	„	2 years,
$\frac{1}{(1+r)^3}$	„	„	£1	„	3 „

and, generally, $\frac{1}{(1+r)^x}$ is the present value of £1 due at the end of x years—x, as before, denoting any number whatsoever.

It is usual to denote $\frac{1}{1+r}$ by v. Hence the foregoing present values will be more conveniently represented by $v, v^2, v^3 \ldots v^x$, respectively. It is obvious that the present value of any other sum will be found by multiplying the present value of £1, due at the end of the same number of years, by the number of pounds in that sum. Copious tables of the present values of £1 due at the end of any number of years, from 1 to 100, together with their logarithms, are given in Mr. Jones's work, Part I.

The indications of the mortality table are represented as follows:—the number shown by the table to attain to any age is denoted by the letter l, with the age attached as a suffix. Thus, l_0, l_1, l_2, l_3, l_x, &c., denote the numbers who attain the successive ages, 0, 1, 2, 3, x, &c., respectively. These symbols will, of course, denote different numbers in the case of different tables. Thus, l_0, representing the number alive at birth, which is called the *radix* of the table, will, in the case of the Carlisle Table, be equal to 10,000, and in that of the Northampton Table, to 11,650; and the values of the other symbols will in like manner vary. But the great advantage of the use of symbols is, that we have no need to distract ourselves with their particular values until we reach the final solution of the problem with which we may be engaged. Also, since the number who die in any year of age—that is, who enter upon that year and do not live to complete it—is equal to the difference between the number who complete that year and the number who completed the previous year, these decrements, as they are called, will be represented as follows:—

l_0-l_1 is the number who die in their			1st year,
l_1-l_2	,,	,,	2nd ,,
l_2-l_3	,,	,,	3rd ,,
$l_{x-1}-l_x$	,,	,,	xth ,,
l_x-l_{x+1}	,,	,,	$(x+1)$th year,

and so on, x representing any age whatsoever.

We are now in a condition to enter upon a description of the Commutation Tables, and their construction, and to show the algebraical properties that belong to them in virtue of that construction. We refer, for illustration, to the table contained in the two following pages.

The table consists, it will be seen, of two side columns con-

Commutation Table—Single Male Life.

Government Rate of Mortality—Interest 4 per Cent.

Age.	D.	N.	S.	M.	R.	Age.
0	10000·0000	189251·4911	3483123·3173	2336·48113	57621·69308	0
1	9431·1674	179820·3237	3293871·8262	2152·26386	55285·21195	1
2	8905·8830	170914·4407	3114051·5025	1989·71684	53132·94809	2
3	8434·3219	162480·1188	2943137·0618	1860·68958	51143·23125	3
4	8008·6623	154471·4565	2780656·9430	1759·42701	49282·54167	4
5	7620·6994	146850·7571	2626185·4865	1679·48959	47523·11466	5
6	7264·0029	139586·7542	2479334·7294	1615·89696	45843·62507	6
7	6933·4451	132653·3091	2339747·9752	1564·72403	44227·72811	7
8	6623·2532	126030·0559	2207094·6661	1521·20284	42663·00408	8
9	6330·0454	119700·0105	2081064·6102	1482·73560	41141·80124	9
10	6051·0605	113648·9500	1961364·5997	1447·21386	39659·06564	10
11	5785·5704	107863·3796	1847715·6497	1414·45703	38211·85178	11
12	5532·5360	102330·8436	1739852·2701	1383·94450	36797·39475	12
13	5291·3245	97039·5191	1637521·4265	1355·52288	35413·45025	13
14	5061·0571	91978·4620	1540481·9074	1328·76796	34057·92737	14
15	4839·8540	87138·6080	1448503·4454	1302·22087	32729·15941	15
16	4626·0186	82512·5894	1361364·8374	1274·53377	31426·93854	16
17	4417·5780	78095·0114	1278852·2480	1244·01695	30152·40477	17
18	4213·8385	73881·1729	1200757·2366	1210·18431	28908·38782	18
19	4014·0326	69867·1403	1126876·0637	1172·44914	27698·20351	19
20	3818·9594	66048·1809	1057008·9234	1131·76185	26525·75437	20
21	3629·2895	62418·8914	990960·7425	1088·97492	25393·99252	21
22	3442·7376	58976·1538	928541·8511	1042·01100	24305·01760	22
23	3264·0617	55712·0921	869565·6973	995·74796	23263·00660	23
24	3094·1061	52617·9860	813853·6052	951·33340	22267·25864	24
25	2932·9909	49684·9951	761235·6192	909·22226	21315·92524	25
26	2780·7540	46904·2411	711550·6241	869·79274	20406·70298	26
27	2639·0772	44265·1639	664646·3830	835·06793	19536·91024	27
28	2505·0301	41760·1338	620381·2191	802·52378	18701·84231	28
29	2378·0529	39382·0809	578621·0853	771·89394	17899·31853	29
30	2257·6521	37124·4288	539239·0044	742·95673	17127·42459	30
31	2143·4127	34981·0161	502114·5756	715·55005	16384·46786	31
32	2034·9211	32946·0950	467133·5595	689·49752	15668·91781	32
33	1932·0753	31014·0197	434187·4645	664·91786	14979·42029	33
34	1834·6160	29179·4037	403173·4448	641·76920	14314·50243	34
35	1742·1788	27437·2249	373994·0411	619·89412	13672·73323	35
36	1654·3493	25782·8756	346556·8162	599·07143	13052·83911	36
37	1570·6596	24212·2160	320773·9406	579·01055	12453·76768	37
38	1490·8889	22721·3271	296561·7246	559·64988	11874·75713	38
39	1414·7080	21306·6191	273840·3975	540·81084	11315·10725	39
40	1342·1355	19964·4836	252533·7784	522·65027	10774·29641	40
41	1273·0650	18691·4186	232569·2948	505·20029	10251·64614	41
42	1207·7753	17483·6433	213877·8762	488·87461	9746·44585	42
43	1145·6257	16338·0176	196394·2329	473·17792	9257·57124	43
44	1086·6545	15251·3631	180056·2153	458·26918	8784·39332	44
45	1030·6715	14220·6916	164804·8522	444·08057	8326·12414	45
46	977·4798	13243·2118	150584·1606	430·53017	7882·04357	46
47	927·3068	12315·9050	137340·9488	417·95256	7451·51340	47
48	879·4023	11436·5027	125025·0438	405·71372	7033·56084	48
49	833·5711	10602·9316	113588·5411	393·70562	6627·84712	49

Commutation Table (continued).

Age.	D.	N.	S.	M.	R.	Age.
50	789·5004	9813·4312	102985·6095	381·69530	6234·14150	50
51	746·9195	9066·5117	93172·1783	369·47976	5852·44620	51
52	705·1776	8361·3341	84105·6666	356·46560	5482·96644	52
53	664·7128	7696·6213	75744·3325	343·12304	5126·50084	53
54	625·4090	7071·2123	68047·7112	329·38523	4783·37780	54
55	587·3514	6483·8609	60976·4989	315·38164	4453·99257	55
56	550·6411	5933·2198	54492·6380	301·26191	4138·61093	56
57	515·2537	5417·9661	48559·4182	287·05295	3837·34902	57
58	481·3810	4936·5851	43141·4521	272·99761	3550·29607	58
59	449·1615	4487·4236	38204·8670	259·29288	3277·29846	59
60	418·5753	4068·8483	33717·4434	245·98204	3018·00558	60
61	389·8164	3679·0319	29648·5951	233·32228	2772·02354	61
62	362·7862	3316·2457	25969·5632	221·28490	2538·70126	62
63	337·2557	2978·9900	22653·3175	209·70782	2317·41626	63
64	312·7669	2666·2231	19674·3275	198·19033	2107·70844	64
65	289·2939	2376·9292	17008·1044	186·74680	1909·51812	65
66	266·8385	2110·0907	14631·1752	175·41815	1722·77132	66
67	244·8012	1865·2895	12521·0845	163·64386	1547·35317	67
68	223·8080	1641·4815	10655·7950	152·06610	1383·70931	68
69	203·8598	1437·6217	9014·3135	140·72583	1231·64321	69
70	184·9140	1252·7077	7576·6918	129·62088	1090·91737	70
71	166·9336	1085·7741	6323·9841	118·75251	961·29649	71
72	149·9696	935·8045	5238·2100	108·20904	842·54398	72
73	134·1890	801·6155	4302·4055	98·19656	734·33495	73
74	119·7040	681·9115	3500·7900	88·87269	636·13839	74
75	106·3778	575·53365	2818·87847	80·15044	547·26570	75
76	94·11924	481·41441	2243·34482	71·98333	467·11526	76
77	83·30894	398·10547	1761·93041	64·79300	395·13193	77
78	73·11608	324·98939	1363·82494	57·80433	330·33893	78
79	63·41526	261·57413	1038·83555	50·91567	272·53460	79
80	54·36266	207·21147	777·26142	44·30211	221·61892	80
81	45·97142	161·24005	570·04995	38·00176	177·31681	81
82	38·23230	120·00775	408·80990	32·03076	139·31505	82
83	31·33915	91·66860	285·80215	26·60808	107·28429	83
84	25·24443	66·42417	194·13355	21·71872	80·67621	84
85	19·83120	46·59297	127·70938	17·27643	58·95749	85
86	15·13649	31·45648	81·11641	13·34445	41·68106	86
87	11·05347	20·403011	49·659934	9·843603	28·336614	87
88	7·702274	12·700737	29·256923	6·917542	18·493011	88
89	5·113615	7·587122	16·556186	4·625125	11·575469	89
90	3·249834	4·337288	8·969064	2·958021	6·950344	90
91	1·946549	2·390739	4·631776	1·779730	3·992323	91
92	1·152956	1·237783	2·241037	1·061004	2·212593	92
93	·6407773	·5970057	1·003254	·5931702	1·1515892	93
94	·3327112	·2642945	·4062485	·3097495	·5584190	94
95	·1605972	·1036973	·1419540	·1504320	·2486695	95
96	·0716510	·0320462	·0382567	·0676627	·0982375	96
97	·0258357	·0062105	·0062105	·0246032	·0305748	97
98	·0062105	·0000000	·0000000	·0059716	·0059716	98

taining the ages, and five inner columns, headed D, N, S, M, and R, respectively. There is another column, headed C, which is essential to the theory of the tables, and the proper place for which, if inserted, would be between columns S and M. But as this column is not required in practice, it is never exhibited. The letters D, N, S, &c., are chosen quite arbitrarily, and have no reference to the signification of the columns at the head of which they are placed.

The number in any column, opposite to any age, is denoted by the letter at the head of the column, with the age attached as a suffix. Thus, the number corresponding to age 20 in column D, for example, is denoted by D_{20}, in column N by N_{20}, and so of the other columns. If the age be x, the corresponding numbers are denoted by D_x, N_x, &c.

The columns are constructed as follows:—Take any age, 20, for example. The number in column D, opposite age 20, is equal to the product of the number represented by the mortality table to attain the age 20, into the present value of £1 due at the end of 20 years. So that the algebraical expression will be $D_{20}=l_{20}v^{20}$.

And, generally, the number corresponding to any age in column D is equal to the number who complete that year of their age multiplied by the present value of £1 due at the end of as many years as are equal to the age. Hence, the general expression for the value of the numbers in column D will be $D_x=l_xv^x$,* x denoting any age.

Hence also, if the age be 0, we have $D_0=l_0v^0=l^0$, since $v_0=1$. That is, the first number in column D is equal to the radix of the mortality table.

Column N is formed from column D, by inserting opposite each age in N the sum of the numbers opposite all the higher ages in D. For example—

$$N_{20}=D_{21}+D_{22}+D_{23}+\&c., \text{ to the end.}$$
$$N_{21}=D_{22}+D_{23}+D_{24}+\&c.$$

So that the expression for the general term of this column is

$$N_x=D_{x+1}+D_{x+2}+D_{x+3}+\&c.$$

x, as before, denoting any age.

* By an expression for the general term of a series, is meant, an expression in which a variable quantity is introduced, and which, by giving any particular value to the variable, gives the term of the series corresponding to that value. Thus, in the above general expression, $D_x=l_xv^x$, x, which denotes the age, is the variable; and if we give to it a particular value, we have immediately the term of the series corresponding to that value. For instance, if $x=20$, the expression becomes

$$D_{20}=l_{20}v^{20},$$

and this is the value in column D corresponding to age 20.

It follows from this that the last N in the table is 0. For, since $N_x = D_{x+1} + \ldots$, if x be the highest age, D_{x+1} and all the following terms vanish, since there are no survivors at the ages denoted by $x+1$, &c. Also, the first N in the table is equal to the sum of all the D's except the first, or $N_0 = D_1 + D_2 + D_3 + \ldots$

Column S is formed by inserting in it, opposite each age, the sum of the numbers in N opposite that age and all the higher ages. The expression for the general term is, therefore—

$$S_x = N_x + N_{x+1} + N_{x+2} + \&c.$$

And it differs from D in including, at each age, the number corresponding to that age in the preceding column. It follows from this that the last S is 0, and that the first S is equal to the sum of all the numbers in column N.

The three columns just described are called the *annuity* columns. The remaining three are called the *assurance* columns. The last named we now proceed to describe.

Column C is formed by inserting, opposite each age, the product of the number who die in the following year of their age by the present value of £1 to be received at the end of a number of years, which is equal to one more than the age in question. Thus—

$$C_0 = (l_0 - l_1)\, v,$$
$$C_1 = (l_1 - l_2)\, v^2,$$

and so on. And the expression for the general term will be

$$C_x = (l_x - l_{x+1})\, v^{x+1}.$$

Column C, as already remarked, is not shown in the table. It is used only for the construction of column M; and M is derived from it precisely as S was derived from N. So that the expression for the general term of this column is

$$M_x = C_x + C_{x+1} + C_{x+2} + \&c.$$

Column R, also, is formed from M, in exactly the same manner as M was formed from C; and hence the expression for its general term is

$$R_x = M_x + M_{x+1} + M_{x+2} + \&c.*$$

We proceed now to demonstrate a few of the algebraical properties, and relations amongst each other, that belong to the

* It will be afterwards seen that there is no necessity for employing those particular powers of v, which have been made use of in the construction of the table. The only condition as to these powers, which is indispensable to the possession by the table of the required properties, is, that their indices shall form an increasing arithmetical series, of which the common difference is unity. The powers that have been chosen possess the advantage of imparting to the expressions for the values of the numbers in columns D and C, a symmetry that would not otherwise have belonged to them.

numbers in the different columns. These will be useful for after reference, with a view to which we shall number them as we go along.

By algebraical properties and relations, we mean those properties and relations that subsist in virtue of the mode in which the table has been constructed, without reference to what the numbers employed in that construction denote.

Among the algebraical relations must be classed those which we have just seen to subsist; which, therefore, we here repeat, for the convenience of reference.

$$\left.\begin{array}{l} D_x = l_x v^x \\ C_x = (l_x - l_{x+1}) v^{x+1} \end{array}\right\} (1)$$

$$\left.\begin{array}{l} N_x = D_{x+1} + D_{x+2} + D_{x+3} + \&c. \\ S_x = N_x + N_{x+1} + N_{x+2} + \&c. \\ M_x = C_x + C_{x+1} + C_{x+2} + \&c. \\ R_x = M_x + M_{x+1} + M_{x+2} + \&c. \end{array}\right\} (2)$$

In what follows we call x the present age; and since, in the above expression, x denotes any age, we may substitute for it $x+n$; the only limitation* that we make with regard to n, being, that it shall not exceed the difference between x and the oldest age in the table, so that $x+n$ may not exceed that age. Making this substitution, then, the above equations will still hold, and we shall have

$$\left.\begin{array}{l} D_{x+n} = l_{x+n} v^{x+n} \\ C_{x+n} = (l_{x+n} - l_{x+n+1}) v^{x+n+1} \end{array}\right\} (3)$$

$$\left.\begin{array}{l} N_{x+n} = D_{x+n+1} + D_{x+n+2} + \&c. \\ S_{x+n} = N_{x+n} + N_{x+n+1} + \&c. \\ M_{x+n} = C_{x+n} + C_{x+n+1} + \&c. \\ R_{x+n} = M_{x+n} + M_{x+n+1} + \&c. \end{array}\right\} (4)$$

It thus appears that the nth term from the present age, in columns S, M, and R, expresses *the sum of the* n*th and following terms* from the same age in columns N, C, and M respectively; and that the nth term from the present age, in column N, expresses *the sum of all the terms after the* n*th* from that age in column D. If we wish *the sum of the* n*th and following terms* in D, we have only to write $x-1$ for x in the first of the expressions (4), which then becomes

$$N_{x+n-1} = D_{x+n} + D_{x+n+1} + \&c. \quad . \quad (5)$$

And, generally, it will be obvious that, to make our summation commence 1, 2, 3, &c., terms earlier or later, we have only to substitute, in the expressions (4), in the former case $(x-1)$, $(x-2)$, $(x-3)$, &c., and in the latter $(x+1)$, $(x+2)$, $(x+3)$, &c., for x.

* Algebraically considered, it is not necessary to make this limitation.

The expressions (2) and (4) enable us to find the sum of the first n terms, either after or commencing with the present age, in the columns D, N, C, and M; for we have only to subtract from the sum of *all* the terms in these columns, after or commencing with the present age, the sum of those after or commencing with the nth from that age. Hence the sum of the first n terms, *after* the present age,

$$\left.\begin{array}{lll} \text{in D is} & N_x - N_{x+n} & \cdot \quad \cdot \quad \cdot \\ \text{N} & S_{x+1} - S_{x+n+1} & \cdot \quad \cdot \\ \text{C} & M_{x+1} - M_{x+n+1} & \cdot \quad \cdot \\ \text{M} & S_{x+1} - S_{x+n+1} & \cdot \quad \cdot \end{array}\right\} (6)$$

And the sum of the first n terms, *commencing with* the present age,

$$\left.\begin{array}{lll} \text{in D is} & N_{x-1} - N_{x+n-1} & \cdot \quad \cdot \\ \text{N} & S_x - S_{x+n} & \cdot \quad \cdot \quad \cdot \\ \text{C} & M_x - M_{x+n} & \cdot \quad \cdot \quad \cdot \\ \text{M} & R_x - R_{x+n} & \cdot \quad \cdot \quad \cdot \end{array}\right\} (7)$$

Also, according to the principle laid down above, if, in the expressions (6) and (7), we substitute $x+k$ for x, we are furnished with expressions for the sum of the first n terms, either after or commencing with the $(x+k)$th.

Thus, the sum of the first n terms, *after* the $(x+k)$th,

$$\left.\begin{array}{lll} \text{in D is} & N_{x+k} - N_{x+k+n} & \cdot \quad \cdot \\ \text{N} & S_{x+k+1} - S_{x+k+n+1} & \cdot \\ \text{C} & M_{x+k+1} - M_{x+k+n+1} & \cdot \\ \text{M} & R_{x+k+1} - R_{x+k+n+1} & \cdot \end{array}\right\} (8)$$

And the sum of the first n terms, *commencing with* the $(x+k)$th,

$$\left.\begin{array}{lll} \text{in D is} & N_{x+k-1} - N_{x+k+n-1} & \cdot \\ \text{N} & S_{x+k} - S_{x+k+n} & \cdot \quad \cdot \\ \text{C} & M_{x+k} - M_{x+k+n} & \cdot \quad \cdot \\ \text{M} & R_{x+k} - R_{x+k+n} & \cdot \quad \cdot \end{array}\right\} (9)$$

If, in the expressions (7), we suppose $n=1$, they become respectively—

$$\left.\begin{array}{l} D_x = N_{x-1} - N_x \quad \cdot \quad \cdot \quad \cdot \\ N_x = S_x - S_{x+1} \quad \cdot \quad \cdot \quad \cdot \\ C_x = M_x - M_{x+1} \quad \cdot \quad \cdot \quad \cdot \\ M_x = R_x - R_{x+1} \quad \cdot \quad \cdot \quad \cdot \end{array}\right\} (10)$$

If, in the expressions (10), we substitute $x+1$ for x, we obtain the same expressions as we should obtain by making n in (6) $=1$. And, in like manner, by substituting in (10) $x+k$, and $x+k+1$ for x, we obtain the same expressions as we should obtain from (9) and (8) respectively, by making $n=1$ in these expressions. The

expressions just referred to, as well as others which may be derived by transposition from those we have given, we do not think it necessary to set down, since their mode of derivation is so extremely simple; and we shall accordingly refer, as we have occasion for them, to the expressions which we have shown to contain them.

Again, since

$$\begin{aligned} N_x &= D_{x+1}+D_{x+2}+D_{x+3}+D_{x+4}+\&c. \\ N_{x+1} &= \quad\;\; D_{x+2}+D_{x+3}+D_{x+4}+\&c. \\ N_{x+2} &= \qquad\qquad\;\; D_{x+3}+D_{x+4}+\&c. \end{aligned}$$

and so on. If we add these equations together, observing that

$$N_x+N_{x+1}+N_{x+2}+\&c.=S_x,$$

the resulting equation will be

$$S_x=D_{x+1}+2D_{x+2}+3D_{x+3}+4D_{x+4}+\&c. \quad . \quad (11)$$

And, in the same way, we should obtain

$$R_x=C_x+2C_{x+1}+3C_{x+2}+4C_{x+3}+\&c. \quad . \quad (11)$$

Sometimes, in the Commutation Tables, the assurance columns are not exhibited. We therefore proceed to show how their place may be supplied by means of the annuity columns.

$$C_x=(l_x-l_{x+1})v^{x+1}=l_xv^{x+1}-l_{x+1}v^{x+1};$$

but

$$l_xv^{x+1}=vD_x \text{ and } l_{x+1}v^{x+1}=D_{x+1};$$

consequently,

$$C_x=vD_x-D_{x+1} \quad . \quad . \quad . \quad . \quad (12)$$

Again, since

$$\begin{aligned} C_x &= vD_x-D_{x+1} \\ C_{x+1} &= vD_{x+1}-D_{x+2} \\ C_{x+2} &= vD_{x+2}-D_{x+3} \\ C_{x+3} &= vD_{x+3}-D_{x+4} \end{aligned}$$

and so on, to the end of life. If we add these equations, observing that

$$C_x+C_{x+1}+C_{x+2}+\&c.=M_x,$$

$$vD_x+vD_{x+1}+\&c.=v(D_x+D_{x+1}+\&c.)=vN_{x-1},$$

$$\text{and } D_{x+1}+D_{x+2}+\&c.=N_x,$$

the resulting equation will be

$$M_x=vN_{x-1}-N_x \quad . \quad . \quad . \quad . \quad (13)$$

The analogous expression for R, in terms of S, and which is obtained in precisely the same way, is

$$R_x=vS_{x-1}-S_x \quad . \quad . \quad . \quad . \quad (13)$$

The expressions just deduced may be exhibited in a somewhat different form, which is, in certain circumstances, more convenient than the other.

Since, by (10),

$$N_x = N_{x-1} - D_x,$$

the first of the foregoing expressions becomes, by substitution,

$$\begin{aligned} M_x &= vN_{x-1} - N_{x-1} + D_x \\ &= D_x - N_{x-1} + vN_{x-1} \\ &= D_x - (1-v)N_{x-1} \quad . \quad . \quad . \quad (14) \end{aligned}$$

The corresponding expression for R_x, obtained in the same way, is

$$R_x = N_{x-1} - (1-v)S_{x-1} \quad . \quad . \quad (14)$$

Of the foregoing expressions, those numbered (1) to (11) are extensively useful, in deducing and simplifying the formulæ in the practical application of the tables; and the remaining expressions, numbered (12) to (14), besides their use in supplying the place of the assurance columns, when required, serve also to verify the tables. We have not space here to give examples illustrative of any of the foregoing formulæ. But these are so very simple, that this does not seem at all necessary.

For the convenience of those who may wish to test the table on pp. 90, 91, by means of the formulæ of verification given above, we subjoin the following elements:—

When the rate of interest is 4 per cent (as in our table),

$$v = \frac{1}{1{\cdot}04} = {\cdot}96153846,$$

and its logarithm is ·98296666.

Also $1 - v = {\cdot}03846154$, and its logarithm is ·58502665.

The formulæ we have demonstrated may appear to some, at first view, both complex and confused. They are, nevertheless, eminently remarkable for their simplicity and symmetry. This, a further and practical acquaintance with them will make abundantly manifest.

We now proceed to show how the present values of benefits may be deduced from the Commutation Tables; but we must first explain what is meant by the present value of a benefit.

As regards the purchaser of a single benefit, it is seldom indeed that he will receive the precise value (using the word in its ordinary sense) of what he has paid. In the case of certain kinds of benefits, indeed, it is impossible that there should, in any case, be an exact

compensation in this sense. For instance, if the benefit be an endowment—that is, a sum of money to be received in the event of the purchaser attaining a certain specified age; if he do not attain that age, neither he nor his representatives will receive anything, and, consequently, the money he has paid will be lost. On the other hand, if he do attain the specified age, the sum to be then received will be more than his payment would have amounted to if it had been put out at interest at the time the bargain was made; else he would not have run the risk of losing it in the interval. In the case of such a benefit, therefore, the purchaser *must* be either a gainer or a loser. If the benefit consist of an annuity, the purchaser *may* live just such a period as that the number of payments he will receive will be exactly equivalent to the money he has paid. If he live a shorter time he will be a loser, and if a longer he will be a gainer. Similar remarks apply to the case of a life assurance: the purchaser *may* die, and the assurance be received at the end of a period, the improvement of his premium or premiums during which would have produced an amount just equal to the assurance. But as it is much more likely that he will die either before or after the period of this equality, so the probability is proportionally greater that he or his representatives will be either gainers or losers.

But while there is this uncertainty in individual cases, there is, in a sufficiently numerous aggregation of cases, a regularity in the occurrences of the events on which the payments of benefits are usually made to depend, which admits of the calculation of the *average* values of those benefits to a very considerable degree of nicety. For, if we assume the mortality which will be experienced by a class of purchasers, sufficiently numerous to secure an average, to be the same as that indicated in the table which we take as our guide (and which table presents the averages of a number of observations), we say, making this assumption, it is evidently quite practicable to name the sum which will require to be contributed by the whole body of purchasers, to afford to each of them a benefit of a certain amount, on the occurrence of a specified contingency, the time or the fact of this occurrence, or both, being uncertain *in each individual case*. And this sum, divided by the number of purchasers, will be the amount which each ought to contribute, since all at the time of purchase are supposed to be equally likely to come into possession of the benefit. It is the last-named amount, obtained in this manner, which is called the *present value*, or *single premium*, of the benefit to be purchased.

The meaning of the term "present value," might, perhaps, have been more briefly explained, by defining it as the sum which would be required from each of a large number of purchasers of benefits of the same kind, so that the seller should be in the end neither a gainer nor a loser.

We now proceed to the application of the tables. But first, we remark, once for all, that in what follows we always suppose the *amount* of the benefit to be purchased to be £1; except, of course, in the cases of an increasing annuity or assurance. In these we suppose that amount only, to which the purchaser becomes or may become first entitled, to be £1. When the present value of a benefit of £1 on any life is found, that of a similar benefit of any other amount, on the same life, will obviously be found by multiplying this present value by the number of pounds in that amount.

PROBLEM I.—To find the present value of an endowment of £1—that is, of £1 to be received at the end of n years provided (x)* be then alive.

The number of individuals now alive, of the given age, according to the mortality table, is l_x; and the survivors of these, at the end of n years, is the number represented by the table to attain the age $x+n$, viz., l_{x+n}. This number is, consequently, the number of pounds which will have to be paid at the end of n years. But, since money makes interest, the present value of this sum, or $l_{x+n}v^n$, contributed now, will be sufficient to provide for the payment of the benefit. And, since all now alive contribute equally, the amount to be contributed by each—that is, the required present value—will be $\frac{l_{x+n}v^n}{l_x}$.†

To adapt this expression to the Commutation Tables, multiply the numerator and denominator by v^x (which will not alter its value), and it becomes $\frac{l_{x+n}v^{x+n}}{l_xv^x}$.

But the numerator of this expression is equal to D_{x+n}, and

* By this symbol is meant, and the phrase may in reading be substituted for it, "a life now aged x years."

† We have assumed above that the number of purchasers of endowments will be the number represented by the mortality table to be alive at the age at which the purchase is made; but the only assumption as to their number which it is *necessary* to make is, that this number will be sufficient to secure an average mortality proportional to that represented in the table. If we had assumed any other number, we should have had to find, by a proportion, the number of survivors at the advanced age. But, by the assumption in the text, this operation is saved. In both cases the result would be the same. The value of a fraction depends, not on the *absolute magnitude* of its terms, but on their *relative magnitude* or *ratio*. The value of 3-4ths is the same as that of 6-8ths or 9-12ths, because the ratio of 3 to 4 is equal to that of 6 to 8 or 9 to 12.

The same remarks will apply to the other benefits.

the denominator to D_x, by (1) and (3). Hence the expression becomes finally $\frac{D_{x+n}}{D_x}$.

From this general expression we obtain at once the solution of any particular case that may be proposed, by giving to x and n the proper values. Thus, if $x=10$ and $n=11$, that is, if the endowment is to be paid on (10) attaining the age of 21, the expression becomes $\frac{D_{21}}{D_{10}}$; and the numerical solution is effected by taking the proper numbers from the table, and operating upon them as indicated by the formula.

If $n=0$, that is, if the endowment is to be paid immediately, then we have, whatever the age may be, $\frac{D_x}{D_x}=1$, that is, to £1, as we evidently ought to have.

Example 1. $x=10$, $n=11$.

$$\frac{D_{21}}{D_{10}}=\frac{3629\cdot2895}{6051\cdot0605}=\cdot59978=12s.$$

And if the amount to be received were £100, the present value would be $\cdot59978\times100=59\cdot978=$ £59. 19*s.* 7*d.*

Example 2. $x=18$, $n=6$.

$$\frac{D_{24}}{D_{18}}=\frac{3094\cdot1061}{4213\cdot8385}=\cdot73427=14s.\ 8d.$$

For £100, the present value would be $\cdot73427\times100=73\cdot427$ $=$ £73. 8*s.* 6*d.*

PROBLEM II.—To find the present value of a life annuity of £1 on (x).

This benefit consists of a series of payments of £1, to be made at the end of 1, 2, 3, &c., years, to the end of life. Its present value, therefore, will be the sum of the present values of the several payments. These present values are, by last problem,

$$\frac{D_{x+1}}{D_x},\ \frac{D_{x+2}}{D_x},\ \frac{D_{x+3}}{D_x},$$

and so on; and their sum is

$$\frac{D_{x+1}+D_{x+2}+D_{x+3}+\&c.}{D_x},$$

which, by (1), is equal to $\frac{N_x}{D_x}$, which is, therefore, the present value required.

Example 1. $x=20$.

$$\frac{N_{20}}{D_{20}}=\frac{66048\cdot1809}{3818\cdot9594}=17\cdot2948=£17.\ 5s.\ 11d.$$

Example 2. $x=60$.

$$\frac{N_{60}}{D_{60}}=\frac{4068\cdot8483}{418\cdot5753}=9\cdot7207=£9.\ 14s.\ 5d.$$

If the annuity be *due*, that is, if the first payment is to be made immediately, then the present value of that payment, that is, $\frac{D_x}{D_x}$, must be added to the above expression, which then becomes

$$\frac{D_x+N_x}{D_x}=\frac{N_{x-1}}{D_x} \quad . \quad . \quad . \quad (10)$$

The same result is obtained by commencing the summation of the series in the numerator of the first expression a year earlier, that is, with D_x, instead of D_{x+1}.

Example.—Required the present value of an annuity *due* of £1 on (20), that is $x=20$.

$$\frac{N_{19}}{D_{20}}=\frac{69867\cdot1403}{3818\cdot9594}=18\cdot2948=£18.\ 5s.\ 11d.$$

Problem III.—To find the present value of an annuity of £1 deferred for n years on (x).

This benefit consists of a series of payments of £1, to be made $n+1$, $n+2$, $n+3$, &c., years hence, to the end of life; and its present value will, therefore, be the sum of the present values of these payments. These present values are found by Problem I., and their sum is

$$\frac{D_{x+n+1}+D_{x+n+2}+D_{x+n+3}+\&c.}{D_x},$$

which, by (4), is equal to $\frac{N_{x+n}}{D_x}$, and this is, therefore, the present value required.

Example.—If $x=20$, and $n=10$, that is, if the annuity is to be entered upon by a person now aged 20, when he attains the age of 30, its present value will be

$$\frac{N_{30}}{D_{20}}=\frac{37124\cdot4288}{3818\cdot9594}=9\cdot7211=£9.\ 14s.\ 5d.$$

If the first payment is to be made n years hence, the requisite

change will be made in the formula, as in last problem, by writing $x-1$ for x in the numerator. The expression for the present value in this case, therefore, will be $\frac{N_{x+n-1}}{D_x}$, which is also, as it evidently ought to be, the expression for the present value of an annuity deferred for $n-1$ years.

If, in the expression $\frac{N_{x+n}}{D_x}$, we suppose n, the period of deferment, equal to 0—that is, that the annuity is to be entered upon immediately—the expression becomes $\frac{N_x}{D_x}$, the same as in last problem, which problem is, therefore, but a particular case of the present; and the following rule will apply to both:—

Divide the number in column N, opposite the age at which the annuity is to be entered upon, by the number in column D, opposite the present age; the quotient will be the present value of the annuity.

A similar remark will apply to the other benefits, and a corresponding rule for all of them might be formed. But, as our space is limited, the present intimation must suffice.

PROBLEM IV.—To find the present value of a temporary annuity of £1 for the next n years, on (x).

This benefit consists of n payments of £1, to be made at the end of 1, 2, 3, ... n years, if x be then alive. The present values of these payments are

$$\frac{D_{x+1}}{D_x}, \frac{D_{x+2}}{D_x}, \frac{D_{x+3}}{D_x}, \ldots \frac{D_{x+n}}{D_x},$$

the sum of which is

$$\frac{D_{x+1}+D_{x+2}+D_{x+3}+\ldots D_{x+n}}{D_x};$$

and this, by (6), is equal to

$$\frac{N_x-N_{x+n}}{D_x},$$

which is, therefore, the present value of the annuity.

This formula might also have been deduced by subtracting from the present value of an annuity for the whole life, that of an annuity deferred for n years, since the present value of an annuity during the next n years, and that of an annuity to commence at the end of n years, are evidently together equal to the present value of an annuity for the whole life. Thus, the present value of

the life annuity is (Prob. II.) $\frac{N_x}{D_x}$, and that of the deferred annuity is (Prob. III.) $\frac{N_{x+n}}{D_x}$. Their difference, or $\frac{N_x - N_{x+n}}{D_x}$, is the present value of the temporary annuity, as before.

Example.—Required the present value of an annuity of £1 during the next 10 years, on (20).

$$\textit{Answer.}\ \frac{N_{20}-N_{30}}{D_{20}} = \frac{66048{\cdot}1809-37124{\cdot}4288}{3818{\cdot}9594} = \frac{28923{\cdot}7521}{3818{\cdot}9594}$$
$$= 7{\cdot}5737 = £7.\ 11s.\ 6d.$$

If this be added to 9·7211, which we found by last problem to be the value of an annuity deferred for 10 years, on a life of the same age, the sum 17·2948 will be the value of an annuity on the whole life; and it corresponds with the value which we deduced for that age by Problem II.

Here we may repeat, once for all, as the remark applies equally to the expressions for all the benefits, that so long as the payments continue the same in number, if they commence a year earlier than we have supposed, the change requisite to be made in the formula is the substitution of $x-1$ for x in the numerator. The above expression thus becomes

$$\frac{N_{x-1}-N_{x+n-1}}{D_x}.$$

Problem V.—To find the present value of a deferred temporary annuity of £1 on (x), the period of deferment being k years, and that of continuance n years.

This benefit consists of n payments of £1, to be made at the end of $k+1$, $k+2$, $k+3$, ... $k+n$ years. The present values of the payments are (Prob. I.)

$$\frac{D_{x+k+1}}{D_x},\ \frac{D_{x+k+2}}{D_x},\ \ldots\ldots\ \frac{D_{x+k+n}}{D_x};$$

and the sum of these expressions is

$$\frac{D_{x+k+1}+D_{x+k+2}+\ldots\ldots D_{x+k+n}}{D_x},$$

which, by (8), is equal to

$$\frac{N_{x+k}-N_{x+k+n}}{D_x};$$

and this is evidently the present value required.

It will be seen that this expression may be derived from that in the last problem, by changing x into $x+k$ or n into $k+n$ in the numerator.

Example.—Required the present value of an annuity of £1 on (20), to be entered upon 10 years hence, and to continue 10 years.

Here $x=20$, $k=10$, and $n=10$.

$$\frac{N_{30}-N_{40}}{D_{20}}=\frac{37124\cdot4288-19964\cdot4836}{3818\cdot9594}=\frac{17159\cdot9452}{3818\cdot9594}=4\cdot4934$$

$$=£4.\ 9s.\ 10d.$$

(*To be continued.*)

On the Value of Mr. Gompertz's Formula for the Number Living, in Terms of the Mortality according to Age, compared with the Value of a similar Formula published in 1832. *By* T. R. EDMONDS, B.A., *formerly of Trinity College, Cambridge.*

IN the *Assurance Magazine* (Oct., 1860, and July, 1861) there have appeared two papers written by me to show in what respect the law of human mortality described by Mr. Gompertz in 1825 differs from the law described by me in 1832. In the same papers I have also shown in what respect the formula of Mr. Gompertz, $y=dg^{p^x}$, for the number living in terms of the mortality according to age, differs from my formula for the same purpose, which is $y=10^{\frac{k^2a}{\lambda p}(1-p^x)}$ or $y=g^{1-p^x}$, on putting $g=10^{+\frac{k^2a}{\lambda p}}$ instead of $g=10^{-\frac{k^2a}{\lambda p}}$, as assumed by Mr. Gompertz.

In the *Assurance Magazine* (April, 1861, and Oct., 1861) there have also appeared two papers, in opposition to mine, which have been written by Mr. Sprague, the accepted advocate of Mr. Gompertz. In the latter paper of Mr. Sprague the opposition has been confined chiefly to the question of the relative value of Mr. Gompertz's formula and my formula of mortality. As this question appears to me to be the only one which can, with any apparently good reason, become the subject of dispute between Mr. Gompertz and myself, I will follow Mr. Sprague's example, and confine my coming remarks chiefly to the comparative merits of the two competing formulæ.

The formulæ to be compared have both been deduced from the same elements, both being deductions from the same differential equation, $dy=-yap^xdx$. This equation can be made to yield, on integration, no more than one correct and complete solution of the problem proposed. I maintain that my formula, $y=g^{1-p^x}$, contains the complete solution. I also maintain that Mr. Gompertz's formula, $y=dg^{p^x}$, does *not* contain the complete solution of the problem, because it includes constants (d and a)

which had been left "indeterminate" by Mr. Gompertz in 1832, the date of publication of my formula.

In consequence of the above two constants having been left undetermined, the formula of Mr. Gompertz involves the serious error of representing the exponent of (g) to be p^x, whilst the true exponent is ($p^x - 1$). Also, the same formula is so defective that it has never been shown to be capable of being *directly* applied to practice in the construction of a new table of mortality from newly-observed rates of mortality according to age in any population.

By means of my formula, with the constants (d and a) determined, the value of (y) may be obtained for any given age directly from newly-observed rates of mortality according to age. Mr. Gompertz, by means of his formula, containing two indeterminate constants, has never been able to do more than interpolate values of (y) between three other assumed values of (y) adopted as correct and transferred from some previously-constructed table of mortality. None of the tables so used as bases by Mr. Gompertz express any definite rate of increasing mortality according to age, nor in their construction have they been regulated by any principle of calculation superior to that commonly designated as that of the "rule of thumb."

Mr. Sprague, the advocate and representative of Mr. Gompertz, has had the opportunity of denying (if he could) the allegations which I had previously made and have now repeated, respecting the defects of Mr. Gompertz's formula, and he has omitted to make such denial. He has not denied that in Mr. Gompertz's formula the exponent of (g) is wrongly stated, nor has he denied that both (d) and (a) were left "indeterminate" by Mr. Gompertz. As to the first two defects, Mr. Sprague observes silence in his last paper. As to the third defect, he makes the allegation, that, for Mr. Gompertz's object, "it was not necessary to make any inquiry respecting the nature of the quantity (a)." This allegation does not differ much from alleging that Mr. Gompertz left (a) an indeterminate constant because he did not wish to know it more particularly.

I had previously stated, as a fact not likely to be disputed, that the object of Mr. Gompertz (as well as myself) was to find an expression for the number of living at any age (x) in terms of the mortality. Upon this Mr. Sprague remarks as follows:—"This was Mr. Edmonds's object, but "was not Mr. Gompertz's. The object of the latter was this—the mor-"tality (not the *annual* mortality) at any age (x) being proportional to "q^x (*i.e.* $=aq^x$), required a formula for the number living at the age (x). "Mr. Gompertz's formula (dg^{p^x}) gives the correct solution of his problem, "and Mr. Edmonds's formula, corrected by the introduction of L_0, gives "the correct solution of his problem." I had also remarked, that, previous to forming the differential equation $dy = -yap^x dx$, it was essential that the nature of the quantity (a) given to represent the mortality for one year should be determined. This remark, which I regarded as an axiom, has been met by a simple denial—"that it is *not* essential that the nature of (a) should be so determined." According to Mr. Sprague, it was not necessary to make any inquiry respecting the nature of the quantity (a), either before the formation of the differential equation or after the integration was completed.

It does not appear to me that these denials by Mr. Sprague can be

rendered consistent with the admission that my formula is correct for my purposes. The original quantities dealt with by Mr. Gompertz and myself are the same, and the final results arrived at by both of us are the same. There is no room for difference between the two formulæ, except in completeness—one formula being free, and the other formula *not* being free from constants which have been left "indeterminate."

It is difficult to account for Mr. Sprague's denial of the apparently obvious truth—that the two formulæ have one common object. I can attribute this denial to nothing else than misconception as to the nature of these constants (d) and (a). Mr. Sprague has fallen into numerous errors with respect to these quantities. These errors I will now proceed to correct.

I would first notice the alleged "curious inconsistency" (p. 35) of my having stated, on the same page, that two errors had been committed by Mr. Gompertz; the first error being that of using the unknown quantity (d) instead of its equivalent in known quantities, $\frac{L_0}{g}$; and the second error being that of retaining the quantity L_0 when it was manifestly superfluous. I am unable to perceive any inconsistency in this statement. The propriety of the former correction is indisputable. Also the propriety of the second correction becomes manifest when it is considered that L_0, being a common multiplier of every value of (y), can yield no information as to the proportions existing between the values of (y) at different ages, and being, therefore, superfluous, can most properly and conveniently be fully represented by unity. I now repeat the "curious inconsistency" as a truth, of which the first part has been admitted (or not denied) by Mr. Sprague, and of which the second part will never be denied by anyone besides Mr. Sprague. The two corrections indicated will convert Mr. Gompertz's formula, $y=dg^{p^x}$ into $y=\frac{1}{g}g^{p^x}=g^{p^x-1}$, which is identical with my formula—having regard to the circumstance that the (g) used by Mr. Gompertz is the reciprocal of the (g) used by me.

Respecting L_0 as distinct from unity, I have twice asserted, in opposition to Mr. Sprague, that it was superfluous and useless in the complete and correct formula of mortality for either of the three periods of human life. I now repeat the assertion, that there is no more reason for inserting L_0 into the formula of mortality than there would be for inserting any specific number of pounds sterling into a formula or table of discount of money, of which the payment is deferred (x) years. The two cases bear the closest resemblance to one another when, in the mortality table, the period of age observed is the short one intervening between "infancy" and "puberty," say from 8 to 12 years, wherein the mortality is constant and at a minimum. In the discount table, if the rate of interest be 3 per cent., each term will be to its next following term as $(1+\cdot03)$ is to 1 : the common ratio of the series will be $(1+\cdot03)^{-1}$, and the $(x+1)$th term will be $(1+\cdot03)^{-x}$ when the first term is unity, say $y_0=1$.

Similarly, in the mortality table, if the mortality be constant and the finite annual decrement be $a=\cdot0063845$, we have only to substitute this value of (a) for ·03, in the formula above given, for the present value of £1 receivable (x) years hence, and we shall then have $y_x=(1+a)^{-x}$ to

represent the number surviving at the age (x) out of every unit of population living at age 0, or when $y_0=1$. The first term of the discount table is always taken to be $y_0=1$, and there is every reason for adopting the same course in constructing a mortality table. The quantity used to denote the finite decrement, whether in mortality tables or in discount tables, is always the difference between the ordinate 1 and the *next greater* ordinate distant by an interval of one year; but in discount tables, as in mortality tables, the finite decrement above described has always reference to the antecedent ordinate $(1+a)$, and not to the succeeding ordinate 1. Mr. Sprague expresses his opinion (p. 39) that the finite decrement in mortality tables ought to be the difference between the ordinate 1 and the *next less* ordinate. If his recommendation were adopted and extended to discount tables, it would ensue that what is now called a discount table at 3 per cent. would in future be called a discount table at 2·91262 per cent.

Mr. Sprague erroneously reasserts that my formula is defective because it does not contain the factor L_0. In contradiction I can state, that, from this formula, which does not contain either L_0 or (d), I have calculated, and afterwards published, five theoretical tables of mortality. I can also state, that, if I had used Mr. Gompertz's formula instead of my own, it would not have been necessary for my purpose to make any use of the constant (d) of that formula. This last fact supplies an excellent proof that the (d) of Mr. Gompertz's formula is superfluous and useless.

All my theoretical tables of mortality have been calculated by summation of the logarithms of the probabilities of living one year, for every year of age. The probability of living one year at any age is independent of L_0, whether my formula or Mr. Gompertz's formula be used. All my tables of mortality have been formed by commencing the summation of the logarithms of the probabilities of living one year at the age of 12 years, which is equivalent to taking unity to represent the number living at the age 12 years. By this means, in my Table A.2, the logarithms of the living at ages 12, 22 and 99, were obtained, and afterwards the numbers corresponding to these logarithms were obtained, these numbers being 1·000000, ·928658, and ·000142. In order to form Table A.1, which represents numbers living at every completed year of age, the decimal point was removed five places to the right, so that the numbers living were written thus, 100000·0, 92865·8, and 14·2. This change in the place of the decimal point was made for convenience, and not from necessity. The chief object of the change was the avoidance of the ciphers, which otherwise would have immediately followed the decimal point at ages above 80 years. The removal of the decimal point one place to the right is equivalent to, but is not really, an operation in multiplication by 10.

In the former of his two papers, Mr. Sprague, in respect of the constant (d) in Mr. Gompertz's formula, remarked (p. 291)—"It is really an essential part of the formula, and *as such is employed by Mr. Edmonds himself.*" In answer, I remarked that there was not the remotest approach to truth in this statement. I added, also, that no attempt was made to adduce any proof of such an improbable statement. The attempt at proof, which was then omitted, is now adduced. This "proof" consists in alleging, that, subsequently to his erroneous statement, I have used (p. 336) the quantity (d) in order to arrive at the formula given by me as the only correct one. In this case I used the quantity in question only to eliminate

it. That is to say, I used the quantity (d) for no other purpose than to prove that it was of no use!

In my last paper I objected to one of the proposed corrections in the published process of integration of Mr. Gompertz's formula, by stating that the equation as corrected (by Mr. Gompertz or by his authority) was more defective, in form at least, than was the equation intended to be corrected. I stated that the proposed correction would leave no new constant on either side of the equation. Mr. Sprague does not deny the justness of my criticism, but he has assumed the appearance of contradicting me, by referring to the place (p. 289) of a correction *made by himself*, such correction being a confirmation of my statement. I had stated that the erroneous (b) had been abolished on one side without the correct (d) appearing, as it ought to have done, on the other side of the equation. Mr. Sprague's correction (p. 290) is, that (d) ought to have appeared in the place designated by me, but that Mr. Gompertz had introduced it at one step later in his process of integration.

This misplacement by Mr. Gompertz of the letter (d) is of high significance, as it serves to show the cause of Mr. Gompertz's failure to determine the nature of the quantity (d), and also serves to confirm a statement made by me in October, 1860 (p. 176), that the integral $y=dg^{p^x}$ was probably known to Mr. Gompertz before the corresponding differential was known to him, and that the published apparent process of integration was little more than a process of differentiation inverted.

The course of proceeding of Mr. Gompertz appears to have been the following:—after he had made the discovery, that in the period of human life extending from the age of 15 to the age of 55 years, the number living at any age (x) was represented by $y=dg^{p^x}$, the first step was to take the hyperbolic logarithms of the quantities on both sides of the equation, whence he obtained the equation—

$$\log.\ y=\log.\ d+\log.\ g\times p^x.$$

The second step was to differentiate the last equation, so as to obtain the equation—

$$\frac{dy}{y}=\log.\ g\times\log.\ p\times p^x dx.$$

The third and last step was to put

$$\log.\ g\times\log.\ p=-a,$$

the quantity (a) being an "indeterminate constant," so far as Mr. Gompertz is concerned, but having been since ascertained by me to represent the annual mortality at the age 0, or at the infinitely small interval of time next the age 0.

The published process of integration by Mr. Gompertz appears to have been derived from the above process of differentiation. The constant (d) was mistakenly supposed to be a simple quantity, easily determinable by arithmetical process, and consisting of no more than one factor, respecting which no information was required. The integration was partially performed, having been performed in the same manner as it would have been if the constant (d) had been absent, and the integral to be arrived at had been $\log.\ y=\log.\ g\times p^x$. By his erroneous first step in the process of

integration, Mr. Gompertz finds that log. y=log. $g \times p^x$; and immediately afterwards declares, without stating any reason for the addition, that log. y=log. d+log. $g \times p^x$. It is only by comparing the process of integration with the foregoing process of differentiation that the misplacement of (d) is accounted for. If the process of integration had been a genuine one, Mr. Gompertz could hardly have failed to inquire for and determine the value of the constant (d) in terms of L_0 and (g). In ordinary cases no serious error would have ensued from the course of proceeding adopted by Mr. Gompertz; but, in the present case, he had the misfortune to stumble on a constant (d), which, containing (g), the second constant of the formula, would, when known, have changed his formula from $y=dg^{p^x}$ into $y=L_0 \times g^{p^x-1}$.

I believe that I am the first person who has described the quantity (α) and used it in the construction of tables of mortality. This newly-described quantity is such that $e^{\alpha}=1+a$, so that α=hyp. log. of $(1+a)$. The quantity (α) represents the decrement of a unit of life, in the infinitely small interval of time next the age 0, multiplied by the infinite number of intervals into which the year is supposed to be divided, and is$=\frac{\alpha}{\nu} \times \nu=\alpha'$. The quantity ($a$) is the finite decrement in one year, and measures the difference between two consecutive ordinates, separated by an interval of one year in age, when the smaller ordinate is represented by unity.

There has been an admission made by Mr. Sprague (and before noticed) which affords indirect evidence that Mr. Gompertz was *not* acquainted with the value or nature of (α). The admission referred to is, that it was not necessary for Mr. Gompertz to inquire what was the value of (α). According to my own experience, this admission is tantamount to proof that he did not know the value of the quantity in question. For, in my own case, after arriving at the knowledge of the correct formula of mortality $y=10^{\frac{k^x\alpha}{\lambda p}(1-p^x)}$, or $y=g^{1-p^x}$, I failed in my first endeavours to construct a theoretical table of mortality, through using the finite annual decrement (when the mortality was constant) to represent (α). My attention having been fixed on the subject, I soon found out the cause of my failure, and afterwards used with success the quantity α=hyp. log. $(1+a)$, instead of the quantity (a) first used by me. If Mr. Gompertz had been acquainted with the true formula of mortality $y=g^{1-p^x}$, and if he had attempted to construct a table of mortality directly from such formula (without the aid of a previously-constructed "rule of thumb" table of mortality), he would probably have succeeded, like me, in obtaining the value of (α) in terms of the finite annual decrement. But it is not alleged that Mr. Gompertz ever made such attempt, or came near the borders of the discovery made by me.

In relation to the comparative values of (α) and (a), I would call attention to what appears to me a "curious inconsistency" contained in a criticism of Mr. Sprague, at page 39. He says of me—"In calculating "the actual mortality, he supposes the rate of mortality constant throughout "the year. This is contrary to the fundamental hypothesis, that the mor-"tality at any age (x), integral or fractional, is ap^x." In reply, I would ask the critic to inform the public (if he can) how such a supposition

can be dispensed with, when the object is to express in finite terms the mortality existing during an infinitely small portion of time at a given age. I would also observe, that although the annual mortality at the precise age (x) is always expressible in exact terms by ap^x, yet this fact is perfectly consistent with the fact of the mortality being constant, as it commonly is in all tables of mortality, for a small interval of age immediately preceding the age of 15 years. At that interval of age (p) becomes equal to unity, and ap^x then becomes equal to (a).

Being the first discoverer of the quantity (a) as existing in all tables of mortality founded upon rates of mortality increasing in geometric progression, I have been obliged to describe it by some name. I have selected the name of "annual mortality," because such name, applied to a point of age, conveys nearly the same idea as is conveyed by the term annual mortality applied to an entire year of age. The annual mortality at any entire year of age is commonly represented by the numbers who have died in that year of age, divided by the average number constantly living in that year of age, during the space of one year. Such annual mortality for an entire year of age will be represented nearly by the annual mortality corresponding to the point of age marked by the middle of the year of age observed. That is to say, the annual mortality during the (x)th year and the $(x+1)$th year of age will be represented *nearly* by $ap^{x-\frac{1}{2}}$ and $ap^{x+\frac{1}{2}}$. These expressions are in accordance with my use of the term "annual mortality," to represent by ap^x the mortality experienced at the precise age (x) intermediate between the (x)th and $(x+1)$th years of age.

The recent paper of Mr. Sprague contains several errors consequent on his imperfect knowledge of the signification of the quantities (a) and (ap^x), and consequent also on his confounding together three perfectly distinct things—the annual mortality during one entire year of age, the annual mortality at the beginning of the year of age, and the probability of a person alive at the beginning dying before the end of the year of age. At page 43, Mr. Sprague says that I have erroneously stated that my law of mortality is such, that if the mortality at the age 25 were 1 per cent., and at age 45 it were 1·80 per cent., then the annual mortality at the intermediate age would be found by interpolating the terms of a geometric series; and that I have also erroneously stated that the annual rate of mortality increases in a geometric progression, of which the common ratio is (p). His words are these:—"Now, bearing in mind the distinction "already pointed out, that if ap^x be the rate of mortality at the age (x), "then the annual mortality at that age will be $1-g^{(p-1)p^x}$, it is obvious "that the annual rate of mortality does not increase in geometric progres-"sion; but, on the contrary, the chance of living a year $(=g^{(p-1)p^x})$ "decreases in a geometric progression."

In this attempt to correct an error which has no existence, Mr. Sprague has committed no less than *four* errors. The first error is that of confounding together the annual mortality at the beginning of a year of age with the probability of dying during the year. The second error is that of using the words "annual mortality at the age x" when he ought to have designated some particular year of age (either the (x)th or $(x+1)$th year), to which the probability of dying in one year is intended to be applicable. The third error is that of saying that the *chance* of living one year decreases in a geometric progression; whilst the fact is, that the *logarithm of the*

chance of living one year decreases in a geometric progression. The fourth error is a verbal one, the words "*but on the contrary*" being used when there is no contrariety, real or apparent. With regard to the first two errors committed by Mr. Sprague, I would remark, whilst using his own language, "that they were not to be expected from the veriest tyro in actuarial science." With regard to his third error, I would remark, that it was not to be expected from anyone having the slightest knowledge of Mr. Gompertz's formula or my formula of mortality.

I have now to call the particular attention of the reader to an extraordinary charge of error brought against me by Mr. Sprague. The alleged error has no existence except in the imagination of Mr. Sprague. It is only by manifest misrepresentation, first of my words, and then of my figures, that the alleged error is supported. I now give the entire passage in which the charge is made. The reader will perceive that the passage is self-contradictory in its most essential particular. At page 39, Mr. Sprague writes as follows:—

. . . . "And there will remain no doubt that he has fallen into two "very distinct and serious errors. In the first place, he has taken to "denote the annual mortality a quantity (a) such that the number living "at the commencement of the year is to the number living at its end as "$(1+a)$ is to 1. Instead of this, the correct assumption is to take (a) "such that the number entering on the year is to the number surviving it "as 1 to $1-a$. This is a fundamental error, and one that was not to be "expected from the veriest tyro in actuarial science. To put the point in "another way:—the annual mortality at any age (x) is found by com-"paring the finite decrement for one year with the number living at the "*beginning* of the year, whereas Mr. Edmonds has compared it with the "number living at the *end* of the year! The practical result of this error "will be, that supposing (a) to be the mortality at the beginning of a "year, Mr. Edmonds makes the actual mortality of the year greater than "(a), instead of less, as it obviously should be. To take the figures given "by him on page 335, if $a=\cdot0063643$, then will the actual mortality of "the year be $1-e^{-a}$ or $\cdot0063440$, instead of $\cdot0063845$ as stated."

With regard to the above extraordinary charge, I would first observe that Mr. Sprague is in error if he believes that the finite decrement for one year when the mortality is constant, in respect of a unit of life at the beginning of the year, as given by me, differs in any respect, except that of form, from the similar finite decrement given by himself. The finite decrement given by me is (a) in respect of $(1+a)$ living at the beginning of the year, and is consequently $\frac{a}{1+a}$ in respect of a unit of life at the beginning of the year. The finite decrement given by Mr. Sprague in respect of a unit of life is $1-e^{-a}=1-\frac{1}{e^{a}}=1-\frac{1}{1+a}=\frac{a}{1+a}$, a quantity which is identical with my own.

I have next to observe that Mr. Sprague has said what is *contrary to the truth* when he states (in contradiction to what he himself had previously stated in the same paragraph) that "Mr. Edmonds has compared the finite decrement for one year with the number living at the *end* of the year." The statement made by me (p. 335), and misrepresented by Mr. Sprague, is the following:—"(a) is the finite decrement for one year, and is such that

"the number living at the beginning of the year is to the number surviving "at the end of the same year as $(1+a)$ is to 1." It appears to me impossible for anyone to fail to perceive from the sentence quoted, that (a) was given by me as the decrement in one year out of $(1+a)$ persons alive at the beginning of the year. Indeed, Mr. Sprague shows that there was no misunderstanding of my words on his part, for he writes thus at foot of page 38:—"Mr. Edmonds supposes the annual mortality to be such "that out of $(1+a)$ persons alive at the beginning of the year, (a) die in "the course of the year, so that the annual mortality is $\frac{a}{1+a}$."

Mr. Sprague having misrepresented my words, proceeds, in the next place, to misrepresent my figures; and having obtained a correct result by his own method, compares it with an imaginary result, which, without any foundation, he alleges to have been given by me. The words used by him, "instead of ·0063845 as stated," are *not true*—no such statement having been made by me. The finite decrement in one year, in respect of a unit of life at the beginning of the year, has been stated by me to be $\frac{a}{1+a}$. On substitution of the value given for (a), we obtain for the value of such decrement $\frac{\cdot 0063845}{1\cdot 0063845}$, which is equal to ·0063440, being the same result as that obtained by Mr. Sprague. This misrepresentation of my figures is the less excusable from the circumstance that Mr. Sprague had before him and was quoting from my published Tables A.1 and A.2. On inspection of these Tables (between ages 8 and 12 years), he might easily have seen that the results, given by him as new, and corrections of my results, are identical with results published by me thirty years ago.

I have been blamed by Mr. Sprague for using the words "homely" and "transcendental" in describing the substance of the law or part of the law of human mortality discovered by Mr. Gompertz. I could find no more suitable words to represent the facts which I wished to communicate. The facts themselves I regarded as of the highest importance in the question of a disputed claim between Mr. Gompertz and myself. I believe that if the facts communicated in the paragraph complained of had been sooner known to Mr. De Morgan and to the public, no charge of "unfair suppression" would ever have been made against me. In that paragraph I stated that the universal law of human mortality was capable of being correctly and precisely expressed by three geometrical series—the common term of each of which being ap^x, and the common ratio (p) having a fixed and determined value in each of the three different periods of human life. I also therein stated that the formula $y=g^{1-p^x}$ was a mathematical consequence of the above-described simple law of mortality, but that the knowledge of this formula did not necessarily either precede or follow the knowledge of the true and exact law of human mortality signified by the expression ap^x.

In the letter from Mr. Gompertz there occurs (p. 297) the following passage, in which my claim is incorrectly described:—

"I wish, in this letter, to notify my thanks to Mr. Sprague, whom I "have not the honour of personally knowing, for his kind and able paper "in vindication of my claim to be the sole independent publisher of a

"theorem which, I am gratified to say, appears of importance to the "scientific world. I say the sole independent publisher, though the fact "of my being the first independent discoverer has not been denied me even "by the gentleman who claims to be the second discoverer, because that "claim, should it be ever proved to be a just one, would not interfere "with me."

In the above passage the question at issue between Mr. Gompertz and myself has been misrepresented. The claim described as mine by Mr. Gompertz is not the claim which I have made. I have not yet made any claim to be either the first or second discoverer of the theorem alluded to—comprised in the formula $y=dg^{p^x}$, and supposed to express correctly and completely the relation at every age between the mortality and the number living. I acknowledged, in my *Life Tables*, published in 1832, that Mr. Gompertz had preceded me, in using, for a similar purpose, a formula greatly resembling my own formula. I did not allude to the defects in Mr. Gompertz's formula, which I then believed to exist, but which I have not had the opportunity of proving to exist until the present moment. As to putting in a claim to be "*second*" discoverer, either of the law of human mortality or of the theorem alluded to, the idea never entered my mind of doing anything so futile.

The claim made by me in 1832 (adopting the phraseology of Mr. Gompertz) amounted to this—that I was the sole independent publisher and first discoverer of the universal law of human mortality as applicable to three distinct periods of human life—*before*, *during*, and *after* maturity; that the mortality at any age (x) in each period was represented by αp^x; and that the values of (p) for the three several periods of life in all populations were $\frac{1}{1\cdot48}$, 1·03, and 1·08 respectively. This discovery was complete before I had any knowledge of the existence of any connexion between such discovery and the theorem of Mr. Gompertz.

Until the present time I have not made any claim which affects Mr. Gompertz's title to the credit of discovering the formula which has been supposed to exhibit, "*for a long period of man's life*," the true and exact relation between the mortality and the numbers living at different ages. Now, however, after having proved the existence of two serious defects in his formula, I am about to make a claim which will affect the value of Mr. Gompertz's claim. The claim which I am about to make is founded on the belief which I entertain that the formula of Mr. Gompertz ceased to be of any value to the public in the year 1832, the date of publication of my formula.

Now, in 1862, I lay claim to the credit of being the first discoverer of the only existing formula which exhibits the true and exact relation between the mortality and the numbers living according to age, in either of the three periods of human life. I have produced abundant evidence of the fact, that the formula of Mr. Gompertz contains two constants which were left "indeterminate" by Mr. Gompertz, but have been "determined" by me. This evidence has been produced before an accepted advocate and representative of Mr. Gompertz. The truth of my allegations respecting the two constants has not been denied, and the representative of Mr. Gompertz has shown himself unable to prove that either of these constants had ever been "determined" by Mr. Gompertz.

Report made by Mr. Samuel Brown to the International Statistical Congress, as to the Institute of Actuaries, &c.

THE mere collection of statistics would be of little service to the public unless they were carefully analysed and compared, and those practical results deduced from them which may tend to benefit society at large. In every branch of statistics there are ardent inquirers into truth, ready to examine the facts collected, with the view of deducing therefrom those laws which seem to govern the mental, the moral, and the social as well as the material world. But there is one class of facts in which, in this country more especially, the results of statistical labours have been turned to the most practical use for the public advantage, and have led to the establishment of Companies of the highest importance, both in a social and commercial point of view. The questions regarding population, the causes of its increase or decrease, marriages, births and deaths, the diseases which end in the latter, and seasons of scarcity or plenty, which affect all alike, engaged at an early period the attention of English as well as foreign statists. Dr. Farr, in his Report on the Programme of this Congress, has ably pointed out how the mathematical school of statistics, commencing with Halley, and the construction of his Life Table, drew attention to the important application of population statistics to estimate the value of human life. There can be no doubt that it was owing to the scientific character of the men who first studied the question of vital statistics that life assurance in this country has been developed to so extraordinary an extent, and is established on so firm a basis. Halley, Demoivre, and Simpson laid the foundations of the science. Dr. Price exposed, with great ability, the false schemes of numerous Annuity Societies, and the ignorance with which they were conducted, and was directly concerned in the establishment, on a sound basis, of the first Society which put to practical use the science of life statistics—the Equitable Assurance Society. The appointment of his nephew, Mr. Morgan, as the Actuary of the Society, ensured its success, for it required both prudence and caution, and high mathematical attainments, to introduce principles then so novel and distrusted. Since then, the system has spread throughout the kingdom, till, by means of various Public Companies and Mutual Associations, a sum considerably exceeding £200,000,000 is secured for the families or representatives of the assured in this country, and which requires at least £6,000,000 in annual payments during the lives of the assured to meet these large accumulations at their deaths. Nor is this the only practical application of these doctrines of life statistics. Reversionary Interest Societies, for purchasing life interests or reversions depending on human life; Annuity Societies, to secure provision for old age or for survivorship in marriage; Endowment Societies, for providing capital to educate or put forward children in life; Friendly Societies and Sick Funds, for affording relief in sickness; all flourish or decay in proportion as the managers of them either understand the science of the probabilities of human life, or consult those who do; or, on the other hand, proceed in ignorance of, or persist in a wilful indifference to, the true laws which govern these events. All these interests, so great and varied, engage the attention of a body of men who, as actuaries of Assurance Companies, are bound to study the scientific principles on which the safety and prosperity of these Companies depend, and by their public character and mathematical education naturally take an interest in all those statistical

questions, in the investigation of which the doctrines of probability can be applied either to discover their laws of operation or to educe useful results.

With the view of combining the knowledge and stimulating the energy of the men engaged in these intellectual pursuits and useful labours, and, still more, of training the younger members of the profession to a more ardent cultivation of the science, the Institute of Actuaries was constituted in January, 1849, and the late John Finlaison, Esq., the Government Actuary, was elected the first President. On the recent and lamented death of Mr. Finlaison, Charles Jellicoe, Esq., was unanimously elected his successor.

The Institute was declared to be founded for the purpose of elevating the attainments and status and promoting the general efficiency of all who are engaged in occupations connected with the pursuits of an actuary, and for the extension and improvement of the data and methods of the science which has its origin in the application of the doctrine of probabilities to the affairs of life, and from which life assurance, annuity, reversionary interest, and other analagous institutions derive their principles of operation. It embraces, as its peculiar province of inquiry, all monetary questions involving a consideration of the separate or combined effects of interest and probability.

It will thus be seen that, under one or other of these heads, it comprises the study of most of the great questions of political or social economy, from the statistics of which the laws of their occurrence or recurrence can be deduced by the doctrines of probability, even though they may not give rise to public Societies to carry them into actual practice.

The Institute consists of Contributing Members, Fellows and Associates, and Non-contributing (namely, Honorary, Foreign and Corresponding) Members. At the present time the number of Fellows and Official Associates is about 66, of the Associates about 80.

Amongst the Honorary Members are enrolled the names of the late Mr. G. R. Porter, of the Board of Trade, whose statistical labours are so well known; Mr. Charles Babbage, F.R.S.; Professor De Morgan; Dr. Farr, F.R.S.; Mr. Benjamin Gompertz, F.R.S.; Mr. Peter Gray; Sir J. W. Lubbock, Bart., F.R.S.; the Right Hon. Lord Overstone, F.R.S.; Mr. John Tidd Pratt, M. Quételet, and Professor Sylvester, F.R.S. The Corresponding Members in Belgium, France, Germany, Holland, Russia, Sweden, and the United States of America, are connected mostly with the great assurance institutions of those countries, and have, many of them, rendered, by their writings or contributions to the *Journal*, valuable service to the literature of the subject. Amongst these may be mentioned MM. Joliat, Lafond, Le Hir, Maas, Pouget, Bruggemann, Hopf (of Gotha), Lazarus (of Hamburg), Masius, Daninos, Dr. Curtis, Professor Silliman, and Mr. Sheppard Homans.

The Institute is governed by a President and Council. The Council consists of Fellows and Official Associates, out of whom the Members elect by ballot, annually, a President, four Vice-Presidents, a Treasurer, and two Secretaries.

One of the most efficient means of carrying out the purposes of the Institute has been the establishment of a system of voluntary examinations; the examiners being Members of the Institute, and a moiety at least of them being Fellows. The examination comprises mathematical theory, vital statistics, computation and construction of tables, bookkeeping, and office

routine. It consists of three parts, separated from each other by at least one year; and no Member is eligible for the final examination until he has attained the full age of 21 years. The first year's examination is confined principally to questions in general mathematics and geometry. The second year's examination comprises more difficult questions in the theory of logarithms, of probabilities, tables of mortality and annuities, and assurances on lives and survivorships. The third year's examination extends to (1) *Life Assurance Finance*—the construction and graduation of tables of mortality, the methods of determining surplus and distributing it in an Assurance Society; (2) *Legal Principles*—Acts of Parliament, partnerships, probates and letters of administration, assignments, bankruptcies; (3) *Statistics*—methods of collecting and arranging data, tests of accuracy, general system of the country's finance, funded and unfunded debt, and fiscal arrangements, taxation; (4) *Currency, Banking and Investments*—currency (metallic and paper), nature of banking, bills of exchange, comparative value of securities, high and low prices, fluctuations in the rate of interest; and (5) *Miscellaneous*—bookkeeping, auditing, valuation of marketable securities, and approximate calculations. Every year since the establishment of the Institute a number of candidates have presented themselves for these voluntary examinations, and several, who have passed, have distinguished themselves by the positions they have taken in life, and by their writings and contributions to the *Journal* of the Institute.

The *Assurance Magazine and Journal of the Institute of Actuaries* was commenced soon after the establishment of the Institute itself. It has now reached the ninth volume, and comprises a variety of articles of the greatest importance and public interest. The contributors to it are mostly, though not exclusively, members of the Institute. Besides original papers and those which are read at the sessional meetings, it contains foreign intelligence, relating to the vital statistics, Assurance Companies (whether for life, fire, or any other branch), Friendly Societies, &c., of all foreign countries; reviews of new works, reports of Companies, correspondence, and the proceedings of the Institute. It would be impossible, in the space allotted to me now, to give any idea of the many valuable original articles, either read at the sessional meetings of the Institute, or prepared expressly for the *Journal*, which are to be found in these nine volumes. Amongst the contributors of some of the most important papers, are Messrs. Jellicoe, Hardy, Hendriks, Higham, Gray, Hopf, Farren, Pinckard, Bunyon, Tucker, Lazarus, Hodge, Day, Porter, &c. Professor De Morgan has also contributed some original arithmetical papers of great value.

At the sessional meetings of the Institute, which are held monthly from November to May, some very interesting papers have been read and discussions held thereon. On all questions of great public interest it has been usual to invite gentlemen, not members of the Institute, who have either written on, or are distinguished for their knowledge of, the subject to be considered; and thus, by the opposition of views, not merely to elicit a lively debate, but to obtain, what should be the result of all discussion, the nearest possible approach to the truth. Amongst the questions of great public interest which have thus been debated, may be mentioned—"The injustice of taxing temporary annuities at the same rate as perpetual annuities." "The rates of mortality, as deduced from the actual experience of Assurance Companies, such as the Eagle, the Economic, the Life Assurance Bank of Gotha, the invalid lives in the Clerical, Medical and General;

risks of climate in America, in the Mutual Life of New York." "The sickness and mortality in Friendly Societies in Great Britain, France and Germany." "Decimal coinage." "The early history of life assurance and annuities, and the recovery by Mr. Hendriks of the lost treatise of the Grand Pensionary, De Witt, on the subject of life annuities." "The progress of the calculus of probabilities." "The true methods of valuing and the marketable values of life interests and reversionary properties." "On the rates of interest for the use of money in ancient and modern times." "On the annual reports of the Registrar-General, and how far the inordinate mortality of the country is controllable by human agency." These, and many other mathematical and professional questions, which, in connexion with life statistics, indirectly but deeply affect the public interest, show how many and varied are the questions opened up for discussion.

The Institute of Actuaries, having so many subjects in common with the Statistical Society, naturally takes the warmest interest in its labours and prosperity. It has for several years worked in harmony, and used the same apartments, and many of its members are also members of the Statistical Society, joining in the debates, aiding the operations, and contributing to the Journal of the latter Society.

It would appear, then, that, by their mental education and practical experience, the profession of actuaries is peculiarly called upon to take a warm interest in the great statistical subjects which have been examined and discussed at this and the previous Congresses. They cannot but feel it their duty and their pleasure to aid in improving the methods of collecting facts and forming statistical tables, in comparing their results in this and other countries, in analysing and correcting conflicting statements which are frequently given as facts, and in endeavouring to deduce from the vast collections which have been already made, or may hereafter be formed by the various Governments represented at this important Congress, those laws which affect the moral, the social, or the physical well-being of all classes of the community. They are ardently anxious to co-operate in a work of such great public advantage, and to assist, by every means in their power, to render practically useful the knowledge which is thus gleaned in the wide field of statistics.

CORRESPONDENCE.

ON THE SIGNIFICANCE OF THE EXPRESSION $\frac{1}{1+a}-(1-v)$.

To the Editor of the Assurance Magazine.

SIR,—It is a trite remark, that we know, or think we know, many things, while we may be very far from apprehending their full significance. I have just met with an instance of this, which appears to me to be not devoid of interest.

We have all been long familiar with the expression, first deduced by Mr. Milne, for the annual premium for assurance—namely, $\frac{1}{1+a}-(1-v)$. This, when attention is called to it, must be admitted to be a remarkable

expression; and yet there has not, so far as I know, any attempt been made to interpret it. Circumstances have led me to attend to it; and I now, in consequence, propose the following

THEOREM.

If a' be the present value of an annuity due of £1, payable during the continuance of any status, then will the annual premium, π, for a benefit of £1, receivable at the end of the year in which that status fails, be determinable by the equation—

$$\pi = \frac{1}{a'} - (1 - v).$$

For, if the arrangement were that the £1 benefit should be receivable *now*, instead of at the end of the year in which the status fails, the equitable premium would obviously be $\frac{1}{a'}$, that is, the annuity due which that £1 would purchase; but inasmuch as the £1 is not receivable now, each payment of the above premium made while the £1 is withheld will have to be diminished by the *discount* (not the *interest*) on £1. Hence the foregoing equation holds.

Attention to the terms of the proposition will, of course, enable us to distinguish the cases in which it is applicable from those in which it is not.

1. It is applicable to assurances on single lives, and on joint lives, of whatsoever number the combination may consist, when the assurance is, in the one case for the whole of life, and in the other for the joint duration. This is well known; but it is not applicable in either of these cases if the assurance is only for a term, inasmuch as the status will not *necessarily* fail, and the benefit consequently will not *necessarily* become payable, during or at the end of the term.

2. It is applicable to an assurance and endowment on either a single life or a combination of any number of joint lives; for here the benefit will necessarily become payable, either during the term, by the failure of the status, or at its termination if the status continue to subsist.

To test this:—We know that the annual premium for this benefit is

$$\frac{M_x - M_{x+n} - D_x}{N_{x-1} - N_{x+n-1}},$$

which, if we substitute in it, for M_x and M_{x+n}, their values in terms of D and N, by the formula $M_x = D_x - (1-v)N_{x-1}$, reduces to

$$\frac{D_x}{N_{x-1} - N_{x+n-1}} - (1-v).$$

And this is of the desiderated form, for the first term is the annuity due, for n years, on (x), that £1 paid now will purchase.

This formula was given (first, I believe) by your correspondents Mr. Sprague and "H. A. S.," on pp. 112 and 117 of your eighth volume; and it was consideration of it, with its analogy to the whole life premium for assurance, that led me into the present inquiry.

3. The property enunciated is applicable to assurances on last survivors if the premium is payable till the benefit becomes due, but not otherwise.

4. It is not applicable to survivorship assurances, for here the benefit does not *necessarily* become due by the failure of the status.

5. It is applicable to the case of a sum payable at the end of a term, and subject to no contingency.

Thus, the sum payable in n years being £1, its present value is v^n. And the present value of an annuity of £1 for n years being $\frac{1-v^n}{r}$, anticipating all the payments a year, we have for that of the same annuity due, $\frac{1-v^n}{vr}$, or $\frac{1-v^n}{1-v}$. Now let π be the premium required—

$$\text{Hence } \frac{\pi(1-v^n)}{1-v}=v^n, \text{ and } \therefore \pi=\frac{(1-v)v^n}{1-v^n}.$$

Adding and subtracting $(1-v)$, we get—

$$\frac{(1-v)v^n}{1-v^n}+(1-v)-(1-v)=\frac{(1-v)v^n+(1-v)(1-v^n)}{1-v^n}-(1-v)$$
$$=\frac{1-v}{1-v^n}-(1-v),$$

which verifies the property, since $\frac{1-v}{1-v^n}$ is, as above, the annuity due of n payments that £1 will purchase.

I am, Sir,
Your most obedient servant,

7, *St. Paul's Villas, Camden Town,*
Dec. 10*th*, 1861.

P. GRAY.

P.S.—Since the foregoing was written, the following additional application of the principle employed has occurred to me:—

THEOREM.

If a' denote the present value of an annuity due of £1, on the continuance of a specified status, then will the present value of £1, receivable at the end of the year in which that status shall fail, be denoted by

$$1-(1-v)a'.$$

£1, payable now, is worth, of course, £1. To defer its payment one year, the equitable consideration would be, the disbursement by the debtor of $1-v$; and, obviously, the deferring of the payment to the failure of the status would be compensated by the disbursement of $1-v$ at the commencement of each year during the subsistence of the status. Hence, the present value of all these disbursements being $(1-v)a'$, that of £1, payable at the end of the year in which the status shall fail, is

$$1-(1-v)a'.$$

We know the truth of this in regard to contingent assurances. But it is also true in the case in which no contingency is involved, that is, when the status is simply a term of years, n. For, in this case, $a'=\frac{1-v^n}{1-v}$. Hence, by substitution—

$$1-(1-v)a'=(1-v)\frac{1-v^n}{1-v}=1-(1-v^n)=v^n.$$

P. G.

ON A METHOD OF ESTIMATING THE INCREASE OF RATE PUT ON ENDOWMENT ASSURANCES TO MEET DETERIORATION.

To the Editor of the Assurance Magazine.

SIR,—Tables of premiums for assurances payable at specific ages (50, 60, &c.), or at death, if before, are now contained in the prospectuses of most Companies, and I doubt not the transactions arising from them are proportionally numerous. In the use of such tables it will often happen that, from the arbitrary increase of rate, founded on adverse medical reports or other causes, it is not obvious how many years are thereby assumed to be added to the age of the assured; and I would submit to you the following simple method of arriving at this by inspection.

Since, for an assurance and endowment payable in n years, to be paid for in n annual premiums,

$$\pi_x = \frac{1}{1+a_{\overline{n-1|}}} + v - 1,$$

then

$$\frac{1}{1+\pi_x - v} = 1 + a_{\overline{n-1|}};$$

and, from the form in which Mr. Sang has tabulated his Short Annuity Values, we have but to substitute, in the last equation, for π_x, the increased premium per £1 (diminished by its proportion for Office commission), and look for the value in the "short annuity" column of the nth page of Sang—the age corresponding thereto, or to the annuity value nearest to it, being the advanced age wanted.

For an example, take a case of an Assurance Office which charges the Carlisle premium at 3 per cent., increased by 10 per cent. for management; the benefit, an assurance and endowment payable at death or on the assured (presently aged 27) attaining the age of 60. Therefore, $n=33$; and suppose that 10*s.* has been added to the Office premium per £100, π, the net premium per £1, is, therefore,

$$\frac{1}{110}(£2.\ 14s.\ 11d. + 10s.) = \cdot 0295075,$$

and

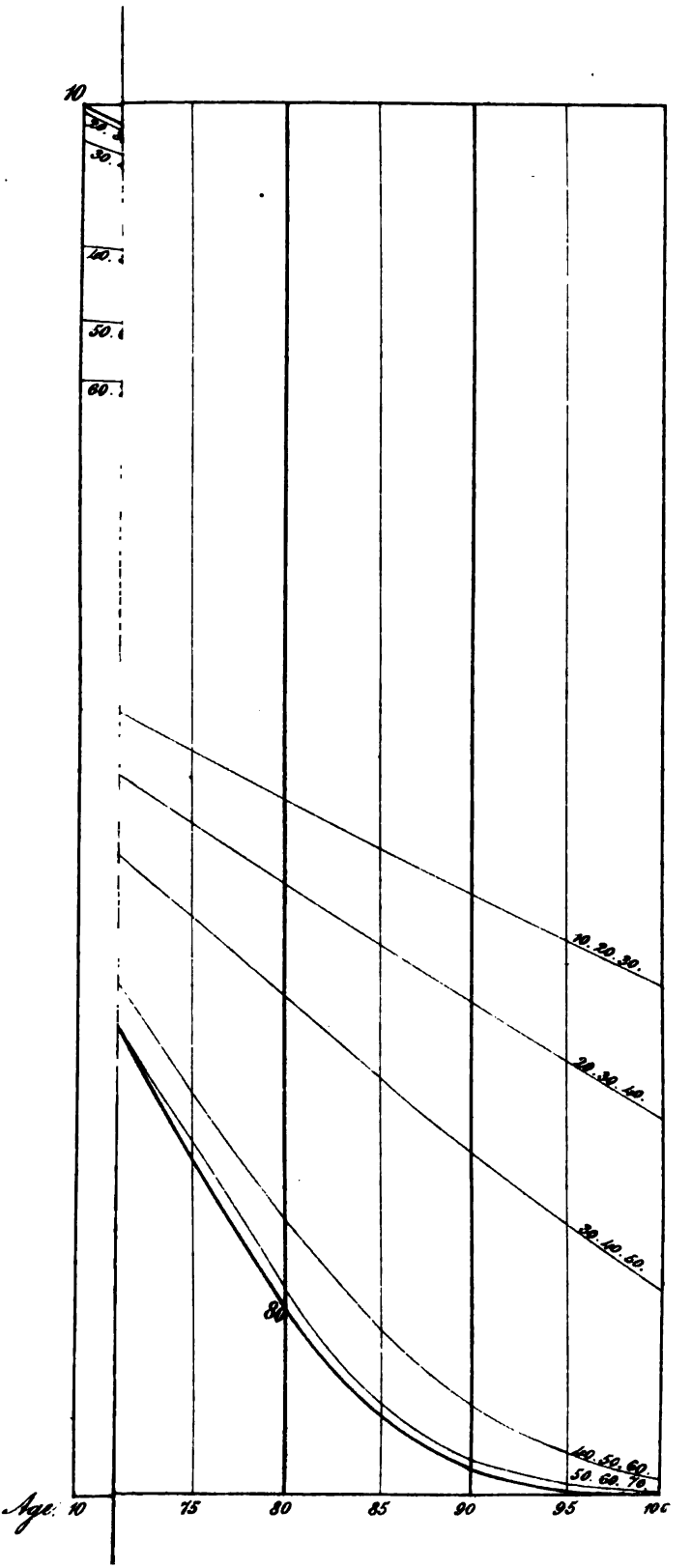
$$\frac{1}{1+\pi-v} = 17\cdot 055.$$

On the thirty-third page of *Sang's Tables*, we find the nearest "short annuity" value to be £17·097, being the amount opposite age 40; and the premium charged by the Office is, therefore, slightly over the premium for assurance and endowment payable in 33 years on a life of the age of 40.

I am, Sir,
Your most obedient servant,

Aberdeen, 16*th December*, 1861. H. AMBROSE SMITH.

P.S.—A wish to compare the results of the approximation for the values of annuities on three joint lives, given in your last Number, with the table in *Jones*, vol. ii., p. 1087, has led me to the discovery that the latter is based on the rate of *five* per cent., and not on *three* per cent., as stated. Perhaps it may be worth your making this known, for the benefit of others desirous of putting the table to a like use. I have re-computed it, and find it correct. H. A. S.


10
20.
30.
40.
50.
60.
10.20.30.
20.30.40.
30.40.50.
40.50.60.
50.60.70.
80
Age:
10
75
80
85
90
95
100

THE

ASSURANCE MAGAZINE,

AND

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

Observations on Gompertz's Law of Mortality and the Dependence between it and Simpson's Rule for finding the Value of an Annuity on Three Lives. By W. S. B. Woolhouse, F.R.A.S., F.I.A., F.S.S., &c.

[Read before the Institute, 30th December, 1861, and printed by order of the Council.]

In the correspondence that has recently taken place respecting the origin of Gompertz's formula expressive of the law of mortality, it has been stated, much to my surprise, that the investigation first given by that gentleman in the *Philosophical Transactions of the Royal Society* for the year 1825 was defective and inaccurate. I do not here propose to reopen the controversy on the subject of the originality of discovery, or to enter upon a renewed discussion of certain other allegations that have been so efficiently disposed of by Professor De Morgan and Mr. Sprague; but, after the statement to which I have more particularly alluded, it is only right that I should avail myself of the present opportunity to communicate the circumstance that Mr. Gompertz kindly presented me with a copy of his valuable Memoir in January, 1839,* in which the various misprints had, in the most legible manner, been previously corrected by himself, and that the processes so corrected are perfectly accurate in every respect. In order that any one may be

* Curiously enough, I wrote the date in the volume at the time of receiving it.

enabled to judge of this for himself, I shall here give a faithful transcription of that portion of the Memoir which contains the mathematical investigation of the formula, as it will occupy but little space.

"If L_x be the number living at the age x; we shall have $aL_x \times q^x\dot{x}$ for the fluxion of the number of deaths $= -(L_x)^{\cdot}$; $\therefore a\dot{x}q^x = -\frac{\dot{L}_x}{L_x}$, $\therefore aq^x = -\text{hyp. log } q \times \text{hyp. log } \frac{L_x}{d}$, and putting the common logarithm of $\frac{1}{q} \times$ square of the hyperbolic logarithm of $10 = \frac{a}{c}$, we have $c.q^x =$ common logarithm of $\frac{L_x}{d}$; d being a constant quantity, and therefore L_x or the number of persons living at the age $x = d(g)^{q^x}$; g being put for the number whose common logarithm is c."

Here there is not the slightest inaccuracy, and the investigation and resulting formula are as complete as could be desired. But in stating thus much, I should, at the same time, carefully bear in mind, that, to many members of the Institute, such confirmatory evidence will not be needed, as it would be preposterous to suppose that Mr. Gompertz, one of the greatest mathematicians of Europe, could in reality bungle in working out the integration of the most elementary form of exponential. The correctness of the final result is indeed a sufficient indication that the intermediate operations, which presented no difficulty, had, by so experienced a hand, been accurately performed.

Mr. Gompertz, who possesses the conscientiousness of a great mind, did not announce his formula as an absolute discovery of the true law of mortality, but, by the computation of various examples from recognised tables, he clearly established for it the important merit of representing, with a remarkable degree of accuracy, the tabular mortality throughout long periods of years; and, for the attainment of this desirable object, it is not likely to be superseded by any other theory equally free from complication.

As the letter d is commonly used as the prefix of a differential, let k be put in place of it, and the formula expressing the number of persons surviving at any age x will be

$$L_x = k(g)^{q^x}.$$

Passing into common logarithms and differences, at uniform intervals of t years, it gives

$$\log L_x = \log k + q^x \log g$$
$$\Delta \log L_x = (q^x - q^{x+t}) \log g = q^x(1-q^t) \log g$$
$$\log(\Delta \log L_x) = x \log q + \log(1-q^t) + \log^2 g$$
$$\Delta \log(\Delta \log L_x) \text{ or } (\Delta \log)^2 L_x = t \log q.$$

Therefore, if the logarithms of the numbers living at equal intervals of age be differenced, the differences will constitute a series in geometrical progression;* and if the logarithms of these last be taken and differenced, the differences so obtained will be constant.

For practical use, these relations are most convenient when exhibited according to the following table:—

Age.	(1) = log L.	(2) = Δ(1).	(3) = log (2).	Δ(3).
x	$\log k + q^x \log g$	$q^x(1-q^t)\log g$	$x \log q + \log(1-q^t) + \log^2 g$	$t \log q$
$x+t$	$\log k + q^{x+t} \log g$	$q^{x+t}(1-q^t)\log g$	$(x+t)\log q + \log(1-q^t) + \log^2 g$	&c.
$x+2t$	$\log k + q^{x+2t} \log g$	&c.	&c.	
&c.	&c.			

If it should be required to construct a table from given values of the constants k, g, q, it will only be requisite to form the columns (1), (2), (3), in a retrogade order, the numbers in column (3) having the constant difference $t \log q$.

As the formula contains three arbitrary constants, it will follow that if the calculated mortality be laid down in a curve, it may be made to pass through any three assigned points, and that the assumption of three such points will be sufficient to determine the values of the constants. But in a curve of mortality that does not conform with Gompertz's law, the computed values of the constants will vary with the position in which the three points are taken; and if the points be assumed to be indefinitely near to each other, the curves will osculate in their vicinity. Thus, in general, any curve of mortality may be considered as accurately represented by Gompertz's formula, if the symbols k, g, q, instead of being constants, are supposed to denote functions of the age x, and to appear as variable parameters that usually undergo but gradual changes in value. As a practical example of these changes, I have calculated the following table from the numbers living according to the Experience of Seventeen Life Offices:—

* Mr. Sprague, in the October Number of the *Journal*, p. 43, states that "the chance of living a year ($=g^{(p-1)p^x}$) decreases in geometrical progression." This is either an inadvertence or a misprint, into which it is immaterial to inquire, as there can be no doubt that the logarithm of the chance of living a year was meant to be expressed. The formula given in the parenthesis is a superposed exponential.

Calculation from Experience Table, ($t=10$).

Age x.	$\log L = \log k + q^x \log g$.	$\Delta \log L = q^x(1-q^t)\log g$.	$\log. = \log(1-q^t) + \log^2 g + x \log q$.	$\Delta = t \log q$.	$\log k$.	$\log g$.	$\log q$.
10	5·0000000						
		0·0302673	8·4809737				
20	4·9697327			0·0474571	5·26212	−0·23498	0·00475
		0·0337622	8·5284308				
30	4·9359705			0·0763912	5·14528	0·12349	0·00764
		0·0402552	8·6048220				
40	4·8957153			0·1245396	5·05718	0·05128	0·01245
		0·0536243	8·7293616				
50	4·8420910			0·2442852	4·96673	0·00749	0·02443
		0·0941124	8·9736468				
60	4·7479786			0·3133640	4·93108	0·00241	0·03134
		0·1936470	9·2870108				
70	4·5543316			0·3472716	4·90610	0·00130	0·03473
		0·4308066	9·6342824				
80	4·1235250			0·3671398	4·57479	−0·00006	0·03671
		1·0032802	0·0014222				
90	3·1202448						

In this table it will be perceived that when Gompertz's curve is made to pass through the points indicated by the ages 10, 20, 30, the constants will be $\log k = 5{\cdot}26212$, $\log g = -0{\cdot}23498 = 9{\cdot}76502$, $\log q = 0{\cdot}00475$; when it is made to pass through the points indicated by the ages 20, 30, 40, the constants are $\log k = 5{\cdot}14528$, $\log g = -0{\cdot}12349$, $\log q = 0{\cdot}00764$, &c. &c.; and in each case the values may be considered as appertaining to the middle age.

Mr. Edmonds, adopting Gompertz's formula as a basis of construction, has, with considerable skill and labour, succeeded in accommodating it to the usual tables of mortality by dividing human life into three periods—viz., infancy, manhood, and old age, over each of which he calculates separately from distinct data. His curve, which is partly theoretical and partly empirical, has, therefore, as it were, two intermediate stations, from which it proceeds on new branches depending on new constants. This abruptness of transition is certainly objectionable; but, although the marked discontinuity at these points does no little violence to abstract notions of mathematical accuracy and consistency, it must be conceded that the results are sufficiently near to the truth for ordinary business purposes.

I shall conclude my observations on this part of the subject by an example in full of the calculation of the curve represented by Gompertz's formula when it is made to pass through three given points; and it will be seen that this can readily be accomplished without the labour of computing the values of the constants that enter into the equation of the curve. The best method of interpolating a table of mortality by means of Gompertz's formula will also be thus indicated.

Let the given points correspond to the ages 20, 30, 40, of the

Experience Table; and let it be required to make the calculation for intervals of five years.

Taking the logarithms of the numbers living at the ages 20, 30, 40, for the first column, and following the precepts at the head of the table, as before described, we get—

Age.	(1) = log L.	(2) = Difference.	(3) = log (2).	Diff. = $t \log q$.
20	4·9697327			
		0·0337622	8·5284308	
30	4·9359705			0·0763912
		0·0402552	8·6048220	
40	4·8957153			

Here $t=10$; and, since the first line of column (3) represents the value of the expression $x \log q + \log(1-q^t) + \log^2 g$, it is evident that, to adapt the numbers to the required interval $t'=5$, we need only to replace the term $\log(1-q^t)$ by $\log(1-q^{t'})$, which will be most readily effected by applying the correction—

$$\log(1-q^{t'}) - \log(1-q^t),$$

and this is done as follows:—

$t \log q$	0·0763912	$\therefore t' \log q$	0·0381956
q^t	1·1923160	$q^{t'}$	1·0919320
$1-q^t$	−0·1923160	$1-q^{t'}$	−0·0919320
		$\log(1-q^{t'})$	8·9634667
		$\log(1-q^t)$	9·2840155
		Correction	9·6794512
		(3) . . .	8·5284308
		(3)′ . . .	8·2078820

This last result will now occupy the line of column (3) that comes immediately after age 20. The subsequent numbers in that column are obtained by successively adding the constant difference $t' \log q = 0{\cdot}0381956$, as above, and those which precede are found by the subtraction of the same constant difference. After completing column (3) in this manner, and inserting the given values of log L for the ages 20, 30, 40, the latter of which will afterwards serve as checks to the accuracy of what has already been done, the required table worked, or filled in, to the left, will finally appear thus:—

Gompertz's Curve through Ages 20, 30, 40.

Age.	log L.	Δ.	(3) = log Δ.
10	4·9980491		
		0·0135360	8·1314908
15	4·9845131		
		0·0147804	8·1696864
20	4·9697327		
		0·0161392	8·2078820
25	4·9535935		
		0·0176229	8·2460776
30	4·9359705		
		0·0192430	8·2842732
35	4·9167275		
		0·0210121	8·3224688
40	4·8957153		
		0·0229438	8·3606644
45	4·8727715		
		0·0250530	8·3988600
50	4·8477185		
		0·0273562	8·4370556
55	4·8203623		
		0·0298711	8·4752512
60	4·7904912		
		0·0326172	8·5134468
65	4·7578740		
		0·0356158	8·5516424
70	4·7222582		
		0·0388900	8·5898380
75	4·6833682		
		0·0424652	8·6280336
80	4·6409030		
		0·0463692	8·6662292
85	4·5945338		
		0·0506320	8·7044248
90	4·5439018		
		0·0552867	8·7426204
95	4·4886151		
		0·0603693	8·7808160
100	4·4282458		

In this way I have made calculations for other points in the curve, and, to show their characters and peculiarities, the whole of the results are briefly comprised in the following tables:—

Logarithm of the Number Living.

Age.	Gompertz's Curve passing through Three Ages.					
	10, 20, 30.	20, 30, 40.	30, 40, 50.	40, 50, 60.	50, 60, 70.	60, 70, 80.
10	5·0000000	4·9980492	4·9888748	4·9535995	4·9261123	4·9031950
15	4·9852797	4·9845132	4·9783440	4·9493325	4·9239556	4·9017688
20	4·9697327	4·9697327	4·9661896	4·9436796	4·9208619	4·8996415
25	4·9533127	4·9535935	4·9521614	4·9361908	4·9164242	4·8964686
30	4·9359705	4·9359705	4·9359705	4·9262699	4·9100586	4·8917361
35	4·9176545	4·9167275	4·9172834	4·9131269	4·9009276	4·8846773
40	4·8983099	4·8957153	4·8957153	4·8957153	4·8878297	4·8741489
45	4·8778790	4·8727715	4·8708221	4·8726488	4·8690415	4·8584453
50	4·8563008	4·8477185	4·8420910	4·8420910	4·8420910	4·8350227
55	4·8335108	4·8203623	4·8089306	4·8016086	4·8034322	4·8000869
60	4·8094410	4·7904912	4·7706578	4·7479786	4·7479786	4·7479786
65	4·7840195	4·7578740	4·7264845	4·6769307	4·6684337	4·6702568
70	4·7571703	4·7222582	4·6755010	4·5828081	4·5543316	4·5543316
75	4·7288134	4·6833682	4·6166573	4·4581167	4·3906588	4·3814239
80	4·6988640	4·6409030	4·5487417	4·2929285	4·1558802	4·1235250
85	4·6672328	4·5945338	4·4703556	4·0740910	3·8191047	3·7388571
90	4·6338252	4·5439018	4·3798847	3·7841802	3·3360209	3·1651084
95	4·5985415	4·4886151	4·2754657	3·4001132	2·6430667	2·3093376
100	4·5612764	4·4282458	4·1549486	2·8913103	1·6490665	1·0329186

Number Living.

Age.	Experience Table.	Gompertz's Curve passing through Three Ages.					
	L.	10, 20, 30.	20, 30, 40.	30, 40, 50.	40, 50, 60.	50, 60, 70.	60, 70, 80.
10	100,000	100,000	99,552	97,471	89,867	84,355	80,019
15	96,636	96,667	96,497	95,136	88,988	83,937	79,757
20	93,268	93,268	93,268	92,510	87,837	83,342	79,367
25	89,835	89,808	89,866	89,570	86,336	82,494	78,790
30	86,292	86,292	86,292	86,292	84,386	81,294	77,936
35	82,581	82,728	82,552	82,658	81,870	79,603	76,679
40	78,653	79,124	78,653	78,653	78,653	77,238	74,843
45	74,435	75,488	74,606	74,271	74,585	73,968	72,185
50	69,517	71,829	70,424	69,517	69,517	69,517	68,395
55	63,469	68,157	66,125	64,407	63,330	63,596	63,108
60	55,973	64,482	61,729	58,974	55,973	55,973	55,973
65	46,754	60,816	57,263	53,270	47,526	46,605	46,801
70	35,837	57,170	52,754	47,370	38,266	35,837	35,837
75	24,100	53,557	48,236	41,367	28,716	24,584	24,067
80	13,290	49,988	43,742	35,379	19,630	14,318	13,290
85	5,417	46,476	39,313	29,536	11,860	6,593	5,481
90	1,319	43,035	34,987	23,982	6,084	2,168	1,463
95	89	39,677	30,805	18,857	2,513	440	204
100	0	36,415	26,807	14,287	779	45	11

The curves corresponding to these last are exhibited in the diagram appended to this paper.

At the earlier ages, the three points, as shown in the diagram, are nearly in a straight line and indicate but little curvature, and in these cases the curve which is made to pass through them resembles that of an ordinary logarithmic curve, asymtotic with the line of abscissas, is comparatively rectilinear, and, for the latter ages, exhibits too great a longevity. When the calculated curve coincides at later ages, and comes into nearer juxtaposition towards the end of life, it will be seen that it then falls considerably below the principal curve at the earlier ages. It may be further noticed that, during any portion of life, one or other of the curves will present a tolerable approximation to the original one for a period of about thirty years. In all the curves a contrary flexure occurs near the age of 70, which is a necessary consequence of the decrement of life attaining its greatest value and afterwards diminishing.

The few remaining observations I have to make have reference to a short and comprehensive paper by Professor De Morgan, inserted in the October Number of the *Journal*, p. 27, in which he deduces Simpson's rule from Gompertz's law of mortality. This paper is very interesting, and the perusal of it has led me to a more extended examination of the subject, which I do not hesitate to lay before the members of the Institute, as the result presents

itself in a manner so marked and decided. Professor De Morgan has, with his usual ability and elegance, proved the accuracy of Simpson's rule on the assumption of Gompertz's law. He has furthermore stated that the same conclusion might be arrived at from a yet more generally expressed law, of which that of Gompertz would appear to be only a particular case. In the following brief discussion of this point, I propose to show that the seeming generality of this last assumption, although apparently a very feasible superstructure, has in reality no foundation whatever, and that the truth of Simpson's rule, which is a necessary consequence of Gompertz's law, cannot exist on any other hypothesis. I allude specifically to the last paragraph of Professor De Morgan's paper, which is as follows:—

"It is not, of course, necessary that the progression of powers should be precisely that of Mr. Gompertz, and the following theorem may easily be demonstrated. If a_n be the chance of a life living n years, and

$$a_n=(a_1)^{p_n},$$

where p_n is not a function of a, then the annuity on any number of lives is not altered in value if, instead of any part of those lives, a single life of equivalent value be substituted."

On first reading the announcement of this theorem, it appeared to me that the function of n, denoted by p_n, instead of being arbitrary, or possessing any considerable amount of generality, was, from the nature of the subject, very much restricted in its application. For, since $a_0=1$, it is obvious that, when n is taken equal to 0 and 1 respectively, the equation must then determine $p_0=0$, $p_1=1$, thus necessarily fixing two values; and if the inquiry be pushed a little further, it will be found that the conditions imposed upon the function are such that it can indeed represent neither more nor less than Gompertz's law, as I shall now proceed to show.

For convenience of notation, let the function p_n be designated by fn; then, since $a_1=\frac{L_{a+1}}{L_a}$, and $a_n=\frac{L_{a+n}}{L_a}$, the proposed equation may be otherwise written thus:—

$$\frac{L_{a+n}}{L_a}=\left(\frac{L_{a+1}}{L_a}\right)^{fn} \quad . \quad . \quad . \quad . \quad (1)$$

Or, dividing by $\frac{L_{a+1}}{L_a}$,

$$\frac{L_{a+n}}{L_{a+1}}=\left(\frac{L_{a+1}}{L_a}\right)^{fn-1} \quad . \quad . \quad . \quad . \quad (2)$$

And as these equations are general, and the value of fn is independent of the age a and varies only with n, by putting $a+1$ for a in (1) we get

$$\frac{L_{a+1+n}}{L_{a+1}}=\left(\frac{L_{a+2}}{L_{a+1}}\right)^{fn}.$$

Also, by making $n=2$ in (2),

$$\frac{L_{a+2}}{L_{a+1}}=\left(\frac{L_{a+1}}{L_a}\right)^{f(2)-1}.$$

Therefore, by substitution,

$$\frac{L_{a+1+n}}{L_{a+1}}=\left(\frac{L_{a+1}}{L_a}\right)^{fn[f(2)-1]} \quad . \quad . \quad . \quad (3)$$

But, by putting $n+1$ for n in (2), we have

$$\frac{L_{a+n+1}}{L_{a+1}}=\left(\frac{L_{a+1}}{L_a}\right)^{f(n+1)-1} \quad . \quad . \quad . \quad (4)$$

Whence, equating the values in (3) and (4), we deduce the following relation for determining the nature of the function characterised by the symbol f, viz.:—

$$f(n+1)-1=fn\{f(2)-1\},$$
$$\text{or } f(n+1)=1+q.fn \quad . \quad . \quad . \quad . \quad (5)$$

in which $q=f(2)-1$ may be treated as a constant factor, being independent of n. The only solution this functional equation admits of is—

$$fn=\frac{q^n-1}{q-1} \quad . \quad . \quad . \quad . \quad (6)$$

This, indeed, is readily ascertained by rigid induction; for, by making, in equation (5), n successively equal to the consecutive numbers 1, 2, 3, &c., we at once find

$$\begin{aligned}
f(2)&=1+q\\
f(3)&=1+q.f(2)\\
&=1+q+q^2\\
f(4)&=1+q.f(3)\\
&=1+q+q^2+q^3\\
\&c. \qquad & \&c.\\
f(n)&=1+q.f(n-1)\\
&=1+q+q^2 \ldots . +q^{n-1}\\
&=\frac{q^n-1}{q-1}.
\end{aligned}$$

The expression originally assumed for a_n therefore becomes

$$a_n=(a_1)^{\frac{q^n-1}{q-1}}, \text{ where } q=a_2-1;$$

and may be written

$$\frac{L_{a+n}}{L_a}=\left(\frac{L_{a+1}}{L_a}\right)^{\frac{q^n-1}{q-1}} \quad . \quad . \quad . \quad . \quad (7)^*$$

* By making $\frac{L_1}{L_0}=10^{-\beta}$, the formula (7) gives

$$\frac{L_x}{L_0}=10^{-\beta\frac{q^x-1}{q-1}},$$

which is by far the most convenient form for calculations in general.

If the age a be taken at birth, or $a=0$, then

$$L_n=L_0\left(\frac{L_1}{L_0}\right)^{\frac{q^n-1}{q-1}},$$

which is equivalent to

$$L_n=k(g)^{q^n} \quad . \quad . \quad . \quad . \quad (8)$$

where $g=\left(\frac{L_1}{L_0}\right)^{\frac{1}{q-1}}$, and $k=\frac{L_0}{g}$, also $q=\frac{\Delta \log L_1}{\Delta \log L_0}$;

and thus we find the law of mortality to be of precisely the form of Gompertz.

It also follows obviously from what has been established, that Gompertz's law of mortality may be directly obtained by assuming, as a hypothesis, the accuracy of the rule by which a status of equivalent value is substituted in the calculation of a joint annuity on several lives.

I have only further to remark, that as the proposed equivalence subsists with respect to each of the several terms of the series representing the annuities, it will obviously hold good whether the annuity be for a term of years or otherwise. This consideration is by no means unimportant, for it has been shown that Gompertz's formula is capable of exhibiting the true law of mortality with sufficient accuracy for long periods, and the exact determination of the value of an annuity on three or more lives for a term of years involves a considerable amount of calculation. It may, therefore, be sometimes desirable to know under what circumstances the convenient rule of substitution can be judiciously applied, and the degree of accuracy that may be expected from it.

On the Tendency of some Systems of Distribution of Surplus to defeat the Object of Life Assurance. By JAMES TERRY, *Esq., of the Hand in Hand Insurance Society.*

[Read before the Institute, 27th January, 1862, and printed by order of the Council.]

THE system pursued by each Office in the distribution of surplus may be divided into three parts, namely—

1. The class of members amongst which surplus is distributed;
2. The actual distribution of it amongst that class; and
3. The interval which elapses between successive distributions.

It is impossible to obtain a comprehensive view of the subject without considering separately these three distinct parts; and, since it forms no part of the present purpose to estimate the relative advantages of different Offices, no attempt will be made to follow the numerous combinations or systems of distribution produced by differences in practice under each of the above heads.

There is no doubt that the methods employed in the distribution of surplus are such as to render the scale of premiums paid in many instances inequitable. Whether we look to the formation of the class amongst which surplus is distributed, or to its actual distribution, or to the interval between successive distributions, there are inequalities in each which nothing but a fortuitous balance of errors can prevent from producing such a result. That such balance seldom obtains will be evident when it is borne in mind that the general tendency of those inequalities is in the same direction. The introduction of an equal period of non-participation for all ages at starting is manifestly an advantage to those who enter at the younger ages; and it has been shown, and will be incidentally exhibited presently, that the actual distribution of surplus tends to benefit the same class, whilst the introduction of a period of several years between successive distributions must be regarded as a series of advantages to the younger entrants, similar to that conferred upon them by the equal period of non-participation at starting. So that, upon the whole, it may fairly be said, that under present systems there is, generally speaking, a pecuniary advantage to be derived from a resort to life assurance when young. This part of the subject has been already treated by several Members of the Institute and others, and it is not my intention to pursue the inquiry as to the equality or inequality of the scale of premiums paid. Any discrepancy under this head may, for the future, be avoided by an alteration in that scale. But there is another inequality in some of the present systems which is essential to them, and which no scale of fixed premiums can remove. However accurately the interest of different members may be adjusted at starting, there is an inequality which develops itself afterwards, and which is the more objectionable, since its tendency is to defeat life assurance by the introduction of the principle of a tontine to an extent that cannot fail to impair the usefulness of many important institutions.

1. *The class of members amongst which surplus is distributed.*

Notwithstanding that membership in a Mutual Office itself creates the right of participation in surplus, the recognition of this right is invariably withheld, either until a certain number of premiums have been paid, or else until those premiums at interest amount to the sum originally assured. The effect of this will be easily seen. If death occur during the period of non-participation, the whole of the surplus belonging to the assured is forfeited; the Office is thus, so far as that amount is concerned, converted into a lottery, in which the survivors of a certain period after admission take all, and those who happen to die in the interval lose all. That portion of the premium which is required to provide the sum assured is applied to carry out the principle of life assurance, and the remainder to destroy it. Of course, the greater the contribution to surplus, or, in other words, the higher the scale of premiums paid and the longer the period of non-participation, the more injurious will be the effect; and in an Office in which participation is withheld until the premiums at interest amount to the sum originally assured, it will be seen how great must be the displacement of life assurance by the introduction of a tontine in the formation of the class amongst which surplus is distributed.

2. *The actual distribution of surplus.*

The original allotment resulting from the particular method employed for the purpose, and the ultimate application of the amounts so allotted, are, of course, distinct operations. Whether the bonus upon a policy be applied to the assured in the form in which it was allotted, or whether that form be altered, is, in point of principle, immaterial. At the same time, as remarked by Mr. Pinckard, the option of receiving the allotted bonus, either in cash or in a reversion, can only be given at the time of issuing the policy. It is for the assured to determine beforehand whether he wishes to secure payment of a fixed sum at death or to lay out a certain sum in premium, otherwise the adjustment in the former case of the premium to the sum assured, or, in the latter, of the sum assured to the premium, which takes place subsequently, and which proceeds upon a total disregard of the state of health of 'the life,' could not properly be carried out without a medical examination.

The present object is to exhibit the original allotment of surplus according to the following six methods, namely:—

1. A reversion in ratio of amount originally assured and number of years since commencement.
2. A reversion in ratio of amount actually* assured and number of years since the previous valuation.
3. A reversion in ratio of total amount of premium paid.
4. Cash in ratio of total amount of premium paid, accumulated at interest.
5. Cash in ratio of value of policy.
6. Cash in ratio of excess of (4) over (5).

With the exception of the first two, all the above more or less involve the scale of premiums charged. In calculating the distribution of surplus by means of them, regard must be had to this circumstance. But there are are so many rates of premium in use, that, out of more than 100, there are not half a dozen alike. However, for the most part, the difference between them is so slight, that the following scale, which is the average of them all, will be found to be exceedingly near the greater number:—

20 . . £1 18 7	40 . . £3 4 8	60 . . £6 18 6
30 . . 2 9 2	50 . . 4 10 7	

Assuming this scale to have been employed throughout, Table I. shows the ratio of distribution between policies on lives of different ages at entry after the lapse of five years by each of those methods; and since it is necessary, for the purpose of comparison, that all the results should be exhibited in the same form, and since surplus is always ascertained in cash, the reversionary bonuses produced by the first three are rendered in their equivalent present values.

That these ratios, exhibited after the first five years, are not materially affected by lapse of time, but will nearly obtain at each succeeding distribution, may be inferred from Table II. showing as before, except that 25 years from admission, instead of five only, are supposed to have elapsed.†

Having exhibited the ratios of distribution so far as they are affected by the ages of the lives only, it becomes necessary to determine them so far as they are affected by the ages of the policies only, because, in point of fact, as will presently appear, the division of surplus is influenced by both these circumstances, and by the latter much more than by the former. If the five policies

* Supposing no bonuses to have been surrendered.

† Thus it will be seen that, as between policies on lives of different ages at entry, but of the same length of standing, the remark of Mr. Jellicoe (*Assurance Magazine*, vol. iii., p. 191), with reference to No. 6 method, that "at each septennial division the same degree, or very nearly the same degree, of injustice is perpetrated as at the first," confirmed, and, indeed, may be extended to all the other methods here spoken of.

in Table I. be supposed to constitute a class, and a distribution to take place among eight such classes, differing from each other only in length of standing—thus, one having been assured 5, another 10, another 15, and so on up to 40 years—Table III. will show the ratio of such distribution by each of the foregoing methods.

The original allotments of surplus resulting from the employment of the foregoing methods having been exhibited, it remains to compare them with those which would have attended a distribution upon recognized principles. It has, indeed, been contended that all appropriation during the currency of a policy is wrong, but practically the correctness of this argument has certainly not been admitted. It is true that the surplus upon a policy depends upon the rates of mortality and interest, and other circumstances, which may prevail throughout its entire continuance, and cannot, therefore, be exactly determined until the policy itself has ceased; but does it follow, therefore, that until then all distribution is wrong? The cost of assuring a sum to be paid at death cannot be ascertained beforehand, but, since the uncertainty as to what that cost will amount to may, for all practical purposes, be regarded as existing only between comparatively narrow limits, Offices have always undertaken the assurance of a particular sum in consideration of the receipt of a fixed premium; and this has been accomplished by stipulating for such an amount of premium as may be regarded as safe to cover the risk incurred, notwithstanding its uncertainty. Without thus relying upon probability, and acting upon a certain degree of security, it is difficult to see how mutual assurance could be carried out; and it is demonstrable that an annual division of surplus upon a policy may be made without lessening that degree of security. For instance, suppose an Office to undertake assurances upon a deposit of 1 per cent. per annum upon the sum assured, in addition to the probable premium, and, at the end of the first year, it be found that the deposit for that year has not been required, where is the error in refunding it, or in remitting the next and every future year's stipulated deposit so long as such an amount remains? If the risk in respect of which a deposit was required for the first year does not materially alter in subsequent years, why should the reserve to meet it be doubled for the second year, trebled for the third, and so on?

Although the *amount* of surplus existing in an Office at the time of making a valuation depends upon the rates of mortality and interest, and other circumstances, which have prevailed in it, yet the *ratio* in which that surplus has been contributed by the

different members, and therefore the ratio in which it should be distributed among them, is clearly independent of those rates, and is determined by the difference between the probable premium and that actually paid at each age.* Employing the Combined Experience class mortality, 3½ per cent., as calculated by Mr. Higham, with a constant addition to the net premium of 5*s.* for expenses, the following exhibits the probable premium for assuring £100:—

20 . . 1·695	40 . . 2·751	60 . . 5·766
30 . . 2·132	50 . . 3·881	

and if the difference between these amounts and the premium actually paid respectively be taken and adjusted to the scale employed in Table I., it will exhibit the ratio in which surplus should be distributed, as shown in column 7 in that table; and this ratio being evidently constant, will apply equally after the lapse of 25 years, as in Table II., column 7, or after any other number of years.

Applying the same test to Table III., it is evident that, as the amount of its contribution to the surplus, accrued since the previous valuation, is not at all affected by the number of years during

* Of course this is true only upon the supposition here made, that the policies are for an equal amount and have been an equal number of years in force since the previous valuation. The share of each contributor being represented by the well-known formula $\phi a^n . \frac{S}{f.\phi a^n}$, it is evident that the ratio between the amounts so represented will not be affected by dividing the above expression by what, under such circumstances, will be the constant quantity $a^n . \frac{S}{f.\phi a^n}$.

Referring to this method, an able writer has remarked (see paper by Mr. Sprague, in *Assurance Magazine*, vol. vii., "On certain methods of dividing the surplus among the assured in a Life Assurance Company, and on the rates of premium that should be charged to render them equitable"), that, "although it is as equitable as any that could be devised, there seems to be one point in its working which is open to some objection—I mean, that, since the successive cash bonuses on each policy will be of about the same magnitude, supposing the state of the Company to remain the same, it follows that the successive reversionary bonuses will form a decreasing series, on account of the increasing age of the life assured. This, of course, is no objection to the theory of the method; but I think it will be admitted, that the public at large, who understand nothing of the theory of life assurance, expect that each reversionary bonus on their policies should be at least as large as the preceding one; and it will probably be thought an advantage if a method of division could be adopted which, while equitable to all parties and simple in its details, should at the same time satisfy the condition mentioned above." The writer then proceeds to indicate two means by which these objects may be, to some extent, attained. The first is, "to distribute only a portion of the surplus that appears as the result of the valuation, and reserve the remainder to accumulate and to be divided at a future time among the survivors of those who have contributed towards it." The second, and, as added, the more practicable one, is to regard the loading as the annual premium for an increasing assurance. In both these cases, however, the effect would be to give an advantage to those who live long, at the expense of those who die soon; and, however desirable it may be to satisfy public expectation, it does not seem to me to be possible for an Office to adopt a system calculated to yield reversionary bonuses of increasing amounts upon a policy, without some departure from the fundamental object of the Association.

which a policy may have been in existence before that period, so the distribution of it should be made regardless of this circumstance, and the amount allotted to each of the classes in Table III. should be the same, whether such class has been assured 10, 20, 30, 40, or any greater number of years, as shown in column 7 in that table.

It is true that, if the surplus in an Office be applied to increase the amount assured upon the same data as those employed in determining the premium, the assured will be entitled to participate in respect of such increase at subsequent divisions. On the other hand, if the same application be made upon "true" or probable data, this will not be the case; but here, the sum assured being increased without any corresponding addition being made to the "loading" or "marginal guarantee," the effect will be to gradually lessen that degree of security under which the Office is acting.

The tabular results given, so far as they involve the scale of premiums paid, would, of course, be affected by an alteration in that scale; but this circumstance would operate less strongly with reference to Table III. Altogether, perhaps, the following conclusions upon the foregoing six methods are warrantable:—

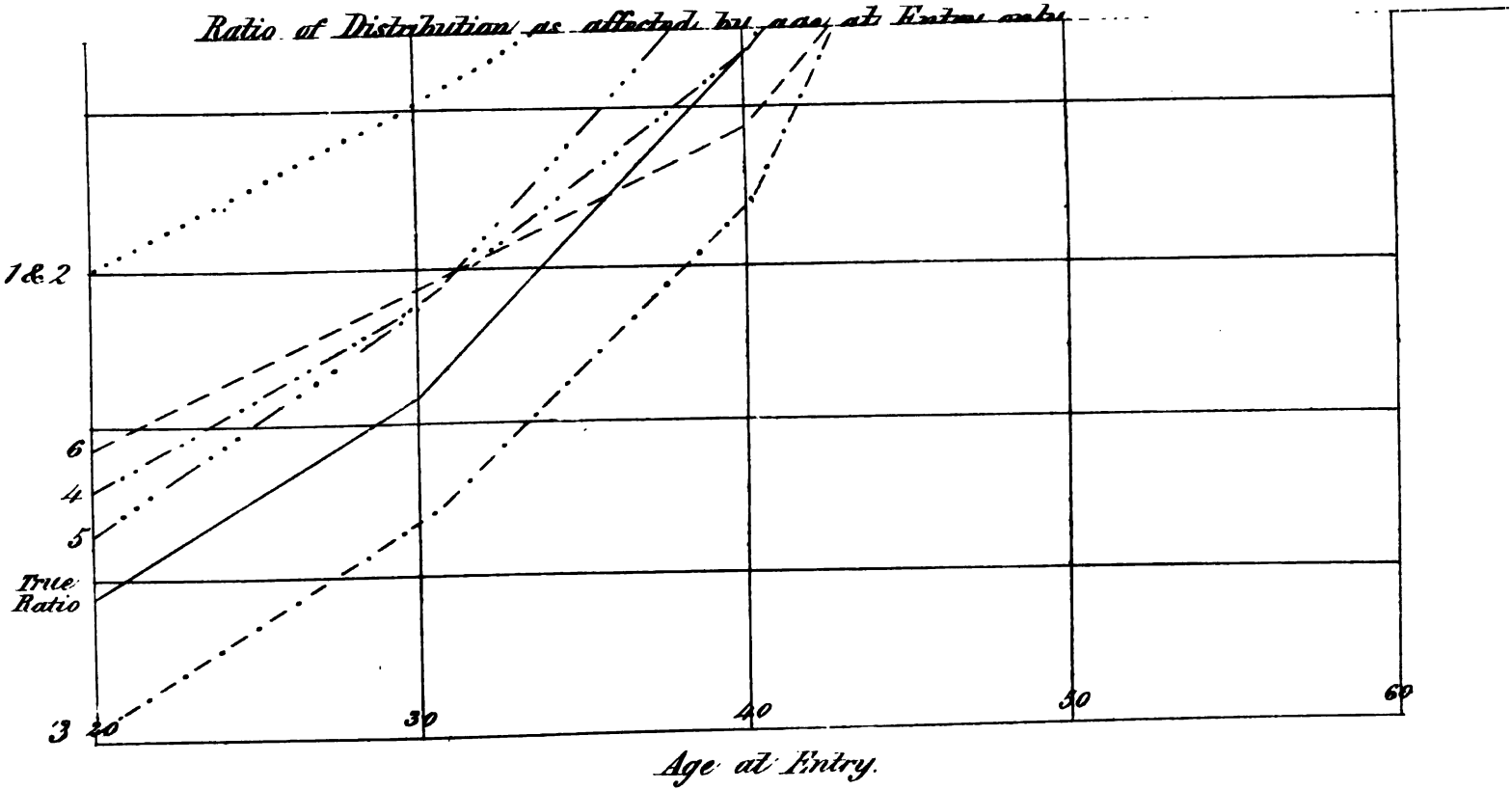
1. As regards the ages at entry of the lives—That there is a general tendency in favour of young ones.
2. As regards the ages of the policies—That there is a general and very powerful tendency in favour of old ones.

When these favourable tendencies are combined, as in the case of an old policy originally effected upon a young life, it will be seen to what an extent the displacement of life assurance must be carried out in the actual distribution of surplus. For instance, Table III. shows that an Office employing the first method would allot to a class that had been assured 40 years twice as much as it would be entitled to (£254, instead of £125 only, out of each £1,000 distributed); and it may be gathered from Tables I. and II., that the amount so allotted would be divided between the policies composing that class in such a manner that a policy originally effected upon a life of 20 would receive five times its proper amount. Had Table III. been calculated for more than 40 years—and by the mortality assumed it might have been continued for the youngest lives for 40 years longer—the results would have been much more unfavourable.

3. *The interval which elapses between successive distributions.*

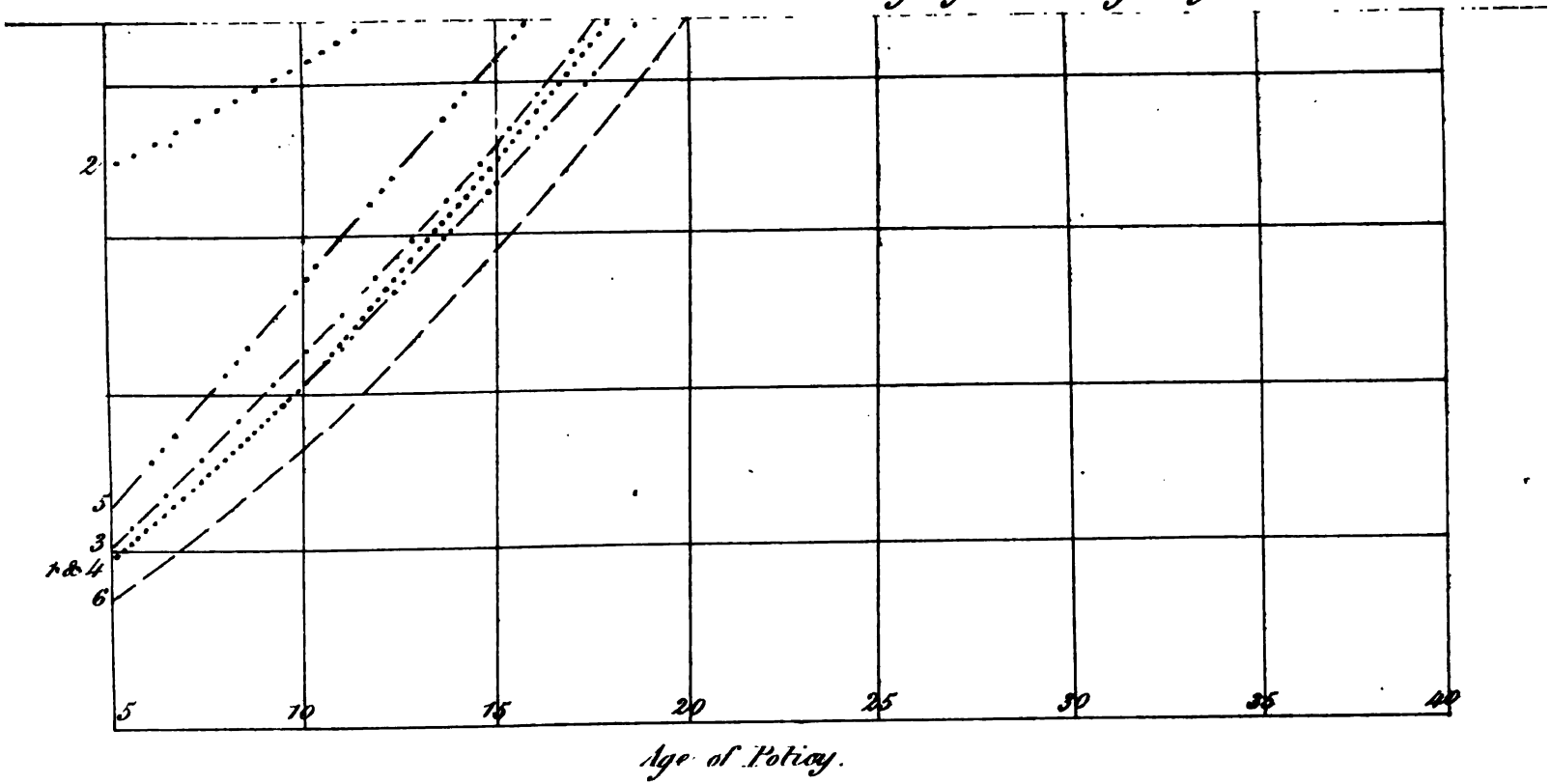
All deferment in the distribution of surplus—involving, as it does, the absorption of a larger amount of capital in carrying on

No 1.


Ratio of Distribution as affected by age at Entry only
1 & 2
6
4
5
True Ratio
3
20
30
40
50
60
Age at Entry.

No. 2.

Ratio of Distribution as affected by age of Policy only.



the business of life assurance than is necessary—is inexpedient. This objection may be urged most strongly against deferring all distribution until a policy has ceased, and attaches, though with less force, to the practice of introducing intervals of 3, 5, 7, and 10 years between successive distributions. Even in its present limited operation, the practice alluded to must alone have encumbered life assurance with a very large amount of useless capital. But its effect is not simply to cause the allotment of surplus to be deferred. The amount accrued during the interval after the first distribution is divided among the survivors of that interval only, and is lost to the assured if the life drops. Those who survive take all, and those who happen to die lose all. This is repeated at every subsequent interval, and thus the same opposition to the principle of life assurance which was created by the introduction of a period of non-participation at starting, is effectually maintained throughout the existence of the policy.

An annual distribution is free from both these objections. It avoids the imposition of a burden which has not only impeded the progress of life assurance, but occasioned considerable abuse of it. It is the obvious and legitimate period which should separate successive divisions.

Although not necessarily involving an annual valuation, some practical objection to an annual distribution may, perhaps, be urged, from a consideration of the labour it would entail, but there is no doubt that the subject admits of considerable simplification. So long as the ratio of distribution coincides with the ratio of contribution, it is not material which determines the other, and there seems to be no reason why the latter should not be made conformable to a convenient mode of distribution. For instance, if the amount of contribution to surplus were the same at each age, one single operation would determine the percentage of allotment, after which the present value of the bonus to be assigned to each policy would be little more than a matter of mere inspection.

However attractive may be its advantages when a resort to life assurance is first made, there is no doubt that this attraction loses much of its force with the assured when by and bye he ceases to be beneficially within its operation. So long as there is a chance of the premature failure of the life, its claims are not likely to be forgotten, but when the continued existence of that life has denied to the assured all pecuniary advantage, and committed him to an indefinite pecuniary sacrifice, he is apt to regard the matter only

in the light of a bad bargain, to be made the best of. In a Proprietary Company the existence of such a feeling, however unprincipled it may be, is not attended with any dangerous consequences, for the assured have no opportunity for its gratification; and where the principle demands that a loss should be sustained, escape from it is rendered impossible. But in a Mutual Society the case is different. The existence of a fund of which the assured are themselves the distributors, the absence of a knowledge on their part of the principles by which they should be governed as such, to which may be added the plausibility of the arguments by which its distribution in favour of the older members is maintained, offer a temptation to the latter to lessen their liability, which has already, on more than one occasion, to some extent been found irresistible. Hence the importance of offering the strongest opposition to the growth of such a feeling in a Mutual Office. How far it prevails in them at the present time it may be difficult to say, but its existence is clearly to be traced in, and it is to be feared that it derives some encouragement from the foregoing methods employed in, the distribution of surplus.

Having considered the three parts which together may be regarded as constituting an entire system of distribution as at present carried out, and having shown that there is a tendency in each to defeat the purpose it is employed to subserve, it may be sufficient to refer to actual results, in order to gather some idea of their *combined* effects.

The following is based upon published statements:—

(1) Sum payable at Death for every £100 originally assured.	(2) Age of the Policy.	(3) Youngest Age at Entry at which Total Premium paid, accumulated at 4 per Cent., would have amounted to the Sum in Column 1.
	Years.	Years.
654·2	65	19
588·2	61	23
555·7	59	24
527·2	57	26
498·7	55	28
470·2	53	29
441·7	51	30
415·7	49	32
392·2	47	33
368·7	45	35

Thus it appears that cases must have occurred in which the sum payable at death has exceeded the accumulated premiums and

4 per cent. compound interest, notwithstanding that the life had far outlived its average after lifetime when assured. It must be admitted that these results have been produced under somewhat exceptional circumstances; but, after making due allowance for such, they yet afford a strong illustration of the extent to which its original intention may and, indeed, must be defeated in an Office which allots to its policies periodical bonuses of continually and rapidly increasing value.

It is, perhaps, premature to expect to find evidence equally strong of the injurious systems pursued elsewhere, because sufficient time can hardly have elapsed to allow of the existence of many very old policies under them—an impediment which is being daily removed; but already, in several instances, the sum payable at death has attained a magnitude equal to nearly 300 per cent. upon the amount originally assured—and this, too, even in cases in which a portion of the profits is handed over to proprietors.

The following may be taken as specimens of the values of *periodical* bonuses allotted at recent distributions for every £1,000 originally assured:—

(1) Value of Bonus allotted.*	(2) Premiums in respect of which Bonus was allotted, accumulated at 3½ per Cent.	(3) Excess of (1) over (2).
948·8	264·5	684·3
883·1	264·5	618·6
328·3	264·5	63·8
293·0	273·4	19·6
277·5	252·9	24·6
266·2	229·7	36·5
264·7	210·3	54·4
246·7	201·6	45·1
225·3	180·1	45·2
169·4	114·0	55·4
160·2	169·4	– 9·2

It is impossible that the bonuses assigned to the above policies can fairly have accrued to them during the interval since the previous valuation. No; the explanation is—that the systems pursued are such that the bonuses apportioned to a policy during its earlier years are inadequate—that this inadequacy is compensated for by the excess of the subsequent ones—and that the latter are rendered still more excessive from the fact that, in many instances, a policy

* Mr. Higham's "Mixed Mortality Lives from among whom the effect of selection has been exhausted." Interest 3½ per cent.

drops before such compensation is made. So that whether or not the assured receives the amount of bonus to which he is entitled, in a great measure depends upon the number of years his policy remains in force; and it is this inequality which is essential to many of the systems generally pursued, and will remain, although the scale of premiums paid, when taken in connection with the benefits to which they entitle the assured, is altogether free from objection.

Enough has been said to show that the whole system of distribution of surplus is strongly pervaded with the pernicious properties of a tontine. The fact is not attempted to be disguised; on the contrary, in the prospectuses issued by some Offices it is brought prominently forward as an additional inducement to the public to enter them; and extreme cases, in which the very object for which they were established has been almost if not altogether defeated, are paraded as evidence of the immense success that has attended the working of those institutions. At the present time especially, when an attempt is being made to revive this injurious form of lottery, the importance of the subject, which has been but imperfectly handled in the present paper, is such as to claim the attention of all who wish to see the principle of life assurance carried out in its integrity.

The total amount assured in this country may, perhaps, be accepted as evidence of the extent to which the public have sought to avail themselves of the advantages of that principle, but it fails as an index of the extent to which those advantages have been secured to them. And this must continue to be the case so long as the transfer from one class to another, which the payment of one portion of the premium effects, is followed by a corresponding re-transfer resulting from the application, or rather the misapplication, of the remainder.

It should always be borne in mind, that in all cases in which the duration of "the life" from the period of its selection coincides with the average duration of life, the principle of life assurance is inoperative; that its usefulness should be in proportion as that average is departed from; and that in extreme cases, either of long or short life, its importance should be most strongly illustrated. It is in these latter that the foregoing methods employed in the distribution of surplus are most defective.

In conclusion, I may refer to the two diagrams appended to this paper for further illustration of the effects of the methods exhibited in Tables I. and III.

TABLE I.—*Showing the Ratio of Distribution of Surplus between Policies on Lives at different Ages at Entry, after the Lapse of Five Years, by each of the following Methods:—Upon the Combined Experience Class Mortality, 3½ per Cent., as calculated by Mr. Higham (except Columns 4, 5, and 6, in which 3 per Cent. and a Rate of Mortality approximately yielding the Scale of Average Premiums payable are employed).*

Age when Assured.	Reversion in Ratio of			Cash in Ratio of			Ratio of Contribution to Surplus.
	Amount originally Assured, and Number of Years since commencement.	Amount actually Assured, and Number of Years since the previous Valuation.	Total Amount of Premium paid.	Premiums Accumulated at Interest.	Value of Policy.	Excess of (4) over (5).	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
20	136	136	62	101	94	107	82
30	161	161	93	129	128	130	114
40	193	193	146	170	185	157	170
50	233	233	248	237	262	216	228
60	277	277	451	363	331	390	406
	1,000	1,000	1,000	1,000	1,000	1,000	1,000

TABLE II.—*Showing the Ratio of Distribution of Surplus between Policies on Lives at different Ages at Entry, after the Lapse of 25 Years, by each of the following Methods:—Upon the Combined Experience Class Mortality, 3½ per Cent., as calculated by Mr. Higham (except Columns 4, 5, and 6, in which 3 per Cent. and a Rate of Mortality approximately yielding the Scale of Average Premiums payable are employed).*

Age when Assured.	Reversion in Ratio of			Cash in Ratio of			Ratio of Contribution to Surplus.
	Amount originally Assured, and Number of Years since commencement.	Amount actually Assured, and Number of Years since the previous Valuation.	Total Amount of Premium paid.	Premiums Accumulated at Interest.	Value of Policy.	Excess of (4) over (5).	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
20	147	147	68	101	111	95	82
30	172	172	103	129	146	120	114
40	202	202	158	170	190	159	170
50	229	229	251	237	242	235	228
60	250	250	420	363	311	391	406
	1,000	1,000	1,000	1,000	1,000	1,000	1,000

TABLE III.—*Showing the Ratio of Distribution of Surplus between Classes of Policies of different Lengths of Standing, from 5 to 40 Years, by each of the following Methods:—Upon the Combined Experience Class Mortality, 3½ per Cent., as calculated by Mr. Higham (except Columns 4, 5, and 6, in which 3 per Cent. and a Rate of Mortality approximately yielding the Scale of Average Premiums payable are employed).*

No. of Years Assured.	Reversion in Ratio of			Cash in Ratio of			Ratio of Contribution to Surplus.
	Amount originally Assured, and Number of Years since commencement.	Amount actually Assured, and Number of Years since the previous Valuation.	Total Amount of Premium payable.	Premiums Accumulated at Interest.	Value of Policy.	Excess of (4) over (5).	
	(1)	(2)*	(3)	(4)	(5)	(6)	(7)
5	19	70	20	19	27	15	125
10	42	83	44	41	55	34	125
15	69	97	71	66	83	58	125
20	100	112	101	95	113	87	125
25	134	129	134	129	142	123	125
30	171	148	171	169	169	168	125
35	211	170	209	214	196	223	125
40	254	191	250	267	215	292	125
	1,000	1,000	1,000	1,000	1,000	1,000	1,000

On the Principles on which the Funds of Life Assurance Societies should be Invested. By ARTHUR HUTCHESON BAILEY, *Esq., Actuary of the London Assurance.*

[Read before the Institute, 24th February, 1862, and printed by order of the Council.]

OF the many valuable contributions by which the transactions of this Institute have been enriched by the labours of Mr. Samuel Brown, not the least interesting is the paper on the investments of Assurance Companies, read before the Institute on the 25th January, 1858. I feel much diffidence in again reverting to this question; but as Mr. Brown, at the outset of his paper, admits that the subject is too extensive to be exhausted by a single discussion, it has occurred to me that an attempt to arrive at some general principles on which the funds of Life Assurance Societies should be invested, and to consider how such principles can best be reduced

* Supposing a permanent rate of addition of £1. 10s. per cent. per annum.

to practice, might, perhaps, be the means of exciting some useful discussion.

Of the two main elements on which all life assurance transactions depend—the rate of mortality and the rate of interest—the latter, I think, affords more scope for the exercise of judgment and skill than the former. From my own observation and experience, I incline to the belief, that, while there are probably some not inconsiderable variations in the rates of mortality experienced by different Offices, these variations are due less to what is called (not very accurately) the selection of lives—the effect of which, I think, is generally over-estimated—than to the differences of class from which the selection has to be made. However this may be, as far as the mortality is concerned, there can be no doubt that the amount of interest realised on the assets can be materially influenced by the degree of judgment and knowledge brought to bear upon the subject. Mr. Brown has given some apt illustrations of how considerably the accumulation of capital is affected by very small differences in the rate of interest.

From the nature of the business of Life Assurance Societies, their financial arrangements can be made under some material advantages, which it is important to keep clearly in view. They engage to pay fixed sums of money at periods generally long distant from the time when the contracts are entered into: in mercantile phraseology, they may be said to accept bills drawn at very long dates. Unlike Fire Insurance Companies, where the profits of years are sometimes swept away by the disasters of as many days, the probable amount of demands on their resources can be calculated, from time to time, within not very wide limits. To such an extent is this the case, that, notwithstanding all the disturbing effects that war, famine, and pestilence have produced, I doubt whether the ill success of any life assurance project has ever been truly attributable to excessive mortality. Further: Life Assurance Societies, unlike banks and commercial enterprises generally, are not exposed to sudden or unusual demands on their resources in times of panic and financial difficulty; and the object of their investments is not to obtain income for the purpose of expenditure, but to accumulate capital for the payment of claims.

From a careful consideration of these peculiarities, I think that the main principles to be observed in investing the funds of a Life Assurance Society are the following, viz.:—

1. That the first consideration should invariably be the security of the capital.

2. That the highest practicable rate of interest be obtained, but that this principle should always be subordinate to the previous one, the security of the capital.

3. That a small proportion of the total funds (the amount varying according to the circumstances of each individual case), should be held in readily convertible securities for the payment of current claims, and for such loan transactions as may be considered desirable.

4. That the remaining and much larger proportion may safely be invested in securities that are not readily convertible; and that it is desirable, according to the second principle, that it should be so invested, because such securities, being unsuited for private individuals and trustees, command a higher rate of interest in consequence.

5. That, as far as practicable, the capital should be employed to aid the life assurance business.

The first two principles may, perhaps, be looked upon as mere truisms. But it should not be forgotten that life assurance investments should not always be regulated by the same considerations as trusts for private and family purposes. In the latter, usually the most important matter is to secure a fixed, or, at all events, an undiminishing income—fluctuations in the value of the capital being less inconvenient.

If these principles generally are sound, it follows, I think, that some modes of investment which are commonly resorted to should be avoided. It may seem a bold assertion, but, nevertheless, I believe it to be true, that the English Funds are altogether unsuited for life assurance investments. For income they offer probably the best security the world has yet seen; but with us that is a secondary consideration; the capital, the security of which is our first object, is subject to very inconvenient fluctuations in value. But it will be urged, that, on account of their ready convertibility, some portion of the assets should always be invested in the Funds. This advantage I believe to be more apparent than real. In the practical working of Life Offices it will commonly be found that, in an easy condition of the money market, the funded property has a tendency to increase, although the purchases must, of course, then be made at prices above the average—because, money being plentiful, eligible securities are then scarce. In periods of pressure, on the other hand, when favourable opportunities for investment are abundant, the funded property is practically inconvertible, from the determination not to effect sales at a loss, except in cases

of absolute necessity. I believe that the Assurance Companies are now much less extensive fundholders than formerly. But there is still a prevalent idea that one half the assets should always be invested in the Funds. Upon this rule the wealthiest of the Offices still acts, and the idea has been so fostered by the large profits realised by some of the older Companies from their purchases at the beginning of this century, that it seems necessary to point out that the risk is now in the opposite direction, and that perfect security is very far from being one of the advantages of the Funds.

The same considerations apply to the Preference Stocks of Railway and other Companies, which are now very popular.

It is, I think, generally admitted that the ordinary stock and shares of trading and other Companies are not eligible for Assurance Societies' investments, as being too speculative. Yet so potent is the influence of the magic words, "Bank of England," that an exception is usually made in favour of Bank Stock, in apparent forgetfulness that the dividends of this great Corporation mainly depend upon the fluctuating and uncertain profits of a particular business—that of banking. How little the exception is justifiable by facts, a glance at the price lists of a comparatively short period will show. It will be found that, in 1840, the price of Bank Stock was marked at 156; while it is now (February, 1862), 244.

Leaving the consideration of what investments to avoid, for the more difficult task of what to select, it seems to me that, for what may be called the working capital, the smaller portion of the assets which is required to be readily convertible, the system of deposits with joint-stock banks and discount establishments affords the greatest advantages. These deposits are always available on demand, or at short notice, without expense, and without risk of depreciation like Exchequer Bills. I think that it is important that the amount to be thus held on deposit should be regulated always by the financial requirements of the Office, and never by the rate of interest. I have frequently had occasion to observe, that the effect of making transfers to permanent investments from deposit accounts, from dissatisfaction with the low rate of interest on the latter, has been to obtain a small increase in the interest at a serious cost to the capital. It will also probably be thought prudent to divide the total amount of deposits among different establishments.

According to the principles attempted to be laid down, the most desirable form of permanent investments seems to be bonds

and mortgages—including in that term every species of loan secured on tangible property; the preference being given to such as require policies as collateral security, as life interests, &c., where assurances are indispensable. I cannot think that it is altogether a justifiable proceeding to insist on assurances being effected which the security does not require, strong as may be the temptation to such a course; it would be better to stipulate at once for a higher rate of interest. Indeed, when a borrower finds the loan which he does require, subjected to a deduction for a premium on a policy of two or three times the amount, which he does not require—accompanied, as is sometimes the case, with a rather pharisaical announcement, that no higher rate of interest than 5 per cent. is ever taken—he must be reminded of the olden time, when the pecuniary necessities of gentlemen of the Charles Surface class were supplied in a currency composed of pictures, wine, and money, in very uncertain proportions.

As, however, the competition for such securities is on the increase, it is impracticable for Offices whose funds are large, to invest more than a part of their assets in this manner. In seeking for other descriptions of mortgages, our principles will lead us to select such as cannot readily be realised, avoiding those that are open to the competition of private trustees. For example, an advance under any of the land drainage Acts, in consideration of a rent charge for a term of years, will command a higher rate of interest than a mortgage in fee on the same estate, although the former charge has priority over the latter. In like manner, securities on the rates of counties and towns, and, generally, loans where the principal is repayable by instalments, are, I think, worthy of attention.

One other class of investments I will mention, which is quite in accordance with the principles laid down, and which seems to me peculiarly adapted for a portion of the funds of Life Assurance Societies—I mean the purchase of reversionary interests. A banker carries on a lucrative business by receiving money on deposit at one rate of interest and employing it at a higher rate; and I believe that a margin of 1 per cent., or less, will suffice to cover risks and expenses, and yield in addition a not inconsiderable profit. Surely, then, an Assurance Company may usefully employ, in the purchase of reversions, part of funds which are not repayable at short notice, but which are to accumulate during the lives of the assured, when it is considered that the margin is probably three times as great as suffices for the operations of the banker. I have,

therefore, never been able to understand how such institutions as Reversionary Interest Societies can have come into existence.

In concluding this hasty and imperfect sketch of a subject on which much difference of opinion exists, and on which a volume might be written, I have purposely avoided entering into details, or attempting to lay down rules by which the merits of any individual security are to be judged. I am aware, too, that other than merely financial considerations must sometimes be allowed their influence—as, for instance, it may be fairly urged that Government securities must be held to inspire confidence with the public. But, I think it will be admitted, that the finances of Life Assurance Societies ought to be managed on some clear general principles, and not, as is sometimes the case, left to depend on the passing impulses of the day, or on ideas derived from other and very different pursuits.

Memoir of the late John Finlaison, Esq., Actuary of the National Debt, Government Calculator, and President of the Institute of Actuaries.

FEW objects can be found more worthy of attention than the life of a man who has spent year after year in an unremitting and successful struggle against the ever-varying difficulties of new sciences; and who, not content with treading the beaten paths, has always sought to take a place among the pioneers of knowledge, neither deterred by the labour, nor dismayed by the risk of failure.

Mr. John Finlaison, late President of the Institute of Actuaries and Government Actuary, was eminently a man of this description. Born at Thurso, in Caithness, August, 1783, he lost his father when only seven years old. His innate energy of disposition, however, aided by the careful tutelage of his mother, procured for him that education for which Scotland affords such facilities; and he showed a keen taste for classical literature, and attained considerable local reputation as a poet. At this time his great mathematical talent was suspected by no one, least of all by himself, who regarded such studies with dislike; but the highly logical bent of his mind was shown in a faculty for systematizing, which he displayed to great advantage as factor to Sir Benjamin Dunbar, of Hempriggs, afterwards Lord Duffus, whose whole estates, with

those of Lord Caithness, were entrusted to his management at the age of 19.

The ability which he thus early displayed induced these kind friends to recommend that he should study for the Scottish bar. He accordingly went to Edinburgh, and qualified himself for that profession; but having visited London in 1804, on business, he became attached to the daughter of the Rev. James Glenn, and receiving from the then Secretary at War, the Hon. Mr. Dundas, afterwards second Lord Melville, the offer of an appointment under the Board of Naval Revision, which enabled him to marry at once, he was induced to relinquish the legal profession and enter the Government service in July, 1805, when Mr. Pitt was Prime Minister. He was shortly afterwards promoted to be first clerk to the Commission, and filled that office till the Board closed its labours in August, 1808. For some time previously he had also acted as secretary to a committee of the Board, represented by one of its members, Admiral Sir William Domett; and in that capacity, although but twenty-three years of age, he framed the eleventh and twelfth reports of the Commission—since laid before Parliament—and was the sole author of the important system therein recommended for the reform of the victualling departments. The accounts of this branch of the service had, for a century past, been so loose, imperfect, and complex, as to amount to a mere nullity; and, moreover, were seldom less than eighteen months in arrear. The effect of the new system was to produce them before the Board in London, simple, perfect, checked and audited, in *three weeks*. And this was accomplished not only without increased labour, but with a reduction of the establishment, which effected an enormous annual saving to the country. A memorial from the officers at Deptford stated, that, in the year when Mr. Finlaison's system first came into operation (the year 1809), the consequent saving from their yard alone was upwards of £60,000. Had the same system been established in the dockyards, many millions of the public money might have been saved.

On the dissolution of the Commission for the Revision of the Navy, the members of the Board expressed their great satisfaction with the ability evinced by Mr. Finlaison; and, in a letter to Mr. Wellesley Pole, then Secretary, they warmly recommended him to the notice of the Lords of the Admiralty, who were at that time much in need of the services of an able accountant. By the special advice of Sir William Domett, Mr. Finlaison was employed by Mr. Wellesley Pole to consider the state of the records and despatches

of the Admiralty; and, if possible, to devise some remedy for the inextricable confusion into which they had fallen.

Ever since the Restoration the records had accumulated without any means of reference, except from the accidental recollection of individuals; hence the production of the entire correspondence upon any one subject was almost impossible. Papers upon the most recent events, even when described and known to be in the office, could with very great difficulty be discovered. In 1805, Lord Barham had appointed an eminent barrister, Mr. Morthland, at a salary of £800 per annum, with two assistant clerks, to devise some remedy for the increasing evil; but, after eighteen months' trial, he completely failed, being deterred by the magnitude and difficulty of the object in view.

Mr. Finlaison undertook the task with great diffidence; he was young in official affairs, as in years; a stranger to the nature of the Admiralty correspondence, and unable to obtain from Mr. Wellesley Pole, who well knew the defect of the office, any idea of the way in which it was to be supplied. Under these discouraging circumstances—after a vain but most diligent search, in the public libraries and elsewhere, for any English or foreign system of assorting correspondence at all applicable to the subject—he could only promise that no personal labour should be spared to accomplish Mr. Wellesley Pole's wish. After nine months of incessant application, he produced a magnificent system of digesting and indexing the records and correspondence of the Admiralty, which, after the lapse of more than fifty years, still works with such perfection, that should information or precedent be needed on any subject under discussion, all that has been received concerning it can be immediately known; or should the name of some individual be called for, all that relates to him may be instantly produced. Under his superintendence, this system so eminently contributed to the dispatch of business, that every letter—although, in time of war, these averaged in number from two to three hundred a day—was answered on the very day of its arrival. The value of such precision and expedition in the conduct of naval affairs can hardly be appreciated by those who have not experienced the want of it. There was not, at that time, any system of official arrangement in any office of state in Europe which could be compared with this invention.

The following account of his work is extracted from a letter written by Mr. Finlaison in 1855, in answer to a request from the Admiralty that he would reconsider the working of his

system, and suggest some improvements which time had made necessary :—

"Among the manuscripts of Samuel Pepys, at Cambridge, will be found 'A Devout Imagination,' a sketch, in fact, of a return intended to show from time to time the nature and amount of the stores in hand in the several dockyards. This renowned Secretary of the Admiralty showed himself no mean adept in the art of arranging a system on the Linnæan principle, although he had died before the birth of its author. His kingdoms were—timber, the metals, fibril materials, and unctuous matter. His genera were the different kinds of each of those. His sub-genera, the raw and manufactured respectively. These last again were ramified into innumerable species and varieties, until you came to the individual article wanted—perhaps a common brass cock for a spirit cask.

"When I entered on my task at the Admiralty, I took the grand idea of Mr. Pepys for the basis of my plan. The Admiralty records were tolerably complete since the period of the Restoration; they consisted of letters and reports addressed to the Board, and of copies of letters and orders issued from it. I soon found that these papers were of three descriptions, and that it would be at once unprofitable and hopeless to attempt to digest the whole. There is, first, a vast correspondence relating purely to the interests of individuals; the discharge of impressed men; applications from officers for employment, for leave of absence, for particular information, &c., which I did not think worthy of record, beside subjects intimately connected with the public service. In the second place, a very large proportion even of papers regarding the public service are worthless as soon as answered; they consist merely of official reports and ephemeral details, such as periodical reports, reports of the arrival and sailing of ships, applications for supplies of stores, payment of ships, &c.

"The remaining portion of the records comprising the public acts of the Board is alone valuable for reference. A digest is made of these only, which form about a third of the whole correspondence; but the papers passed over can always be found, if needed, by means of the index, as will presently be seen.

"The purpose of any system of reference must be to supersede the use of individual memory—to instantly invest a new Board of Admiralty with the very same facilities for the despatch of business which had previously been at the command of their predecessors. The principle of my method is briefly this:—All information sought for must relate either to a person or a thing. The index, therefore, is an alphabetical list of proper names of persons and ships, in which the history of each name may be traced, though concerned in fifty different things; while the digest is a systematic list of words in which any subject may instantly be brought under review, though fifty persons have taken part in it. It was in the arrangement of this list that I found Mr. Pepys suggestion to be, like the poetry of Homer, a source of inspiration. One hundred and four chapters embraced all capital topics, such as firearms. I divided the chapters into sections as numerous as the different kinds of each of these, and placed in each section all discussions upon one and the same thing. The Minié rifle, for instance, ought to be found in one chapter and section of the digest, unencumbered by discussions on other sorts of gun-barrels or on firearms in general. In the one case a lord hears that something has happened—he knows not *what*

nor *when*—to a particular person or ship; he is instantly informed by means of the index. In the other case, a lord hears there are heavy complaints—he knows not *when* or *from whom*—of bad victuals. But, on reference to the digest, he is at once put in possession of the whole matter.

"In each of these books the utmost brevity is required. Immediately the letters reached me from the Board—generally the morning after they were received and acted on—I marked each, if of a memorable nature, in pencil, with the number of the chapter and section in the digest to which it belonged, as 51–1; having a sheet containing a plan of the digest as a guide. I also marked any proper name, as well as the signature, with a large cross ×. The letters then passed to the clerk of the index to be stamped, marked and numbered, for the different cabinets in which they were finally to be placed. This done, the clerk, whenever he saw a cross, without reading the letters, entered the name referred to in his index, together with the digest mark, 51–1, which referred to the subject-matter, and the numbers by which the letter itself might be found. It was estimated that in this way he made annually forty thousand entries. From him the letter went to the clerk of the digest, who opened the chapter and section referred to, and wrote therein an exact précis of the letter, and its answer already engrossed upon it by the lord whose department it concerned, together with the numbers referring to its place of deposit. If of an ephemeral nature, the letter was, of course, not marked for, and not entered in, the digest. The letters being indexed and digested, were, by the same two clerks, placed in their respective places. To a third clerk and myself fell the duty of producing any of the papers when wanted, and of compiling for the Board statements or memoirs on any topic whatsoever upon which information or precedent might be desired.

"To give the Board ocular proof, in the shape of an actual model, that there could be no doubt of the success of this plan, I took the whole correspondence of the year 1803, the first year of the war: assisted by one clerk, I made, in three months, an index of every name and ship mentioned therein. With my own hand I made a précis of all the *memorabilia* found in every single letter—and in that year these were not less than twenty-five thousand; placing experimentally in the margin some significant title. Revising these last carefully, generalisation—thus rising from analysis to synthethis—ensued, and my plan was developed and acted upon.

"A second division of clerks was appointed to digest and index, on the same principles, the records of former years since 1792, and thus the chain of naval history is complete from that period to the present time."

Baron Dupin, a member of the French Institute, gives a clear account of this system, while describing the Admiralty, in his work entitled *Voyages dans la Grande Bretagne*. He recommends it as a model for the use of his Government, which proves that it was highly esteemed in France. Louis XVIII., after his return to France, transmitted to Mr. Finlaison, in 1815, the order of the Fleur-de-lys—a gracious acknowledgment of services rendered him in connection with this subject. The invention was shown to other royal personages by Lord Melville, then First Lord of the Admiralty, who permitted them to carry away plans for the use of their

respective Governments. Among these were the Grand Dukes Constantine and Nicholas, the Archdukes John and Lewis of Austria, and the Grand Duke Michael of Russia, who visited the Admiralty at various times between the years 1816 and 1818; and to each of whom Mr. Finlaison presented an elaborate account of his method, and received testimonials of their satisfaction. He was honoured by the particular notice of the Grand Duke Nicholas, since Emperor of Russia, who sent him an autograph letter of thanks.

To further the efficient working of the system, Mr. Finlaison, at Mr. Wellesley Pole's strong recommendation, was, in 1809, appointed Keeper of the Records. A few months later, a library having been formed, consisting of about 5,000 volumes, he was made librarian, and became thus the general depositary of all the information, whether in print or manuscript, in the Admiralty. This necessarily entailed his being general referee for supplying that information to each member of the Board, to the public and private secretaries, and to every clerk in the office. Moreover, he became, in consequence, reporter and précis writer on all difficult and complicated inquiries arising from day to day. This highly confidential and important service—than which nothing can be more arduous—he fulfilled for twelve years. While he occupied this position, many members of the Board availed themselves of his freely-offered services, in matters foreign to his appointed duty and performed out of office hours. The following are a few instances in which his ability was remarkably displayed on important questions.

In 1810, Lord Mulgrave, then First Lord of the Admiralty, was called to account for his naval administration by Lord Melville, under the mask of a question on troop-ships. Lord Mulgrave, who had shown Mr. Finlaison much previous kindness, now requested him to prepare the materials for his defence in both Houses of Parliament. Mr. Finlaison accordingly, after three months of zealous and successful exertion, produced such an amount of information, calculation, and reasoning, as enabled Lord Mulgrave triumphantly to clear his conduct as a statesman, and obtained from him many expressions of gratitude and promises of protection, which, however, were not redeemed when needed in after years.

In 1811, by Mr. Barrow's desire, he compiled an account of the enemy's force, involving the analysis and comparison of more than eight hundred dispatches, as well as of many thousand foreign

and English newspapers. He formed a digest out of all this intelligence for every port and arsenal in Europe; and framed a list of every foreign ship of war, with particulars of their age, force, &c., and of the increase and loss of the enemy's naval power during the previous six years. This list experience showed to be perfectly correct, and it was even relied on in Parliament as accurate. Such information had never before been obtained with even tolerable authenticity.

In the same year Mr. Yorke employed him to investigate the abuses of the sixpenny revenue at Greenwich Hospital, which was a fund instituted in 1696, and intended to support the out-pensioners by a tax of 6*d.* a month on all seafaring men. He compiled, with great labour, a report on the subject, in which he showed that 10,000 additional pensioners might be constantly maintained on that establishment, and also that the allowance of the existing pensioners might be much increased by including the class of fishermen, boatmen, and mercantile sailors among the taxpayers, as well as by the reform of many abuses in the office and by the abolition of sinecure places.

For some years past petitions had been repeatedly sent up by the various departments of the Government, praying for a revision of their salaries. The matter having been more than twice forced upon the notice of Parliament, Mr. Finlaison was, in 1813, directed by Mr. Croker to inquire fully into the case of the Admiralty departments, and to apply systematic principles of justice to the consideration of the subject, which had never before been done. After six months of close attention, he completed the elaborate report upon which was founded the present system of salaries in the Admiralty.

In 1814 he compiled the first official Navy List, a work of immense though humble labour, the accuracy and usefulness of which is now well known. It was issued monthly, and as long as Mr. Finlaison remained at the Admiralty he undertook the troublesome duty of correcting and editing it.

From 1817 to 1818 he was occupied in framing one of the most useful records ever formed in any nation, a biographical register of every commissioned officer in the Navy—in number about 6,000—describing their services, merits and demerits. This laborious work he engrafted on to his system of the digest and index; and its value is obvious when it is considered that it affords the means of deciding, with perfect justice, on the claims of individual officers; and, in the event of any expedition

being undertaken, renders it easy to select the *élite* and flower of the Navy, or to fix at once upon those officers who, from previous local experience or other causes, would be most fitted for the service—a matter of great importance.

In 1819 his labours were still further increased by the resolution of the Lords of the Admiralty to apply his method of arranging papers to all the hydrographical information that had ever been received at the office, in manuscript, print, or charts, and to consign these last also to his custody. Deeply impressed with the importance of the measure, and fully aware of the immense benefit to hydrographical knowledge which should result from it, he undertook the task with energy and enthusiasm, though as the whole of his time was engrossed by the routine of his daily duties, the extent of geographical and nautical study which the nature of the service demanded, was entirely acquired in his few moments of leisure.

Mr. Finlaison had thus, during the ten years of his official career, accomplished no fewer than ten important and laborious public services, of which some were in their nature ephemeral, though of the utmost consequence at that time, while others—for instance, the Admiralty system and the Navy List—having stood the assaults of adverse criticism, and the more formidable tests of time and of two great wars, continue unchanged in design to the present day. He also introduced into the Record Office, and strictly enforced, a custom of civility towards the public and a readiness to impart information, which, had it been universally carried out, would effectually have prevented the "Circumlocution Office" from acquiring celebrity.

At this time the reputation which he had gained induced Mr. Barrow, and several other distinguished members of the Royal Society, to recommend that he should become a candidate for election into that body. He, however, declined to do so, from motives which had their origin in the peculiar independence of his disposition. He preferred that his name should be known to scientific men from his works, and not from his being casually thrown among them. He undertook, also, about this period, to frame a digest of the library of the Royal Society upon the plan of the Admiralty system. An outline of this plan was laid before the council and approved, but the immense call upon his time in matters of state, in 1819, compelled him reluctantly to abandon the work.

The position of librarian at the Admiralty having brought

before his notice many valuable State papers relating to the American rebellion, he was, in 1813, induced to attempt the completion of Sir Redhead Yorke's Naval History, which was intended to form part of Campbell's *Lives of the Admirals*. Mr. Finlaison carried out his design in part, by continuing the History down to the year 1780. His remarks on the political aspect of the American question show both a very wide and accurate extent of reading, and also a knowledge of the true principles of commercial and colonial policy, very unusual at the time when he wrote; at the same time his analysis of Rodney's inglorious action with De Guichen, off Martinique, on the 17th of April, 1780, shows an acquaintance with naval strategy which gained him very high commendation from Sir W. Domett and other eminent naval officers. This portion of the work was printed for private circulation, but, for many reasons, its publication was delayed; and afterwards, when his attention was turned to the subject of life assurance, he found himself unable to spare the time necessary for completing the history, and finally abandoned its further progress.

In 1815, Dr. Barry O'Meara, having been appointed physician to Napoleon at St. Helena, commenced a correspondence with Mr. Finlaison, his private friend, on the subject of the Emperor's daily life. Copies of these celebrated letters were given by him to Mr. Croker, and by him transmitted to the Government and the Prince Regent. Dr. O'Meara wrote under the positive assurance that these letters should never be suffered to become public; and, in 1824, Mr. Finlaison burned them all by his desire. Some of the copies, however, fell into other hands, and were published in 1853, in a book entitled *Napoleon at St. Helena, and Sir Hudson Lowe*. Intelligence of the proposed publication of this work having reached the ears of Lord Clarendon, he at first thought of suppressing it by an injunction; but before instituting proceedings, he requested Mr. Finlaison to read over the proof sheets and give his opinion as to the danger of permitting its issue. Having, with some difficulty, obtained the proof sheets from the publisher, Mr. Finlaison found that the author had not obtained possession of any documents of importance; and Lord Clarendon, judging that it was not a book likely to attract much attention, withdrew his opposition.

He now completed a work upon which he had been zealously employed since 1812. This was the fund for the maintenance of the widows and orphans of all who were employed in the civil departments of the Royal Navy. He first suggested it in a letter

to the heads of the departments, and at once obtained Mr. Croker's full co-operation and warmest approbation. Mr. Finlaison then first directed his attention to the principles of life assurance, a science which was little understood. It was upon this study that his future labours were to be spent, and on the investigation and discovery of its laws that his eminent reputation was to depend—a reputation that gained him honourable notice in the history of his country.*

Starting with small knowledge, he worked assiduously and ably for seven years, during which time he mastered the principles of the subject, constructed the necessary tables, matured the whole plan of the undertaking, and obtained the means of forming a Society to execute it. Through Lord Melville's intervention, his generous efforts terminated successfully, by the establishment of the fund by Order in Council, 17th September, 1819.

Many other Societies were framed on this model: among others, the Naval Medical Supplemental Fund, for providing an annual pension, of not less than £40 additional to that allowed by Government, to the widows of medical officers. This charity, although commenced subsequently to the preceding, and in imitation of it, was completed before it, and established under Government authority in August, 1817. Mr. Finlaison took the greatest interest in this Society, which owed its existence and subsequent prosperity entirely to himself. His financial skill enabled him to construct its valuable system of accounts, which he gradually perfected by the experience of several years; and his management as secretary, gave the Society, in the twelfth year of its existence, an annual income of £9,000, with trifling expense to individual members. The unworthy and ungenerous treatment he then received from its directors, who not only refused to remunerate him for a new and elaborate system of tables, but endeavoured to injure his character in the eyes of the Chancellor of the Exchequer, by the grossest falsehoods, compelled him to throw up the ungrateful office of secretary and manager, after he had, during twelve years, devoted much of his time and talents, almost gratuitously, to the service of the Society. In a letter written at the time of his resignation, he speaks thus of the stability of his scheme:—

"Many similar Societies have been dissolved through their unskilful construction, while this institution, to which I can triumphantly refer as the

* *Vide* Lord Macaulay's *History*, vol. i., p. 283.

only one in the empire which has its foundation deeply laid in scientific skill, is established on such principles of cohesion, the result of long and patient investigation, as that neither time nor accident, except the suicidal act of its own rulers, can in the slightest degree affect its efficacy."

It is worth recording that this "suicidal act" did take place; and that, owing to the mismanagement of its directors, the Society gradually sank, till, singular to say, it died in the same year with its founder and benefactor.

The success of the charity thus founded by Mr. Finlaison, together with his subsequent services in an investigation into the general condition of Friendly Societies, upon which he was employed by a Select Committee of the House in 1824, naturally gave rise to a private practice among the various Benefit Societies which speedily sprang up in all parts of England. He constructed tables for many of these, furnished the scheme of some, and entirely constituted others.

From the beginning of the year 1819, Mr. Finlaison, though he still retained the office and performed the duties of Keeper of the Admiralty Records, found himself required to devote all the time and skill at his command to the service of the Treasury, which then stood in great need of an able financier. During the previous year, arrangements had come under the consideration of Government for the payment of all half-pay and pensions by some means beyond the current service. Lord Melville was well aware of Mr. Finlaison's value as a trustworthy public servant, and he also knew that he had acquired considerable reputation at the Admiralty by his knowledge of the principles of finance. He, therefore, advised the Chancellor of the Exchequer (Mr. Vansittart, afterwards Lord Bexley) to entrust to Mr. Finlaison the very confidential service of maturing a scheme for meeting this expense. This was to raise a fund, by resuscitating an old corporation, so as to be in some sort a rival to the Bank of England. In the year 1808 two remarkable modifications had been introduced into the plan for the redemption of the National Debt. The first, brought in by Mr. Percival, was the great operation of granting life annuities chargeable on the Sinking Fund; the other was the savings banks system, soon afterwards commenced by Mr. Rose. Upon both these schemes Mr. Finlaison was now desired to comment, in connection with the "dead weight," by which name the half-pay and pension expenditure was familiarly known, from the accident of Lord Castlereagh so

calling it in the course of a debate. We shall say nothing at present of the report on the savings banks, which led to the commutation of the half-pay and pensions, nor of Mr. Finlaison's services in connection with this measure. They were subordinate to the work suggested by his second report, and to which, for nearly ten years, every faculty of his mind was devoted. His report on the life annuity system was laid before Mr. Vansittart on the 1st September, 1819; and the learning, ability, and extensive acquaintance with every bearing of the subject which it displayed, proved it to be the fruit of many years' previous study. In it Mr. Finlaison first directed the attention of the Chancellor of the Exchequer to the ruinous loss sustained by the Government in granting life annuities to the public at prices immensely below their value. This loss, which had already, in eleven years, amounted to nearly two millions, began from the circumstance that the Treasury had no actuary attached to its service. In the year when the life annuity system was instituted, 1808, there was but one eminently scientific man in practice as an actuary in all England, Mr. W. Morgan, of the Equitable Life Assurance Office; and there was but one English observation on the mortality of mankind—viz., that formed on the bills of mortality in Northamptonshire by Dr. Price, the uncle of Mr. Morgan, expressly for the benefit of the Equitable. How beneficial it was had appeared from Mr. Morgan's Address of the 24th April, 1800, in which he published a statement of the experience of the Equitable during the previous thirty years, which showed that the real decrement of life under the age of thirty was *one-half* that indicated by their tables; at other and higher ages the difference was *two-thirds* and more. Their gains were accordingly enormous. It is obvious that the tables which were so gainful to the Equitable, who, in the shape of premiums, *purchased* life annuities, must be ruinous to the public revenue which was to *sell* them. Nevertheless, the Treasury, in profound ignorance of this palpable truth, applied to Mr. Morgan to furnish tables for the newly-instituted Government annuities; who, accordingly, computed them on the basis of those very Northampton tables he had just proved so inaccurate; and the Government not only accepted his advice, but persisted in that erroneous measure of value for twenty years. Mr. Finlaison's report was not printed till 1824, when it was laid before the committee of finance; but Mr. Vansittart's confidence in the correctness of the Northampton tables was greatly shaken by the revelations it con-

tained, and he at once directed Mr. Finlaison to spare no labour in the investigation of the true law of mortality prevailing in England among the class of annuitants. He was thus plunged into the abyss of a new science, and in the watches of many a night slowly but surely traced out the fundamental laws of life.

He attempted at first to procure the necessary data from the records of the births, marriages, and deaths of the nobility of Great Britain; but, after consuming eighteen months in the trial, he found it would involve too wide a range of reading for his immediate purpose; and, although he had made considerable progress in this laborious research, he was obliged reluctantly to suspend it, and to turn to the Exchequer offices of England and Ireland. Here he found the very materials for his purpose in the records of the various tontines from 1695 to the great tontine in 1789, which, together with the annuitants of the Sinking Fund, and an observation made by himself on the Chelsea and Greenwich pensioners, afforded him the means of comparing more than 19,000 deaths. The incalculable advantage that these data afforded him arose from the fact, that in every instance the age of the deceased was delivered on oath; and, in the case of tontines, that of the surviving co-partners also. In another respect these observations had possibly an advantage, namely, that being taken only from the higher classes, they excluded those lives which could not have any concern with life assurance.

The only observation that had hitherto been made in England, in addition to the old Northampton tables, was the work of his contemporary, Mr. Milne, of the Sun Fire Office, whose data were only 900 deaths of both sexes indiscriminately, and whose ages were most imperfectly conjectured. Some idea may be formed from this fact of the superior extent of Mr. Finlaison's researches, and of the means he had of attaining perfection in his results. But the gradual conviction forced upon Mr. Finlaison, that none of his predecessors in political arithmetic had ever attained correctness, obliged him to repeat his work twice, thrice, even four times, before the truth of the new results became unquestionable; and the time which, during four years, he devoted to the intense labour of calculation, very considerably exceeded an average of *twelve* hours a day. With less enthusiasm in the cause, his task would not have been concluded in any reasonable period.

The methods of adjustment which he employed for this purpose are given in his report, 1829. From this it appears that

nineteen observations were adjusted from the formula (Milne's notation)—

$$ {}_1a=\sqrt[3]{\sqrt[5]{{}_1a_3 \cdot {}_1a_2 \cdot {}_1a_1 \cdot {}_1a \cdot {}^1_1a} \times \sqrt[5]{{}_1a_2 \cdot {}_1a_1 \cdot {}_1a \cdot {}^1_1a \cdot {}^2_1a} \times \sqrt[5]{{}_1a_1 \cdot {}_1a \cdot {}^1_1a \cdot {}^2_1a \cdot {}^3_1a}}; $$

while two more, being those from which the tables of life annuities were calculated, were adjusted from the formula—

$$ {}_1a=\frac{1}{25}(5\,{}_1a+4\,{}_1a_1+4\,{}^1_1a+3\,{}_1a_2+3\,{}^2_1a+2\,{}_1a_3+2\,{}^3_1a+{}_1a_4+{}^4_1a). $$

The result of his studies was the discovery that the average duration of human life had sensibly increased during the century. These tables also were the first which exhibited the difference between male and female mortality, although it had been loosely remarked for many years previously. Their accuracy in this respect has been impugned, and, in the case of advanced ages, appears to require some modification; but the recent elaborate and very extensive researches made by his son and successor, Mr. Alexander Finlaison, have shown that the error is small, and that he had attained almost as great accuracy as was possible with the data at his disposal.

The following extracts from the reports of 1829 and 1860 will exhibit the difference in a few cases.

The results in Mr. John Finlaison's report are extracted from observations 9 on female life, and 20 on male life, pages 44 and 58. Those in Mr. Alexander Glen Finlaison's are taken from observations 14 and 15, pages 140 and 144.

Males.

Age.	1829. Mortality per Cent.	1860. Mortality per Cent.	Difference.
3	1·24862	0·97842	·27020
8	·60403	·64628	– ·04225
13	·52586	·59678	– ·07092
18	·93132	1·04714	– ·11582
23	1·41513	1·36790	·04723
28	1·27163	1·17380	·09783
33	1·24605	1·17550	·07055
38	1·31414	1·32440	– ·01026
43	1·35342	1·36200	– ·00858
48	1·42012	1·54340	– ·12328
53	2·14940	2·08640	·06340
58	2·96085	2·59970	·36115
63	3·55167	3·45860	·09307
68	5·26964	5·24080	·02884
73	7·22627	7·53160	– ·30533
78	9·79843	10·99010	– 1·19167
83	16·22553	16·23580	– ·01027
88	30·95338	21·95490	8·99848
93	46·00000	31·91890	14·08110
98	100·00000	71·42857	28·57143

Females.

Age.	1829. Mortality per Cent.	1860. Mortality per Cent.	Difference.
3	1·09779	1·07493	·02286
8	·57219	·61279	– ·04060
13	·47732	·61345	– ·13613
18	·83950	·82793	·01157
23	·86238	·82857	·03381
28	·90490	·86084	·04406
33	·99023	·95716	·03307
38	1·11019	1·02467	·08552
43	1·15152	1·10770	·04382
48	1·19281	1·27960	– ·08679
53	1·28756	1·51620	– ·22864
58	1·69708	1·87270	– ·17562
63	2·27646	2·62970	– ·35324
68	3·39883	4·06110	– ·66227
73	5·47601	6·07950	– ·60349
78	8·75801	9·42750	– ·66949
83	11·38146	14·45910	– 3·07764
88	16·05611	20·67350	– 4·61739
93	34·81818	29·32050	5·49768
98	65·00000	71·42857	– 5·42857

By the end of the year 1823, Mr. Finlaison had completed a set of tables on single lives, constructed on the basis of his recent discoveries, and adapted to the purposes of the Sinking Fund. In these he gave the value of immediate and deferred annuities at nine rates of interest, payable half-yearly, for every year and half year of human existence; and it may be observed, that no table of half-yearly annuities had till now been completed in this or any other country. He afterwards calculated a similar system of tables for two joint lives.

The Institute of France took considerable interest in the inquiry upon which Mr. Finlaison was engaged. The Academy of Sciences transmitted many questions on the subject for Mr. Finlaison's consideration; and, in return for the information contained in his answers, they did him the honour to send him a letter of thanks.

The savings bank system, and the report which Mr. Finlaison was desired to make on it, have already been mentioned as contemporary with the report on the annuity system, which led to such important results. His services in connection with the former subject may now be briefly enumerated in the order of their occurrence: they were performed during the progress of his calculations for the tables of life annuities, at the expense of intense mental toil. Before the close of 1819, Mr. Finlaison had furnished the Chancellor of the Exchequer with a very elaborate statement of the

age of each individual in the receipt of naval half-pay or pensions —14,000 persons, thence deducing the decrement of life among them. In 1821 Mr. Harrison employed him for several months in computations relative to the Superannuation Act, and expressed himself much pleased with the exertions and skill displayed by Mr. Finlaison; and in 1822 he was occupied in considerations relative to the commutation of the naval and military half-pay and pensions—which measure, suggested by him in his report on the savings banks, was finally established by the negotiations with the Bank of England, in 1823, for its acceptance of the charge for public pensions in consideration of the "dead weight" annuity. All the calculations connected with this subject were performed by Mr. Finlaison with such unsparing exertion and arithmetical skill as should neither be forgotten nor undervalued—especially as it was plainly stated in the House of Commons, that, in the whole establishment of the Bank, there was not one person capable of computing the new annuity at the fractional rate of interest agreed upon. This measure having passed into a law, it became necessary to ascertain the real decrement of life among the pensioned and half-pay officers of the Army and Navy. The result of Mr. Finlaison's investigations presented a most perfect view of the extent and progressive diminution of this department of the national engagements, similar to the view of the *rentes viagères* annually published by the French Government. Mr. Finlaison had previously completed the observation on the decrement of life among the Greenwich and Chelsea pensioners, already mentioned as forming part of the data for his law of mortality. It was formed from a registry of more than 70,000 persons, including 8,000 deaths, and presented important matter for the consideration of the Chancellor of the Exchequer.

The business of the savings banks was now committed to the National Debt Office, and an actuary and check-officer was needed. The Government, therefore, resolved to remove Mr. Finlaison from the Admiralty, to fill the newly-created office in the Treasury; and, on the 1st January, 1822, he was appointed Actuary and Principal Accountant of the Check Department of the National Debt Office, the duties of which position he performed for twenty-nine years.

From the first moment of his undertaking the subject of life assurance, until the production of the report of 1829, he continued constantly impressing upon the members of the Government the enormous loss which the country sustained by persisting in using the erroneous tables; but, so far from meeting with the encourage-

ment and commendation which might have been expected, he received nothing but neglect and contempt. Persons in high position refused to see him; and, when he had positively forced himself into their presence, would give no heed to his statements. And yet, when the matter came before Parliament, these were the very people who publicly declared that "they could never get any account from him!"

The anxiety which he suffered during the time when he was devoting his best energies to the service of his country, may be traced in his letters. He writes to Mr. Walpole, Secretary to the Chancellor of the Exchequer, in 1824, of the extreme cruelty "of leaving me to expend my talents, time, and labour, like a galley slave, at the rate of sixteen hours a day, eleven of which were constantly employed on the calculations for the life annuities; and this without being able, by the most fervent eloquence or the most pathetic complaints, to extract one shilling from the Treasury, as the records of that office will show, between Lady Day, 1819, and the month of April, 1821, save the sum of £500; and out of these, my private resources, I had to pay the clerks employed, to find them accommodation, to find an extensive collection of books, to defray the expense of stationery and all other contingencies that might arise."

Again, in a letter to a noble lord, dated March, 1828, praying for his assistance to defend his reputation in the House of Commons, when the correctness of a recent report by him, on the tremendous loss the Government was still sustaining in the matter of life annuities, was impugned by the ministry, he writes:—"In the simplicity of my heart I fancied that a service which went to prevent the loss of millions of the public money would have been rewarded by honour and profit. I have been fatally undeceived: my salary has been stopped; I have met with nothing but envy, malice, and hatred, for my discovery: and, although I have every year reiterated my warnings, not a jot of attention was paid me till now, when my statements are called extravagant. This is the same imputation on an *actuary's* character as cowardice is on that of an *officer*, and cannot be endured."

It was the accidental production of a letter from Mr. Finlaison, addressed to the Secretary of the Treasury, Mr. Herries, and dated 30th April, 1827, that occasioned the inquiry alluded to above. It was inadvertantly laid before Lord Althorp's committee of finance by the Treasury themselves, in March, 1828. On the motion of Earl Grey, this letter was printed by order of the House of Lords,

and it was then shown that the annuitants of the first year, ending September, 1809, had already received back principal and interest of all that they had paid, and ought, therefore, in justice, to be all dead. But, far from being so, four out of every ten were still alive, and likely to live for many years. The same letter established, that, in the previous year, 1827, the loss sustained by the public revenue was advancing at the rate of £8,000 PER WEEK! and it further proved that this loss was concealed by the method of preparing the yearly accounts. This discovery caused the immediate suspension of the life annuity system, which was remodelled upon the basis of Mr. Finlaison's tables, and resumed in November, 1829.

In the same year Mr. Finlaison produced his report on the evidence and elementary facts on which his new tables of life annuities were founded, printed by order of the House of Commons, September, 1829. This important parliamentary document contained twenty-one new observations of the law of mortality, and one of the law of sickness prevailing among the labouring classes in London.

In the recent report on life annuities by Mr. Alexander Glen Finlaison, it is shown "that the loss saved to the Government by the adoption of Mr. Finlaison's tables, was, in five years, £390,501; by which it may be estimated that the substitution of Mr. Finlaison's tables for the old Northampton rates has already made a difference to the country of nearly £3,000,000 sterling."

In the year 1830 Mr. Finlaison was occupied in the investigation of the affairs of the London Life Assurance Association, and drew up a report containing many valuable suggestions for their benefit. By the end of the year 1831, he had, in conjunction with Professor De Morgan, completed an elaborate report on the affairs of the "Amicable"; and, in the succeeding year, he furnished them with a new set of tables. He also computed the tables for the Royal Naval and Military Life Assurance Society, which was established in 1837. Mr. Finlaison had a large share in the constitution of this Society, where he held, till his death, the office of consulting actuary. It may be mentioned here also, that, in 1839, he furnished the scheme and prepared the tables of the New York Life Assurance and Trust Company, the first institution of the kind established in the United States.

As the Government adviser on all measures involving political arithmetic, Mr. Finlaison's counsel and powers of calculation were

in constant requisition; and, from the year 1830 till his retirement from official life in 1851, there were few sessions of Parliament in which he was not called as a witness before some committee for the consideration of some important financial measure; few Crown commissions to which he did not render some important service—of which that in 1849, on church leases, under Lord Harrowby, may be specially mentioned; and scarcely a single department of the Government which did not apply to him for advice and assistance.

In 1833, by direction of Viscount Goderich, he commenced extensive computations of the duration of slave and creole life, with reference to the emancipation of the slaves on the West Indian plantations. In 1835 Mr. Finlaison performed the calculations necessary to effect the West India loan of £15,000,000, for compensating the slave-owners; and, in answer to certain resolutions on that loan, moved by the late Mr. Hume on the last day of the session of that year, he prepared a report early in 1836, which, as a Parliamentary paper on the higher finance and the funding system, is well worthy of perusal. Mr. Hume's reputation for financial knowledge was so completely annihilated by it, that he was never afterwards regarded as an authority on matters of finance, Mr. Finlaison having proved him to be completely in error on thirteen important questions. His masterly treatment of the subject did not fail to be duly appreciated by Lord Monteagle, by whom the loan was effected.

The demands made on Mr. Finlaison's mental powers had been so great throughout the progress of the measure for the abolition of slavery, that, at its conclusion, he became seriously ill. Great fears were entertained for the safety of his over-wrought mind, and he obtained permission from Lord Monteagle to reside in the country, and thenceforward was obliged to exercise more caution in his devotion to the public service. About this time, being a widower, he married the daughter of Mr. Thomas Davis, of Waltham Abbey.

His valued friend, Dr. Southwood Smith, the father of sanitary reform, having submitted to him the experience of the London Fever Hospital, of which he was physician, for the ten years preceding 1834—an observation which included nearly 6,000 patients—Mr. Finlaison was enabled to make a calculation of the mortality of fever, which presented some curious and instructive results. These were published, in 1835, by Dr. Southwood Smith, in a work entitled the *Philosophy of Health.*

His professional researches were still assiduously carried on, and his knowledge and counsel were called for, to an extent greatly beyond what is generally known, in the measures emanating from the Ecclesiastical Commissioners. In proof of this, his numerous reports may be adduced, on the means of improving church property by the abolition of fines, in 1835; on the great question of church leases, and the steps leading to the "Appropriation Clause," in 1836; and on the church-rate question, in 1837.

There was one service in connection with these measures to which Mr. Finlaison was wont to refer, with some consciousness of superior talent, as one of pre-eminent merit. Some time before the Parliamentary contest on the subject of church rates, Mr. Finlaison, in November, 1836, attended the Cabinet, and had the honour of explaining to the assembled Ministers, that he could demonstrate, by the mere force of arithmetic, and without any survey, the rental of the episcopal and capitular estates, at a minimum. Earl Russell, who was present, might still possibly remember how incredulous many members of the Cabinet were, and that Lord Sidmouth pronounced such a thing to be a flat impossibility. But Mr. Finlaison did prove it, in two memorandums to Lord John Russell, dated January, 1837, which were found to be unassailable. His opinion was fully relied on, and this most confidential service was matured in secresy, until the month of April, 1838, when his reports were laid before Parliament.

Mr. Finlaison was consulted on certain points connected with the establishment, in 1837, of the registration of births, deaths, and marriages; and the closeness of his estimate of the deaths to be registered the first year—falling within twelve of the whole number recorded, 334,000—attracted much notice when mentioned in the Registrar-General's First Annual Report. He also predicted the number which would be found at the ensuing Census of 1841, with the object of enabling the Registrar-General to estimate the number of clerks to be provided, and he came within 1,200 of the whole female population of Great Britain.

The last Committee of the House of Commons before which he attended as a witness was that which sat, in 1848, upon Feargus O'Connor's Land Scheme. In the same year he sent in to the Treasury the second of two reports on the Act of the previous session for lending money to the Irish landlords, with the tables necessary to give effect to it. None but a very able actuary

could have computed them, from the complexity of some of the conditions.

In 1847, the Institute of Actuaries having been formed, Mr. Finlaison was unanimously elected President, and filled that post till the day of his death. It is not necessary to enlarge upon the affectionate regard which he entertained for all the members, nor upon the uniform kindness which they exhibited towards him. These are best shown by the letters which passed between them in 1851, when, being attacked by a dangerous, and, as it was thought, incurable disease, he tendered his resignation.

"*To the Vice-Presidents and Members of the Council.*

"*26th April*, 1851.

"My dear Sirs,—Having, for many months past, been afflicted with a disease, which, in all probability, will soon be mortal, I feel that I should make a bad return to the Institute for the honour they have so repeatedly conferred on me, in choosing me their President, if I now continued to hold that distinguished office; and especially in such a time as the present, when I am totally unable to acquit myself of its duties.

"I request you, therefore, to tender my respectful resignation of the office of President, with my grateful acknowledgments for the kindness and deference which, on every occasion, I have received at the hands of its members.

"To yourselves, my dear Sirs, collectively and individually, without exception, I never can find words to express the sense I entertain of your kindness to me whenever and wherever we have met; and let it not repent you that, however unmerited on my part, you have shown me such kindness, when I inform you that I receive it as a testimonial from the *élite* of my brethren that my life has hitherto not been unprofitably spent, but that I have, in some degree, enlarged the boundaries of a science that, even in its infancy, promises to be of the utmost benefit to mankind.

"Farewell.

"Your affectionate brother and servant,

"John Finlaison."

"John Finlaison, *Esq.*, *President of the Institute of Actuaries.*

"*3rd May*, 1851.

"Dear Mr. President,—We are deputed by the Council to acknowledge the receipt of your letter of the 26th ult., and to assure you that its contents have been the occasion of very sincere regret to every member. Although aware that you had been for some time suffering from indisposition, the Council generally were not in the least conscious that your malady was of so severe or so alarming a character, and they have received the information with deep and unfeigned sorrow.

"We are requested to convey to you warm and earnest hopes that your valuable life may be spared for many years to come. We are also to convey to you the urgent request of the Council that you will continue to

retain the Presidency of the Institute, and remain the head of that profession which you have been in so great a measure the means of creating, and for which you have so zealously and faithfully laboured throughout your useful life.

"Believe us, dear Sir,

"With much respect,

"Your faithful servants,

"JENKIN JONES, } *Hon. Secs.*"
"ROBERT TUCKER, }

In August, 1851, Mr. Finlaison finally retired from his position as Actuary of the National Debt and Government Calculator, after a confidential and devoted service under the Crown of nearly half a century, forty-two years of which were passed as chief clerk of one or other of two most important public departments—a service unalloyed by the slightest censure.

The last nine years of his life were passed in comparative ease and tranquillity. He applied himself, with unimpaired energies, to his favourite studies of Scriptural chronology and the universal relationship of ancient and modern weights and measures. His researches, which were exceedingly profound on both subjects, led him to form opinions decidedly adverse to the introduction of the decimal system of coinage and metrology into this country. He was on the point of completing a work in which his knowledge of numerical relations gave him unusual advantages, and, in combination with his native mental abilities, promised to render such a treatise of the utmost value to the student of biblical literature, when he was unexpectedly attacked by congestion of the lungs, and, after a brief illness of one week's duration, he passed away, on the 13th April, 1860, in his seventy-seventh year, from a scene in which he had long taken no useless part.

The reader of the foregoing outline of the chief events of Mr. Finlaison's active life, can hardly fail to be struck with the energy and unceasing industry which enabled him to accomplish so many entirely different undertakings. That an orphan boy should raise himself, by his own industry, to acquire a European celebrity, is a matter for high praise; but it is more surprising that a man, whose taste and studies had been entirely devoted to literature, should, by mere perseverance, gain a mastery over an abstruse science to which he had a positive dislike at starting.

The poetical talent which he possessed as a boy, he cultivated with great success, and produced some lyrical poems of considerable merit. He was also an enthusiastic mason, and attained a

high rank in that mysterious fraternity. He bore the chivalrous character of a past age. Incapable of acting with meanness, he could hardly be brought to believe it in others; and he suffered through life from the implicit confidence which he placed in every one. Injustice or oppression he would never endure; and for the oppressed he has not only afforded his time and energy, but has risked making powerful enemies, and on one occasion hazarded his whole fortune on behalf of a perfect stranger. He was affectionate in disposition, witty and animated in conversation, cool in moments of danger; and he died as he had lived, with the courage and resignation of a Christian gentleman.

On the Construction and Use of Commutation Tables for Calculating the Values of Benefits depending on Life Contingencies. By Peter Gray, *Esq.*, F.R.A.S.*

(Continued from page 104.)

PROBLEM VI.—To find the value of a life annuity on (x), of which the first payment is to be £1; the second, £2; the third, £3; and so on, increasing annually, by the amount of the first payment, to the end of life.

This benefit may be conceived to consist of a series of annuities of £1, of which the first is to be entered upon now; the second, one year hence; the third, two years hence; and so on, an additional annuity being entered upon every year, till the end of life. The present values of these annuities are, by Problems II. and III.,

$$\frac{N_x}{D_x},\ \frac{N_{x+1}}{D_x},\ \frac{N_{x+2}}{D_x},\ \&c.;$$

and the sum of these expressions, which will be the required present value, is, by (2),

$$\frac{N_x+N_{x+1}+N_{x+2}+\&c.}{D_x}=\frac{S_x}{D_x}.$$

The same result will be obtained by viewing the benefit in another light. Thus, the first payment is £1, to be received a year hence; its present value, therefore, is $\frac{D_{x+1}}{D_x}$. The second payment is £2, to be received two years hence; its present value, therefore,

* Extracted from the *Mechanics' Magazine* for 1842.

is $\frac{2D_{x+2}}{D_x}$. In like manner, the present values of the third, fourth, &c., payments will be respectively

$$\frac{3D_{x+3}}{D_x},\ \frac{4D_{x+4}}{D_x},\ \&c.$$

and the sum of these expressions, which will be the present value of the benefit, is by (11), $\frac{S_x}{D_x}$, as before.

Example.—Required the present value of an increasing annuity of £1, £2, £3, &c., on (30).

$$\textit{Answer.}\ \frac{S_{30}}{D_{30}}=\frac{539239\cdot0044}{2257\cdot6521}=238\cdot8494=£238.\ 17s.$$

PROBLEM VII.—To find the present value of a deferred increasing annuity on (x), that is, of an annuity to be entered upon n years hence, and whose first payment is to be £1; the second, £2; the third, £3; and so on, increasing every year, by the amount of the first payment, to the end of life.

This benefit may be considered as a series of deferred annuities of £1, of which the first is to be entered upon n years hence; the second, $n+1$ years hence; the third, $n+2$ years hence; and so on, an additional annuity being entered upon every year, till the end of life. The present values of these annuities are, by Problem III.,

$$\frac{N_{x+n}}{D_x},\ \frac{N_{x+n+1}}{D_x},\ \frac{N_{x+n+2}}{D_x},$$

and so on; and the sum of these expressions, which will be the present value required, will be, by (4), $\frac{S_{x+n}}{D_x}$.

Example.—An annuity, whose successive payments are to be £1, £2, £3, &c., is to be entered upon by (20), 10 years hence, that is, when he attains the age of 30. Required its present value.

$$\textit{Answer.}\ \frac{S_{30}}{D_{20}}=\frac{539239\cdot0044}{3818\cdot9594}=141\cdot2005=£141.\ 4s.$$

PROBLEM VIII.—To find the present value of a temporary increasing annuity for n years on (x), the first payment being £1; the second, £2; and so on, increasing each year by the amount of the first payment, till the end of the term, when all payments cease.

This benefit consists of a series of temporary annuities of £1, of which the first is to be entered upon now, to continue n years, and then to cease; the second is to be entered upon one year hence, to continue $n-1$ years, and then to cease; and so on to the

nth annuity, which will be entered upon $n-1$ years hence, continue one year, and then cease—that is, will make but a single payment.

Now, the present values of these annuities are (omitting, for brevity, the common divisor D_x), by Problems IV. and V.,

$$\begin{array}{rl} \text{Of the 1st,} & N_x - N_{x+n} \\ \text{2nd,} & N_{x+1} - N_{x+n} \\ \text{3rd,} & N_{x+2} - N_{x+n} \\ \cdots & \cdots\cdots \\ \text{Of the } n\text{th,} & N_{x+n-1} - N_{x+n}. \end{array}$$

The sum of these is, (7),

$$S_x - S_{x+n} - n N_{x+n};$$

and, inserting the divisor, we have for the required present value,

$$\frac{S_x - S_{x+n} - n N_{x+n}}{D_x}.$$

Example.—Required the present value of an increasing annuity of £1, £2, £3, &c., on (55) for the next 10 years.

Answer. $$\frac{S_{55} - S_{65} - 10 N_{65}}{D_{55}} = \frac{60976{\cdot}4989 - (17008{\cdot}1044 + 23769{\cdot}292)}{587{\cdot}3514}$$

$$= \frac{20199{\cdot}1025}{587{\cdot}3514} = 34{\cdot}390 = £34.\ 7s.\ 10d.$$

Problem IX.—To find the present value of an arrested increasing annuity on (x), that is, of an annuity whose successive payments are to be £1, £2, £3, and so on, increasing £1 every year till n payments have been made, the nth payment being thus £n, which is also to be the amount of each subsequent payment till the end of life.

The difference between this benefit and that which formed the subject of the last problem is, that while, in the case of the benefit referred to, all payments were to cease at the end of n years, in the present case the increase only is arrested, and the annuity continues to pay £n a year during the remainder of life. If, therefore, to the present value of the benefit in the last problem we add the present value of an annuity of £n, deferred for n years, the sum will be the present value required. That is—

$$\frac{S_x - S_{x+n} - n N_{x+n}}{D_x} + \frac{n N_{x+n}}{D_x} = \frac{S_x - S_{x+n}}{D_x}.$$

Example.—Required the present value of an annuity of £1, £2, &c., on (55), the increase of which is to be arrested after 10 years.

Answer. $\frac{S_{55}-S_{65}}{D_{55}}=\frac{60976\cdot4989-17008\cdot1044}{587\cdot3514}=\frac{43968\cdot3945}{587\cdot3514}$

$=74\cdot8589=£74.\ 17s.\ 2d.$

We have now to deduce the expressions for the present values of the assurance benefits.

PROBLEM X.—To find the present value of an endowment assurance of £1 on (x); that is, of £1 to be received n years hence, provided (x) shall have died in the preceding year, viz., in the nth year from the present time.

Of the l_x individuals, whose present age is x years, represented by the mortality table to be now alive, l_{x+n-1} survive $n-1$ years, and l_{x+n} survive n years; consequently, $l_{x+n-1}-l_{x+n}$ is the number who die in their nth year; and it is also the number of pounds which will have to be paid, at the end of n years, to the representatives of those who thus die. The present value of this sum, therefore—that is, the sum which, put out at interest now, would in n years just amount to the first-named sum, is what must be advanced now to provide for this payment. This present value is $(l_{x+n-1}-l_{x+n})v^n$. And since all the l_x individuals now alive are equally interested, all of them contribute equally to this amount. The contribution of each will be, therefore—

$$\frac{(l_{x+n-1}-l_{x+n})v^n}{l_x},$$

and this is the required present value.

As in Problem I., multiply the numerator and the denominator of this expression by v^x, and it becomes

$$\frac{(l_{x+n-1}-l_{x+n})v^{x+n}}{l_x v^x}.$$

But the numerator of this expression is equal to C_{x+n-1}, and the denominator to D_x, by (1) and (3). Hence, the expression for the required present value, by the Commutation Table, is

$$\frac{C_{x+n-1}}{D_x}.$$

If $n=1$, that is, if the assurance is to be received a year hence, provided the death of x take place in the present year, the expression becomes $\frac{C_x}{D_x}$. If $n=2$, it is $\frac{C_{x+1}}{D_x}$, and so on.

Example.—Required the present value of an endowment assurance of £1 on (30), provided he die in his 40th year.

Here $x=30$, and $n=10$. Hence, the present value is $\frac{C_{39}}{D_{30}}$.

Since C is not exhibited, we may use for it either of the expressions (10) or (12). Taking the first, the formula becomes

$$\frac{M_{39}-M_{40}}{D_{30}}=\frac{540\cdot81079-522\cdot65022}{2257\cdot6521}=\frac{18\cdot16057}{2257\cdot6521}=\cdot008244=2d.$$

If the sum to be received be £100, its present value will be

$$\cdot008244\times100=\cdot8244=16s.\ 6d.$$

We have seen that when $n=1$, that is, when the sum assured is to be received a year hence, provided (x) be then dead, the formula for the present value becomes $\frac{C_x}{D_x}$. But, by (12), $C_x=vD_x-D_{x+1}$. Hence, by substitution, the formula in this case becomes

$$\frac{vD_x-D_{x+1}}{D_x}=v-\frac{D_{x+1}}{D_x};$$

and this expression affords a proof of the correctness of the formula in this problem. For, since v is the present value of £1, to be certainly received a year hence, and $\frac{D_{x+1}}{D_x}$ is (by Problem I.) the present value of £1 to be received at the same time provided (x) be then alive, the difference between these two is evidently the present value of £1 to be received a year hence if (x) be then dead.

Problem XI.—To find the present value of a life assurance of £1 on (x); that is, of £1 to be received at the end of the year in which (x) dies, whensoever that event may happen.

We saw, by the last problem, that $\frac{C_x}{D_x}$ is the sum which must be contributed now by each of the l_x individuals to provide for the payment of £1 to the representatives of each of those of their number who die in the first year; and that $\frac{C_{x+1}}{D_x}$ is the sum each must contribute to provide for a like payment to the representatives of each of those who die in the second year. And, in like manner, by making n successively equal to 3, 4, &c., to the end of life, in the formula deduced in last problem, we should obtain, for the amounts to be contributed now to provide for the deaths that take place in the 3rd, 4th, &c., years respectively,

$$\frac{C_{x+2}}{D_x},\ \frac{C_{x+3}}{D_x},\ \&c.$$

And the sum of all these expressions is evidently the total sum which each of the l_x individuals must now pay to entitle his representatives to £1 at the end of the year in which he dies.

This sum is

$$\frac{C_x + C_{x+1} + C_{x+2} + \&c.}{D_x},$$

which, by (2), is equal to $\frac{M_x}{D_x}$; and this is, therefore, the required present value.

Example.—Required the present value of a life assurance of £1 on (20).

$$\textit{Answer.}\quad \frac{M_{20}}{D_{20}} = \frac{1131{\cdot}76185}{3818{\cdot}9594} = {\cdot}296353 = 5s.\ 11d.$$

If the amount assured be £100, its present value will be

$${\cdot}296353 \times 100 = 29{\cdot}6353 = £29.\ 12s.\ 8d.$$

PROBLEM XII.—To find the present value of a deferred assurance of £1 on (x); that is, of £1 to be received at the end of the year in which (x) dies, provided that event take place after n years.

The present value of this benefit, similar to that of the benefit in Problem III., is evidently equal to the sum of all the terms after the nth in the series of last problem, which expresses the present value of the same benefit for the whole life. This sum is, by (4), $\frac{M_{x+n}}{D_x}$; which is, therefore, the present value required.

Example.—Required the present value of an assurance of £1, deferred for 10 years, on (20).

$$\textit{Answer.}\quad \frac{M_{30}}{D_{20}} = \frac{742{\cdot}95673}{3818{\cdot}9594} = {\cdot}194544 = 3s.\ 11d.$$

If the assurance be £100, its present value will be

$${\cdot}194544 \times 100 = 19{\cdot}4544 = £19.\ 9s.\ 1d.$$

PROBLEM XIII.—To find the present value of a temporary assurance of £1 for n years on (x); that is, of £1 to be received at the end of the year in which (x) dies, provided that event take place during the next n years.

The present value here is evidently the sum of the first n terms in the general series of Problem XI., which sum is, by (7),

$$\frac{M_x - M_{x+n}}{D_x};$$

and this is, therefore, the present value required.

Example.—Required the present value of a temporary assurance of £1 for the next 10 years on (20).

$$\textit{Answer.}\quad \frac{M_{20} - M_{30}}{D_{20}} = \frac{1131{\cdot}76185 - 742{\cdot}95673}{3818{\cdot}9594} = \frac{388{\cdot}80512}{3818{\cdot}9594} =$$

$${\cdot}101809 = 2s.$$

If the sum assured be £100, its present value is

$$\cdot 101809 \times 100 = 10\cdot 1809 = £10.\ 3s.\ 8d.$$

If the present value just found be added to that found by the last problem, their sum, ·296353, is the present value of an assurance for the whole life, as found by Problem XI., which it evidently ought to be.

Problem XIV.—To find the present value of a deferred temporary assurance of £1 on (x); that is, of £1 to be received at the end of the year in which (x) dies, provided that event take place in the n years following the next k years.

The present value required in this problem is evidently the sum of n terms of the general series of Problem XI., beginning with the $(x+k)$th; which sum is, by (9), equal to

$$\frac{M_{x+k}-M_{x+k+n}}{D_x};$$

and this is, therefore, the present value required.

Example.—Required the present value of an assurance of £1 on (30), to be received at the end of the year of death, provided that event take place between the ages of 40 and 50.

Here $x=30$, $k=10$, and $n=10$.

$$\therefore \frac{M_{x+k}-M_{x+k+n}}{D_x} = \frac{M_{40}-M_{50}}{D_{30}} = \frac{742\cdot 05673-522\cdot 65027}{2257\cdot 6521} = \frac{220\cdot 30646}{2257\cdot 6521}$$

$$= \cdot 097582 = 1s.\ 11\tfrac{1}{2}d.$$

If the sum assured be £100, its present value will be

$$\cdot 097582 \times 100 = 9\cdot 7582 = £9.\ 15s.\ 2d.$$

Problem XV.—To find the present value of an increasing life assurance on (x), which is to be £1 if death take place in the first year, £2 if in the second, £3 if in the third, and so on—the assurance increasing each year that the payment is deferred, by the amount of the first year's assurance, till the end of life.

The present value here is found exactly in the same way as that of the corresponding annuity benefit in Problem VI.; for this assurance may be conceived to be made up of a series of assurances of £1, of which the first is to be entered upon immediately, and the others at intervals of one year, until the life shall have ceased to exist. The sum of the present values of all these assurances will evidently be the present value required. Now, the present value of the first is, by Problem XI., $\frac{M_x}{D_x}$; and of the second, third, fourth, &c., by Problem XII.,

$$\frac{M_{x+1}}{D_x},\ \frac{M_{x+2}}{D_x},\ \frac{M_{x+3}}{D_x},\ \&c.;$$

and the sum of all these—that is, the present value required in the problem—is, by (2),

$$\frac{M_x+M_{x+1}+M_{x+2}+M_{x+3}+\&c.}{D_x}=\frac{R_x}{D_x}.$$

The same expression would be obtained by adding together the present values of the assurances for the separate years. Thus, the present value of £1 to be received at the end of one year, if death take place in that year, is, by Problem X., $\frac{C_x}{D_x}$; of £2 to be received at the end of two years, if death take place in the second year, $\frac{2C_{x+1}}{D_x}$; of £3 if death take place in the third year, $\frac{3C_{x+2}}{D_x}$, and so on. And the sum of these is, by (11), $\frac{R_x}{D_x}$, as before.

Example.—Required the present value of an increasing assurance of £1, £2, £3, &c., on (30).

$$\textit{Answer.}\ \frac{R_{30}}{D_{30}}=\frac{17127\cdot42459}{2257\cdot6521}=7\cdot5866=£17.\ 11s.\ 9d.$$

PROBLEM XVI.—To find the present value of an increasing assurance, deferred for n years, on (x), which is to pay £1, £2, £3, &c., at the end of the year in which (x) dies, according as that event shall take place in the $(n+1)$th, $(n+2)$th, $(n+3)$th, &c., year from the present time.

By reasoning precisely similar to that employed in Problem VII. for finding the present value of the corresponding annuity benefit, and which it seems, therefore, unnecessary to repeat, the expression for the present value of the benefit in the present problem is found to be $\frac{R_{x+n}}{D_x}$.

Example.—Required the present value of an increasing assurance of £1, £2, £3, &c., deferred 15 years, on (30).

$$\textit{Answer.}\ \frac{R_{x+n}}{D_x}=\frac{R_{45}}{D_{30}}=\frac{8326\cdot12414}{2257\cdot6521}=3\cdot6890=£3.\ 13s.\ 9d.$$

PROBLEM XVII.—To find the present value of a temporary increasing assurance of £1, £2, £3, &c., for n years, on (x).

The solution of this problem is strictly analogous to that of Problem VIII., and the required present value is found to be

$$\frac{R_x-R_{x+n}-nM_{x+n}}{D_x}.$$

Example. $x=30$, $n=5$.

Ans. $$\frac{R_{30}-R_{35}-5M_{35}}{D_{30}}=\frac{17127\cdot42459-(13672\cdot73323+3099\cdot47060)}{2257\cdot6521}$$
$$=\frac{355\cdot22076}{2257\cdot6521}=\cdot15734=3s.\ 2d.$$

PROBLEM XVIII.—To find the present value of an arrested increasing assurance of £1, £2, £3, on (x); that is, of an assurance which is to be £1 if death take place in the first year, £2 if in the second year, £3 if in the third, and so on to the nth year, when the assurance will be £n, at which amount it is to continue for the remainder of life.

The present value here, by a mode of proceeding similar to that in Problem IX., will be found to be

$$\frac{R_x-R_{x+n}}{D_x}.$$

Example. $x=30$, $n=5$.

Answer. $$\frac{R_x-R_{x+n}}{D_x}=\frac{R_{30}-R_{35}}{D_{30}}=\frac{17127\cdot42459-13672\cdot73323}{2257\cdot6521}$$
$$=\frac{3454\cdot69136}{2257\cdot6521}=1\cdot5302=£1.\ 10s.\ 7d.$$

The foregoing comprise all of what may be called the *simple* benefits, and some (we mean the increasing benefits) which perhaps do not, in strictness, belong to that category. Previous to making a few general and miscellaneous remarks upon them, we present the following synoptical table of the results we have obtained. We omit, for brevity and distinctness, the common divisor D_x.

The present Value of an	To be received in n years, is	For Life, is	For the first n years, is	After the first n years, is	For n years after k years, is	Uniform after n years, is
Endowment of £1 on (x)	D_{x+n}					
Endowment assurance of £1 on (x)	C_{x+n-1}					
Annuity of £1 on (x) ..	..	N_x	$\left\{\begin{matrix}N_x-\\N_{x+n}\end{matrix}\right\}$	N_{x+n}	$\left\{\begin{matrix}N_{x+k}-\\N_{x+k+n}\end{matrix}\right\}$	
Assurance of £1 on (x)	.	M_x	$\left\{\begin{matrix}M_x-\\M_{x+n}\end{matrix}\right\}$	M_{x+n}	$\left\{\begin{matrix}M_{x+k}-\\M_{x+k+n}\end{matrix}\right\}$	.
Increasing annuity of of £1, £2, &c., on (x)	..	S_x	$\left\{\begin{matrix}S_x-\\S_{x+n}-\\nN_{x+n}\end{matrix}\right\}$	S_{x+n}	$\left\{\begin{matrix}S_{x+k}-\\S_{x+k+n}-\\nN_{x+k+n}\end{matrix}\right\}$	$\left\{\begin{matrix}S_x-\\S_{x+n}\end{matrix}\right\}$
Increasing assurance of £1, £2, &c., on (x)	..	R_x	$\left\{\begin{matrix}R_x-\\R_{x+n}-\\nM_{x+n}\end{matrix}\right\}$	R_{x+n}	$\left\{\begin{matrix}R_{x+k}-\\R_{x+k+n}-\\nM_{x+k+n}\end{matrix}\right\}$	$\left\{\begin{matrix}R_x-\\R_{x+n}\end{matrix}\right\}$

On looking at the expressions in the above table, the following analogies present themselves, attention to which will prevent mis-

takes in practice. We confine our remarks to the numerators, and it must be remembered that the denominator *in each case* is the D corresponding to the present age.

1. When a benefit for the whole life is to be entered upon immediately, the quantities in the numerator have for their signature the present age.

2. When the benefit is not to be entered upon immediately, the quantities in the numerator (which, as regards the columns made use of, are the same as in the expressions for the corresponding benefits to last the whole life), have for their signature the age at which the benefit is to be entered upon.

3. When the benefit is to be entered upon immediately, and to last for a term of years only—for example, n years—the expressions, as regards the uniform benefits, are derived from those for the same benefits to last the whole life, by writing $N_x - N_{x+n}$ and $M_x - M_{x+n}$ for N_x and M_x respectively; and as regards the increasing benefits, by writing $S_x - S_{x+n} - nN_{x+n}$ and $R_x - R_{x+n} - nM_{x+n}$ for S_x and R_x respectively. If we write $S_x - S_{x+n}$ and $R_x - R_{x+n}$ for S_x and R_x, we adapt the formulæ to the case in which the increase only is arrested at the end of n years.

4. The expression for an annuity benefit is changed into that for the corresponding assurance benefit, by writing C, M, and R, for D, N, and S respectively—the signatures remaining the same, except in the case of C, the signature of which must be diminished by unity.

Since the expressions we have deduced are fractions, the terms of which consist of numbers in the Commutation Table all of the same dimension, none of them rising above the first degree, it follows that, if all the numbers in the table be either multiplied or divided by any number whatsoever, the numerical values of the expressions will not be affected.

Hence the truth of the remark in the Note, page 93, is established.

And hence, also, it follows, that the position of the decimal point in the Commutation Tables is perfectly arbitrary. The position assigned to it in our table is that corresponding to a radix of the mortality table of 10,000, although the mortality actually made use of corresponds to a much higher radix, as stated in the previous part of this paper, p. 87.

The general problem solved in the preceding pages is, What is the *present value* of a benefit whose *amount* is £1? Suppose the problem reversed, and that it is required to find what *amount* of

benefit is equivalent to a *present value* of £1. The following proportion evidently subsists in all cases:—The present value of a benefit whose amount is £1 is to the present value of a like benefit of any other amount, as the amount of the first to the amount of the second. Hence, calling a the present value of the first benefit, and £1 that of the second, we have $a : 1 :: 1 : \frac{1}{a}$. That is, we should be furnished in each case with an expression which is the reciprocal of that which we have obtained for the present value of the same benefit when its amount is £1. Thus, $\frac{D_x}{N_x}$ will be the amount, or annual rent, of a life annuity to be entered upon immediately, and $\frac{D_x}{M_x}$ the sum to be received at the end of the year of death, in consideration of a present payment of £1, by an individual now aged x years. So, also, $\frac{D_x}{S_x}$ will be the first, and $\frac{2D_x}{S_x}$, $\frac{3D_x}{S_x}$, &c., the second, third, &c., annual payments of an increasing life annuity whose present value is £1.

Hence the table on page 177 furnishes us also with a solution to the general problem in this form; for, to apply it to any particular case, we have only to form a fraction having for its *numerator* D_x, and for its *denominator* the expression in the table corresponding to that case. But this belongs more properly to another branch of the subject.

Before concluding this portion of the present paper, we make a remark, which perhaps might with equal propriety, have been introduced elsewhere. It is as to the principle on which the preceding problems have been solved.

Each benefit consists of a payment or payments, to be made at a certain specified time or times, provided a state of things, also specified, shall then have place; such state of things being, where a single life is concerned, either the existence or the non-existence of that life, coupled, in the latter case, with the further condition, that its failure shall have taken place within a certain specified period. Problems I. and X. (using the former or the latter according as the first or the second of the states of things mentioned is that on which the receipt of the benefit is made to depend) enable us to find the present value of the payment for any or each year of existence; and the sum of the present values for each year that the benefit is to last is evidently the whole present value of the benefit. Thus, the present value of a life annuity of £1 is the

sum of a series, whose terms express respectively the present value of £1 to be received at the end of one year, of two years, of three years, &c., from the present age to the end of life. The present value of an annuity of £1 to last n years, is the sum of the first n terms of the same series; and that of an annuity of the same amount, to be entered upon n years hence, and to last during the remainder of life, is the sum of all the terms after the nth.

(*To be continued.*)

INSTITUTE OF ACTUARIES.

PROCEEDINGS OF THE INSTITUTE.

First Ordinary Meeting, Session 1861-62.—*Tuesday, 25th November,* 1861.

The President in the Chair.

The minutes of the annual general meeting were read and confirmed.

The Secretary announced various donations to the library.

Messrs. R. B. Markby and T. J. Searle, duly nominated at the last ordinary meeting, were unanimously elected Associates of the Institute.

Mr. W. Spens read a paper "On the mortality experience of the Scottish Amicable Life Assurance Society."

Thanks were voted to Mr. Spens, and the meeting adjourned to Monday, 30th December, 1861.

Second Ordinary Meeting, Session 1861-62.—*Monday, 30th December,* 1861.

The President in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced various donations to the library.

The undermentioned gentlemen, duly nominated at the last ordinary meeting, were unanimously elected members of the Institute, viz.:—

Official Associate—H. Harben, Esq.

Associates.

Mr. J. N. Smith.	Mr. S. C. Thomson.
„ J. B. Bourne.	„ T. Dence.

The President announced, that out of twelve candidates for the matriculation examination (1861), six had passed, in the following order of merit :—

Mr. J. N. Smith.	Mr. T. J. Searle.
„ D. A. Bumsted.	„ S. C. Thomson.
„ F. J. C. Taylor.	„ M. P. Christie.

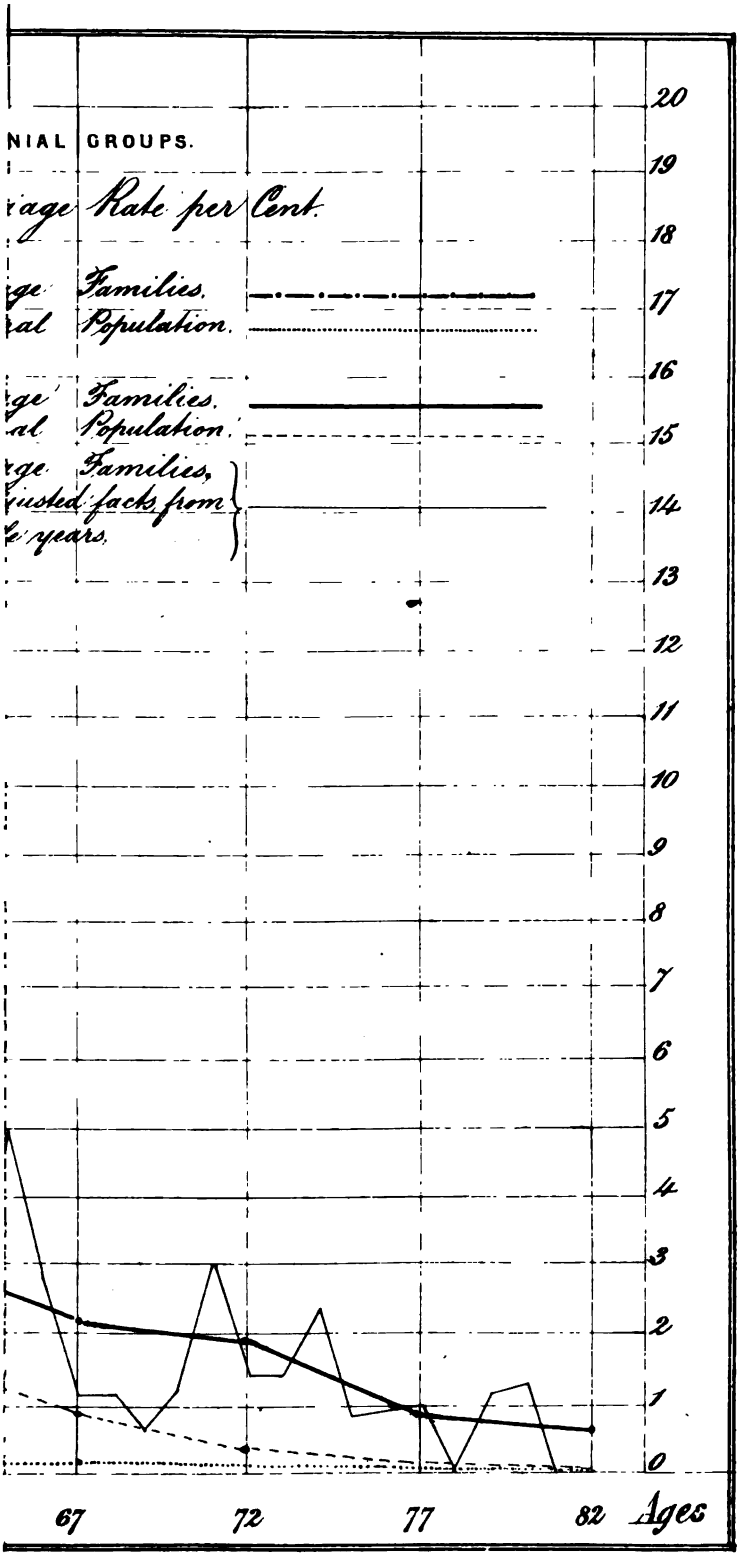
That seven candidates had presented themselves for the second year's examination, of whom three had passed in the following order :—

Mr. W. M. Makeham.
„ R. P. Hardy.
„ John Finlaison.

And that only one candidate had presented himself for the third year's examination, viz., Mr. Marcus Nathan Adler, M.A., who had also passed.

Mr. W. S. B. Woolhouse then read a paper "On Gompertz's law of mortality, and the dependence between it and Simpson's rule for calculating an annuity on three lives."

Thanks having been voted to Mr. Woolhouse, the meeting adjourned to Monday, 27th January, 1862.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THE

ASSURANCE MAGAZINE,

AND

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On the Statistics of First and Subsequent Marriages among the Families of the Peerage, considered specially with reference to the Calculation of Premiums for Assurances against Issue. By ARCHIBALD DAY, *Esq., Actuary of the London and Provincial Law Assurance Society.*

[Read before the Institute, 24th March, 1862, and printed by order of the Council.]

IN the month of January, 1859, I had the honour to read before the Institute of Actuaries a paper upon the determination of the rates of premium for assuring against issue. Since that period the subject has lost none of its interest; on the contrary, its importance is yearly increasing, and it bids fair to form an important branch of assurance business. It is true that, out of the numerous inquiries made at Assurance Offices respecting the rates of premium for these risks, not 20 per cent. become the subject of policies absolutely granted, still it is desirable that there should be some sort of consistency in the premiums quoted. It would appear that considerable differences of opinion exist among actuaries upon the subject; and I feel, therefore, that I owe no apology to the Institute for departing from our usual routine subjects of rates of interest and mortality, and introducing once more the question of

issue policies and the statistics of marriage. On the former occasion we considered the case which is usually presented to us officially—namely, that in which a husband and wife are both living, but where the prospect of children by the existing marriage (hitherto unfruitful) is so very remote that it may be disregarded, and the assurance required is generally against the risk of issue to the husband by his present or any future wife. It is clear, from a consideration of the case, that, next to the question of survivorship between husband and wife, the most important feature is the probability of the husband's *re*marriage; and this is a matter concerning which we have at present but little information. But although the probabilities of marriage cannot be calculated with the same degree of precision as those of death and survivorship—the decease of every man being a certain event, whereas his marriage may be affected by various circumstances, and is, to a great extent, under his own control—yet it may be shown that a general law of marriage really does exist, and that, too, sufficiently definite for ordinary purposes. It is, however, unnecessary for me here to enter upon the argument, the subject having been exhaustively treated by abler pens than mine; and I need only refer to the labours of M. Quetelet, and to the more familiar papers of Mr. Samuel Brown, which are to be found in the early volumes of the *Journal* of the Institute.

The opinion has long been prevalent that a distinctive marriage rate would be found to prevail among different classes of the community, and that the aristocracy and landed gentry would exhibit a law differing materially from that of the labouring population. As the former are essentially the class whose reversionary interests frequently involve the necessity of issue policies, it seemed highly desirable that, as actuaries, we should endeavour to obtain some definite information as to the operation of that law—if, indeed, it should be found to exist at all.

The rate of mortality of this class, it will probably be considered, was satisfactorily shown in the results of the investigation made by my friend Mr. Bailey and myself upon the families of the peerage, which was presented to the Institute in the course of last Session; and it is from the confidence with which those observations inspired me in the accuracy and sufficiency of the data, that I have been induced now to make use of the same materials for determining the rate of marriage among the families of the aristocracy.

The facts on the present occasion have been extracted from the same book of the peerage as before—namely, that of Lodge—

assistance being also obtained from an older volume of Debrett. The date up to which the observations reach is 31st December, 1855, and they are almost all included within a period of 100 years preceding. It should be remarked that, in the observations upon the marriages of bachelors, no cases were extracted from the records of a family if the information as to the remainder was defective; that is to say, that if, for instance, in a particular family the necessary particulars as to the eldest son happened to be given, and they were deficient as to the other sons, that case was, perhaps, with an excess of caution, omitted from the list. On the other hand, as the facts relating to second and subsequent marriages were comparatively scanty, resource has occasionally been had to some of the collateral branches, but in instances only where the families were of such distinction that little risk of error was involved. The probabilities of the marriage of spinsters, and of the remarriage of widows, have not been omitted through any lack of interest in this—to the fairer portion of the community—all important question; but as this paper has been prepared with the avowed intention of applying the results to a particular case, dependent rather on the other sex, the consideration of their claims must be deferred to a more favourable opportunity.

Referring, then, to the male lives alone, the following are the results of the observations, which extended over 3,666 cases:—

	Bachelors.	Widowers.	Total.
Married	1,540	438	1,978
Died (unmarried). . . .	445	368	813
Existing on 31st December, 1855 (unmarried) . .	736	139	875
	2,721	945	3,666

It will be noticed that more than 50 per cent. of the whole number contracted marriage during the period under observation; and although the number of facts appears to be small, it will be remembered that tables of mortality have been founded upon a less number. The total number of deaths in the Carlisle Table was 1840 only, which is exceeded by the number of marriages before us; and the comparison is more effective when it is borne in mind that the chance of marriage, unlike that of death, extends over a portion only of the term of man's life. Few observations, upon a class at any rate, can boast of being more extensive; and I venture to submit, that the number of facts, when grouped, as will

be shown hereafter, in quinquennial periods of age, will be sufficient to give results upon which dependence may be placed. The total number of years during which the probability of marriage has been observed is 50,272; viz.—

For bachelors	42,383·5
„ widowers	7,888·5
	50,272·0

I should here explain that no distinction has been made between second and subsequent marriages—that is to say, a person who has become for the second time a widower has been regarded in the same light as the survivor of one wife only. I have not kept a record of the number of these cases, but, speaking from memory, I think that three came under observation who had each ventured upon a fourth marriage.

I now submit, in the following Tables I. and II., the facts observed under every year of age for bachelors and widowers, together with the annual marriage rate deduced therefrom. Owing, however, to the small numbers living at some ages, there are great inconsistencies in the results, and it has been thought desirable to show the facts also in groups of five years, from which the marriage rate deduced forms a table of great regularity. The rate for intermediate ages has been obtained from the first differences of the logarithms. The headings of the different columns will probably sufficiently explain their construction.

In Table I., perhaps, the first fact which arrests attention is, that, out of 2,721 males completing the age of 15, there are not 100 left at the age of 60 living in a state of celibacy; and that, out of that number of cases remaining, not one bachelor contracted a marriage after the age of 59. It is much to be regretted that the numbers under observation above that age are so small, since no marriage rate can be obtained from them. Of course, it does not follow that no marriages of bachelors take place in this class above 60, but I can find no facts from which to deduce any conclusions. According to the table of first marriages among the general population, in my previous paper, there were, for the quinquennial group between 55 and 60, but 3·5 marriages per 1,000, and from 60 to 65, 1·5 per 1,000 only. It might, however, be anticipated that the marriage rate in the peerage families at these ages would be greater, because, on a reference to the two previous quinquennial groups, it will be found to have been in

excess. This may, perhaps, appear more clearly hereafter, on reference to the diagram which accompanies this paper. The rate is apparently greatest at the age 27, being nearly 9 per 1,000; and the rate for the quinquennial group from 25 to 30 is greater than any other—perhaps, however, it would not have been anticipated that it would have been so nearly twice as great as between 20 and 25—while it very little exceeds that at the next higher ages, 30 to 35. It will be further noticed that, between the ages 35 and 40, the rate is greater than between 20 and 25. These results will, I think, warrant the conclusion that the marriages of the aristocracy do not, on the average, take place at such early ages as those of the general population. To this point I shall again have an opportunity of referring.

From Table No. II. (Widowers) it will be seen that the probabilities of widowers remarrying are throughout greater than those of the first marriages of bachelors, being nearly twice as great in the quinquennial groups 25–, 30–, and 35–, and continuing in an increasing ratio up to 50. From this point the comparison fails, in consequence of the deficiency in the numbers of first marriages. There is great regularity in the diminishing probabilities of marriage among the widowers, which, however, will best be seen by reference to the diagram, representing the annual marriage rate per cent., the average of each group of 5 years being taken to represent the rate at the middle age in the group. This is denoted by the dark even line, the irregular one showing the rate for every age from the unadjusted facts. The lower dark line exhibits the marriage rate for bachelors, also derived from the quinquennial groups; this, however, fails before the age of 60. The light dotted lines represent the marriage rate of bachelors and widowers respectively in the general population, and the comparison is shown in Table No. III., p. 190.

TABLE I.—*Peerage Families (Bachelors).*

Age (x).	Completed the Age (x).	Died between the Ages x and $x+1$.	Existing on 31st Dec., 1855, between the Ages x and $x+1$ (Bachelors).	Sum of two preceding Columns.	Number who might contract Marriage between Ages x and $x+1$.	Married between the Ages x and $x+1$.	Annual Marriage Rate.	Number who might contract Marriage. (Quinquennial Groups.)	Number Married. (Quinquennial Groups.)	Annual Marriage Rate. (Quinquennial Groups.)	Age (x).
15	2,721	9	31	40	2701·	1	·00037				
16	2,680	17	31	48	2656·						
17	2,632	15	29	44	2610·	2	·00077	13050·	25	·00192	17
18	2,586	17	23	40	2566·	4	·00156				
19	2,542	14	36	50	2517·	18	·00715				
20	2,474	20	46	66	2441·	25	·01024				
21	2,383	41	38	79	2343·5	94	·04011				
22	2,210	25	29	54	2183·	105	·04810	10865·	458	·04213	22
23	2,051	18	27	45	2028·5	114	·05620				
24	1,892	17	29	46	1869·	120	·06421				
25	1,726	21	24	45	1703·5	127	·07455				
26	1,554	23	30	53	1527·5	111	·07267				
27	1,390	11	15	26	1377·	122	·08860	6921·5	533	·07700	27
28	1,242	14	24	38	1223·	99	·08095				
29	1,105	14	15	29	1090·5	74	·06786				
30	1,002	18	19	37	983·5	73	·07422				
31	892	6	16	22	881·	55	·06243				
32	815	5	14	19	805·5	58	·07200	4048·5	289	·07137	32
33	738	7	15	22	727·	55	·07565				
34	661	5	14	19	651·5	48	·07368				
35	594	3	8	11	588·5	35	·05947				
36	548	10	10	20	538·	33	·06134				
37	495	6	10	16	487·	27	·05544	2466·5	135	·05473	37
38	452	5	10	15	444·5	22	·04949				
39	415	4	9	13	408·5	18	·04406				
40	384	6	7	13	377·5	23	·06093				
41	348	3	9	12	342·	11	·03216				
42	325	4	10	14	318·	15	·04717	1596·	63	·03947	42
43	296	3	12	15	288·5	6	·02080				
44	275	2	8	10	270·	8	·02963				
45	257	1	10	11	251·5	6	·02386				
46	240	2	5	7	236·5	6	·02537				
47	227	3	6	9	222·5	6	·02697	1112·5	22	·01977	47
48	212	2	10	12	206·	2	·00971				
49	198	1	8	4	196·	2	·01020				
50	192	2	8	10	187·	1	·00535				
51	181	1	5	6	178·	4	·02247				

53	165	4	8	12	159·	3	·01887				
54	150	5	4	9	145·5	1	·00687				
55	140	2	6	8	136·	2	·01471				
56	130	5	9	14	123·	1	·00813				
57	115	2	3	5	112·5	1	·00889	573·5	6	·01046	57
58	109	4	5	9	104·5						
59	100	3	2	5	97·5	2	·02051				
60	93	1	2	3	91·5						
61	90	4	3	7	86·5						
62	83	4	2	6	80·	..	..	401·5			
63	77	3	2	5	74·5						
64	72	4	2	6	69						
65	66	1	4	5	63·5						
66	61	3	2	5	58·5						
67	56	5	3	8	52·	..	..	259·			
68	48	2	4	6	45·						
69	42	2	2	4	40·						
70	38	2	3	5	35·5						
71	33	..	4	4	31·						
72	29	3	2	5	26·5	..	..	137·5			
73	24	1	..	1	23·5						
74	23	2	2	4	21·						
75	19	2	2	4	17·						
76	15	..	1	1	14·5						
77	14	2	..	2	13·	..	..	66·5			
78	12	..	1	1	11·5						
79	11	1	..	1	10·5						
80	10	..	..	..	10·						
81	10	..	2	2	9·						
82	8	1	1	2	7·	..	..	36·			
83	6	1	..	1	5·5						
84	5	1	..	1	4·5						
85	4	..	1	1	3·5						
86	3	..	2	2	2·						
87	1	..	..	..	1·	..	..	8·5			
88	1	..	..	..	1·						
89	1	..	..	..	1·						
90	1	..	..	..	1·						
91	1	..	..	..	1·						
92	1	..	..	..	1·	..	..	3·5			
93	1	..	1	1	·5						
	42,974	445	736	1,181	42383·5	1,540		42383·5	1,540		

TABLE II.—*Peerage Families (Widowers).*

Age (x).	Completed the Age (x).	Died between the Age x and $x+1$.	Existing on 31st Dec., 1855, between the Ages x and $x+1$. (Widowers.)	Sum of two preceding Columns.	Number who might contract Marriage between Ages x and $x+1$.	Married between the Ages x and $x+1$.	Annual Marriage Rate.	Number who might contract Marriage. (Quinquennial Groups.)	Number Married. (Quinquennial Groups.)	Annual Marriage Rate. (Quinquennial Groups.)	Age (x).
20	1	..	..	..	1·						
21	3	..	..	..	3·						
22	3	..	..	..	3·	..	..	28·	5	·17858	22
23	9	..	..	..	9·	1	·11111			·17224	
24	12	..	..	..	12·	4	·33333			·16614	
25	21	..	..	..	21·	2	·09524			·16025	
26	28	..	..	..	28·	5	·17857			·15458	
27	38	1	..	1	37·5	7	·18667	194·5	29	·14910	27
28	48	..	1	1	47·5	10	·21053			·14390	
29	61	1	..	1	60·5	5	·08264			·13888	
30	72	2	..	2	71·	15	·21127			·13404	
31	75	..	1	1	74·5	8	·10738			·12936	
32	86	..	2	2	85·	9	·10588	408·5	51	·12485	32
33	87	1	2	3	85·5	9	·10526			·12051	
34	93	1	..	1	92·5	10	·10811			·11632	
35	104	1	2	3	102·5	18	·17561			·11227	
36	100	1	2	3	98·5	12	·12183			·10837	
37	110	2	3	5	107·5	7	·06512	554·5	58	·10460	37
38	118	1	1	2	117·	10	·08547			·10357	
39	131	3	1	4	129·	11	·08527			·10255	
40	137	1	..	1	136·5	18	·13187			·10153	
41	149	..	3	3	147·5	9	·06102			·10053	
42	157	1	..	1	156·5	12	·07668	763·5	76	·09954	42
43	164	2	2	4	162·	20	·12346			·09460	
44	163	3	1	4	161·	17	·10559			·08990	
45	166	5	5	10	161·	13	·08074			·08543	
46	167	4	..	4	165·	8	·04848			·08119	
47	175	2	4	6	172·	18	·10465	829·5	64	·07715	47
48	168	2	3	5	165·5	17	·10272			·07314	
49	168	2	2	4	166·	8	·04819			·06933	
50	171	8	3	11	165·5	11	·06646			·06573	
51	172	5	5	10	167·	7	·04192			·06231	
52	175	5	4	9	170·5	13	·07625	846·5	50	·05907	52
53	175	3	2	5	172·5	12	·06956			·05529	
54	178	1	3	4	171·	7	·04094			·05176	

55	179	4	3	7	175·5	7	·03989			·04845	
56	180	6	3	9	175·5	8	·04558			·04535	
57	176	1	1	2	175·	10	·05714	871·5	37	·04246	57
58	180	8	4	12	174·	8	·04598			·04029	
59	174	3	2	5	171·5	4	·02332			·03823	
60	180	3	3	6	177·	11	·06215			·03628	
61	174	7	..	7	170·5	5	·02933			·03443	
62	184	6	..	6	181·	4	·02210	887·5	29	·03268	62
63	183	10	..	10	178·	5	·02809			·03012	
64	184	3	3	6	181·	4	·02210			·02775	
65	193	13	7	20	183·	9	·04918			·02558	
66	179	8	5	13	172·5	5	·02899			·02357	
67	181	5	3	8	177·	2	·01130	874·5	19	·02173	67
68	184	18	1	19	174·5	2	·01146			·02112	
69	176	12	5	17	167·5	1	·00597			·02052	
70	168	9	2	11	162·5	2	·01231			·01995	
71	171	12	4	16	163·	5	·03067			·01939	
72	162	11	7	18	153·	2	·01307	743·	14	·01884	72
73	146	12	4	16	138·	2	·01449			·01597	
74	135	12	5	17	126·5	3	·02372			·01354	
75	124	17	4	21	113·5	1	·00881			·01148	
76	115	12	4	16	107·	1	·00935			·00973	
77	105	12	3	15	97·5	1	·01026	485·	4	·00825	77
78	96	12	4	16	88·					·00799	
79	84	8	2	10	79·	1	·01266			·00773	
80	78	11	1	12	72·	1	·01389			·00749	
81	68	11	1	12	62·					·00725	
82	58	4	1	5	55·5	..	..	285·	2	·00702	82
83	57	9	..	9	52·5	1	·01905				
84	48	10	..	10	43·						
85	43	11	1	12	37·						
86	32	8	1	9	27·5						
87	24	7	1	8	20·	..	..	105·	0	..	87
88	16	5	2	7	12·5						
89	10	4	..	4	8·						
90	6	1	..	1	5·5						
91	5	1	..	1	4·5						
92	4	4	..	4	2·	..	..	12·	0	..	92
	8,142	368	139	507	7888·5	438	..	7888·5	438		

TABLE No. III.

Age.	Probability of Marrying in a Year.			
	Bachelors.		Widowers.	
	General Population.	Peerage Families.	General Population.	Peerage Families.
15–	·00464	·00192		
20–	·11209	·04213	·30766	·17858
25–	·12209	·07700	·35791	·14910
30–	·07851	·07137	·28627	·12485
35–	·04558	·05473	·20313	·10460
40–	·02798	·03947	·14075	·09954
45–	·01448	·01977	·08858	·07715
50–	·00705	·01074	·05711	·05907
55–	·00349	·01046	·03201	·04246
60–	·00152	..	·01745	·03268
65–	·00146	..	·00862	·02173
70–	·00031	..	·00316	·01884
75–	·00059	..	·00100	·00825
80–	·00000	..	·00067	·00702

The column of "General Population" I take from my previous paper; it was derived from the number of marriages in the year 1851 according to the Registrar-General's returns, compared with the population according to the census of that year. The number of marriages in which the ages of both husband and wife were known was 56,347, and it was assumed that, in the cases of those whose ages were not known, the numbers in each quinquennial group would bear the same proportion to the total number as in the instances where the ages were given, and they were accordingly raised in that proportion. From this it appears that the first marriages of the aristocracy are at a much later age than those of the general population—the probability of marriage between 15 and 25 being less than one-half, and not much exceeding one-half between 25 and 30. At this period they become about equal, and for the remainder of life, so far as the table extends, the marriage rate of the peerage families is in excess.

The same peculiarity attaches to the second and subsequent marriages, the probability of remarriage appearing, under 40, to be only half, about 50 to be equal to, and above that age greater, in an extraordinary degree, than in the general population. The very great differences at the older ages in this class of second marriages would seem to require some observation, the distinction in rank seeming insufficient to account for it, although the attraction of a title may go some way to explain the readiness with which the aged nobility occasionally obtain second partners; and it appears to me possible that the error, if any there be, may be in the construction of the former table; for as it seems probable that

the proportion of persons in advanced years who would decline to acknowledge their ages at the time of marriage would be greater than of younger men, the distribution of the marriages of those whose ages were not given should not have been made in the same proportions.

That the marriages of the males of the peerage families take place later in life may again be shown by a comparison (see Table IV.) of the percentages of marriages at different periods of life with other tables previously published; and I allude especially to those in Mr. Brown's paper, *Assurance Magazine*, vol. vii.

TABLE No. IV.—*Proportion per Cent. of Marriages.*

Ages.	Peerage Families.	England (S. Brown).		Belgium.	Massachusetts.	Poorer Classes (St. George's-in-the-East).	Peerage Families.	1,027 Peers (Sadler).
	First and subsequent Marriages.	1846-7-8.	1851-2-3.	1841-5.	6¾ Years to January, 1857.	Statistical Society's Journal.	First Marriages only.	First Marriages only.
Under 30	53·08	76·77	75·62	52·75	75·01	85·00	65·97	62·81
30 to 45	33·98	18·31	19·22	39·93	19·54	14·06	31·63	27·75
45 to 60	9·50	4·03	4·25	6·05	4·12	·94	2·40	7·88
60 and upwards	3·44	·89	·91	1·27	1·33	0·00	0·00	1·56
	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

These Tables, with the exception of Mr. Sadler's (Col. 8), refer to both first and subsequent marriages; but as there are, perhaps, in the present observations, a larger proportion of widowers than in the others, I have added also the (Col. 7) the percentage of first marriages alone. Col. 1, then, shows the percentages of marriages according to the families of the peerage; Cols. 2 and 3 represent England as deduced by Mr. Brown; Col. 4 is from M. Quetelet's observations in Belgium; Col. 5 from the Registration Tables of Massachusetts; Col. 6 is derived from a Report of a Committee of the Statistical Society upon the poorer inhabitants of St. George's-in-the-East in 1845; and Col. 8 was compiled by Mr. Sadler, and is to be found in the second volume of his work on the *Law of Population*. It will be observed that the greatest similarity exists between the marriages in Belgium and the peerage families, for which at present I am unable to offer any satisfactory explanation; and that the greatest differences are between the two extremes in the social scale, the peerage and the poor of St. George's-in-the-East. The number of facts from which the latter table was

formed was 1,488. Mr. Sadler's facts are decidedly confirmatory of my own observations, but he has succeeded in obtaining greater numbers at the advanced ages. In his book, above referred to, he makes constant reference to his synoptical register of the peerage in support of his arguments against Mr. Malthus, but I am not aware that he has published it separately. Upon this part of my subject I need hardly, perhaps, dwell farther, and I pass on to some considerations as to the ages and condition of the ladies who became the second wives of the noble widowers.

The number of ladies whose ages at marriage were ascertained is 111 only, or 25 per cent. upon the whole number, a proportion so small that the following table cannot, I fear, be considered of much practical value.

TABLE No. V.—*Second Marriages of Widowers, with Ages of Wives.*

Ages of Husbands.	Ages of Wives.								
	15—	20—	25—	30—	35—	40—	45—	50—	55—
20—	1								
25—	1	5							
30—	2	9	1	1					
35—	4	7	5	2	2				
40—	2	8	7	4	2	1			
45—	3	3	7	1	3	1			
50—	1	1	4	..	2				
55—	..	2	3	3	1	1			
60—	..	1	..	2	..	..	1	..	1
65—	1	..	1	1	..	..	1	1	
70—	..	..	..	..	..	1			
75—									
80—									
	15	36	28	14	10	4	2	1	1

The ages of the husbands and second wives, it will be seen, are given in quinquennial groups, as before; and I would draw attention to two conclusions which may be deduced therefrom, which have some bearing on the question of assurances against issue, viz.—

1. That in no instance did the age of the second wife exceed that of her husband.
2. That in two instances alone did widowers contract marriage with ladies who had passed the ordinary period of child-bearing, assuming that to be the age of 50 years.

Of the 438 remarriages of widowers, it appears that 355, or 81 per cent., were contracted with spinsters; 81, or 18½ per cent.,

with widows; and of the remaining 2, one had been divorced from a previous husband, and the condition of the other, who was a foreign lady, could not be traced.

The duration of time between the death of a first wife and a subsequent marriage is a question of some interest, and I have accordingly constructed a table (No. VI.) of the average difference in years between the two events.

TABLE No. VI., *showing the Average Number of Years between Death of Wife and Remarriage of Widower.*

Age of Widowers.	Number of Widowers who remarried.	Number of Years between Death of Wife and Remarriage.	Average Number of Years.
20—	12	45·	3·75
25—	61	272·5	4·47
30—	62	369·	5·95
35—	72	359·	4·99
40—	66	290·	4·39
45—	49	196·5	4·01
50—	42	137·	3·26
55—	33	108·5	3·29
60—	21	76·5	3·64
65—	9	30·5	3·39
70—	7	11·5	1·64
75—	3	13·5	4·50
80—	1	·5	·50
	438	1910·	4·36

In numerous instances the period of mourning was very limited, while in two or three particular cases it might have been said, with Hamlet—

> "The funeral baked meats
> Did coldly furnish forth the marriage tables."

It will be seen that the average period between the two events is rather less than 4½ years, but there is but little regularity in the results at different ages. It is a noticeable fact, that, out of 438 second and subsequent marriages, no less than 51, or 11·6 per cent., took place within a year from the termination by death of the first; and 113, more than 29 per cent. of the remainder, were contracted in the second year. In many instances it is to be presumed that virtual divorces had previously taken place, as we could otherwise hardly conceive the possibility of a second marriage contracted within *one week*. The extreme case on the other side is that of a nobleman who remained a widower for a period of 37½ years and remarried at the advanced age of 68.

I had cherished a hope that in this investigation I might have been able to obtain some trustworthy information as to the fruitfulness of second marriages, as it has an important bearing upon the class of assurances against issue; but it would have been worse than useless to have attempted it with the few facts under observation. The ages of the parents form an important item in the investigation, and, unfortunately, there are but 111 cases in which the ages of both husband and wife are known; then, before the particulars could be complete for consideration, the duration of the marriage must have terminated by the death of one parent; and a third important consideration presents itself in the question, whether there had been children by the previous marriage. In the face of these difficulties I have, for the present, felt obliged to relinquish the attempt. Mr. Sadler, speaking generally, states that the average number of children from second and subsequent marriages is 2·75, and he observes that the children of these marriages are in a greater proportion females. How far his observations may be entitled to confidence, in the absence of the data upon which he grounded his conclusions, I am not prepared to give an opinion. I am informed that some new facts have very recently been published by the Registrar-General of Scotland, which may have an important bearing on the subject, especially as he infers that the proportion of unfruitful marriages increases with a higher rank in life; but this part of the question may well stand over for further consideration.

I may now apply some of the foregoing results to the practical question of issue premiums, and I have accordingly constructed Table VII. upon the same principle as on the former occasion, the assurance being for the sum of £1 payable at the end of the year in which a husband whose wife is now living may, after her decease, contract a second marriage. I need not enter upon the details of its construction, as they are already recorded in the *Assurance Magazine*, vol. viii.; suffice it to say, that the value of the benefit in the nth year consists of the product of the respective probabilities of the death of the wife before the end of that year, and of the survivorship of the husband, multiplied by the annual marriage rate at the advanced age—the whole discounted by the present value of £1 at the end of n years. A summation of these values for every age produces the required single premium. The elements of the table, with the exception of the marriage rate, are the same as before—namely, English Life Table No. 1—and a rate of interest £3 per cent.

TABLE No. VII.—*Present Value of £1 at the end of the Year in which a Husband shall contract a Second Marriage.*

Age of present Wife.	Age of Husband.											
	50.		55.		60.		65.		70.		75.	
	Peerage Families.	Former Observations.	Peerage Families.	Former Observations.	Peerage Families.	Former Observations.	Peerage Families.	Former Observations.	Peerage Families.	Former Observations.	Peerage Families.	Former Observations.
45	·0637	·0276	·0375	·0123	·0208							
50	·0782	·0328	·0458	·0145	·0251	·0056	·0128	·0018				
55	·1042	·0446	·0618	·0196	·0342	·0076	·0172	·0024	·0077	·0006		
60	..	..	·0842	·0276	·0473	·0108	·0244	·0035	·0111	·0009	·0039	·0002
65	..	..	..	..	·0643	·0150	·0338	·0049	·0156	·0013	·0055	·0003
70	..	..	..	..	..	..	·0456	·0068	·0215	·0018	·0077	·0004
75	..	..	..	..	..	..	..	..	·0289	·0025	·0106	·0006
80	..	..	..	..	..	..	..	..	..	..	·0144	·0008

From what has been said, it will at once have been anticipated that a very considerable increase in the rates formerly quoted has taken place, more especially for cases in which the lives are of advanced age; and since at those ages the annual marriage rate is 3, 4, and even 8 times greater, and as it enters into calculation for every year of life from 82 downwards to the present ages of the proposed assured, the extent of increase, though at first sight startling, will be seen to be entirely justified. An examination of the above table will show that more risk attaches to these transactions than, perhaps, we have lately been disposed to allow, the business having hitherto consisted chiefly of receiving premiums unaccompanied by the payment of corresponding claims; it is, however, submitted, that the limit of the risk has now been shown—the probabilities of remarriage can hardly be expected to exceed those of the peerage families, and there is still in favour of the Assurance Companies the possibility of no issue by the second marriage; in some instances also the risk is confined to the birth of male issue only, and in many there is the further chance of that issue dying before the parent, or before attaining the age of 21 years.

I have calculated, as a final table, the single premium for an assurance payable in the event of the subsequent marriage of a widower, the predecease of the wife having already taken place. This, it is considered, calls for no special remark, the case seldom occurring in practice without considerable modifications.

TABLE No. VIII.—*Single Premium for the Assurance of £1 payable on the Second Marriage of a Widower. (English Life Table; Interest, £3 per Cent.)*

Age.	Single Premium.	Age.	Single Premium.
50	·5463	65	·1471
55	·3666	70	·0879
60	·2397	75	·0384

In closing this paper I cannot but feel that its defects are numerous—that more minute details might have increased its value—and that many deductions as to the social condition of an important class might have been drawn by a more philosophical mind; but, while apologising for my shortcomings, I trust that my subject may have been one of interest, and that some little practical value may attach to my second attempt to determine rates of premium for a special class of risks.

Observations, by WILLIAM SPENS, *on Mortality Experience, supplementary to those contained in No. XLVI. of the Assurance Magazine.*

I HOPE, from time to time, to be able to give some further tables bearing on the subject, and to state how far I think subsequent experience and new information confirm or modify the interim conclusions which I ventured to submit in the present state of our knowledge; and I shall be very glad that my recent observations may receive attention, and correction where required, from critics abler than their author.

I have not observed any abstract exhibiting distinctly the general divisions of the 83,905 policies embraced by the Experience of Seventeen Offices; and, indeed, such abstract can only be made out by inferring some of the details from others. I now submit a table of this nature.

TABLE No. I.—*Abstract of Data of Experience of Seventeen Offices, embracing* 83,905 *Policies.*

	(1) Number of Policies.	(2) Total Numbers exposed to a Year's Risk at each Age.	(3) Total Deaths.	(4) Average Duration of Policies.	(5) Percentage of Mortality on Col. 2.	(6) Discontinued.	(7) Existing.
Male Lives (Town), (B1)	16,097	89,601	1,190	5·6	1·3	4,457	10,450
Female Lives (Town), (B4) ..	1,448	6,848	134	4·7	1·8	447	867
D3, deduced from	17,545	96,449	1,324	5·5	1·4	4,904	11,317
London Equitable	21,398	262,210	5,144	12·3	2·0	9,324	6,930
London Amicable	4,618	55,087	1,792	11·9	3·2	535	2,291
Other Town Experience necessarily inferred to produce next line	5,141	59,914	914	11·7	1·5		
Combined Town Experience, Table E	48,702	473,660	9,174	9·7	1·9		
Country, D4 (B2 and B5) .	13,335	72,985	1,158	5·3	1·6	3,284	9,393
Table G, deduced from	62,537	546,645	10,332	8·7	1·9		
Irish, D5 (B3 and B6)	9,236	51,621	1,446	5·6	2·8	3,038	4,752
Other Experience necessarily inferred to produce next line.	12,132	113,897	2,003	8·6	1·8		
Total Experience	83,905	712,163	13,781	8·5	1·9		
Total Experience inferred ..	17,273	173,811	2,917	10·1	1·7	4,162	10,194
						25,247	44,877

The marked excess of mortality exhibited by the experience among the females and Irish lives is very distinctly shown in this abstract, and I think if the experience had shown nothing else than this, it would have been of the greatest practical importance to Offices. The attention thus directed to the necessity of more caution in regard to these risks has, I am confident, saved many tens of thousands of pounds to Assurance Offices.

Table D3 of the experience of Offices shows the number exposed to risk of male lives (town) and female lives (town) combined. I have separated "male lives (town)," and shown them, in Table No. II., with the total expected deaths by the English Life Table No. 2,* which have been calculated in detail. I have also divided the lives at risk, and the expected deaths during each year of the assurances, as per Tables III. and IV. These do not agree with the approximate statements I have made in the previous Number of the *Magazine* in reference to deaths among "male lives (town)" during and after the first six years of the assurances, owing to a misunderstanding of the position of the "existing" in that portion of the experience tables on which the calculations were founded; and I am glad to have this opportunity of making the correction, though it does not make any alteration on my opinion of the general conclusions.

TABLE No. II., *showing, in the case of "Male Lives (Town)" of the Experience of Offices, the Total Numbers exposed to the Risk of Mortality, the Actual Deaths, and the Expected Deaths by the English Life Table No. 2.*

Age.	Number Exposed to the Risk of Mortality.	Deaths.	
		Actual.	Expected by English Life Table No. 2.
Under 20	1,027	5	6·603
20 and under 25	2,373	20	20·072
25 " 30 ...	7,463·5	51	68·914
30 " 35	13,666·5	102	139·628
35 " 40	16,647·5	151	191·565
40 " 45	15,536·5	176	208·925
45 " 50 .. .	12,680	167	202·721
50 " 55	9,200	166	177·689
55 " 60	5,825·5	145	146·434
60 " 65	3,168·5	95	117·697
65 " 70	1,443	66	75·221
70 " 75	439	32	31·647
75 " 80	106·5	11	11·447
80 " 85	24·5	3	3·849
	89,601	1,190	1,402·412

* It may be noted, that, throughout, it is the "male" table which is referred to.

TABLE No. III., *showing, in the Case of the "Male Lives (Town)" of the Experience of Offices, the Numbers exposed to the Risk of Mortality during each Calendar Year of the Assurances, and the Actual Deaths.*

Ages at Entry.	1st Year. (half.)		2nd.		3rd.		4th.		5th.		6th.		7th.		8th.		9th.		10th.		11th.	
	At Risk.	Died.	At Risk.	Died.	At Risk.	Died.	At Risk.	Died.	At Risk.	Died.	At Risk.	Died.	At Risk.	Died.	At Risk.	Died.	At Risk.	Died.	At Risk.	Died.	At Risk.	Died.
Under 20	202	1	353½	2	269½	2	207	2	166	1	134½	2	112½	..	98½	..	84½	1	66	..	51	1
20 and under 25	602½	1	999	12	774	9	616	4	516½	1	422½	3	341	4	279½	1	231	..	197½	2	159½	1
25 „ 30	1,298	6	2,259	11	1,886	17	1,600½	17	1,372½	6	1,223	10	1,048	8	891	12	748	5	634	11	523½	5
30 „ 35	1,535	8	2,717½	19	2,317	22	1,960	14	1,698	15	1,467½	19	1,279	13	1,091	20	925½	12	785	11	658½	8
35 „ 40	1,435½	11	2,496½	24	2,100	23	1,758½	9	1,520½	15	1,350	11	1,186	21	1,008	14	838½	9	711½	7	593½	5
40 „ 45	1,132½	4	2,016	22	1,695	15	1,466	22	1,253½	10	1,084	22	938	12	806	18	681	8	581½	19	476	7
45 „ 50	789	10	1,383½	11	1,170	14	991	12	844½	16	734½	16	624½	17	526	10	442½	7	378½	8	329	8
50 „ 55	500½	3	865½	13	737	27	614	13	515	9	445	17	384	11	316½	7	256½	6	218	5	186	2
55 „ 60	304	5	520	13	443½	7	374½	16	312	5	270½	6	225	9	184½	3	151½	15	122½	6	102	6
60 „ 65	134½	3	227½	11	174½	4	144½	4	124	7	101	2	87½	2	76½	7	58½	3	47½	3	41	4
65 „ 70	44½	..	72½	4	59	4	45	4	33	1	30	3	20½	1	17	2	13	1	12	1	10	1
70 „ 75	12	1	18	..	14	1	10	..	10	1	8	..	6	2	4	..	3	..	3	..	2	1
75 and upwards	3½	1	8	..	2	..	2	..	2	..	1½	..	1	..	1	..	1	1	..	..	..	..
	7,993½	54	13,931½	142	11,641½	145	9,789	117	8,367½	87	7,272	111	6,253	100	5,299½	94	4,434½	68	3,757	73	3,132	49

TABLE No. III. (continued).

Ages at Entry.	12th.		13th.		14th.		15th.		16th.		17th.		18th.		19th.		20th.		21st.		22nd.		23rd.		24th.		TOTALS.	
	At Risk.	Died.	At Risk.	Died.	At Risk.	Died.	At Risk.	Died.	At Risk.	Died.	At Risk.	Died.	At Risk.	Died.	At Risk.	Died.	At Risk.	Died.	At Risk.	Died.	At Risk.	Died.	At Risk.	Died.	At Risk.	Died.	At Risk.	Died.
Under 20	38	..	29½	..	17	..	8	..	5	..	5	..	2	..	..	..	..	..	..	..	..	..	..	..	..	..	1,849·5	12
20 and under 25	129½	1	96½	..	45	1	20	1	9	..	5	1	2	..	1	..	1	..	1	..	1	..	..	..	..	..	5,450	42
25 „ 30	440	3	350½	2	211	..	107	2	59	1	37	1	22	..	16	..	8½	..	6	..	3	..	1	..	..	..	14,744·5	117
30 „ 35	556	8	450½	11	296	2	150½	1	85	2	47	1	34	..	20	1	12½	..	9	..	7	..	4	..	4	..	18,109·5	187
35 „ 40	507	4	427	9	279	3	146	..	87	3	39	..	28	..	20	..	19	1	13	1	11	..	10	..	7	..	16,591·5	170
40 „ 45	390½	6	318	6	222½	3	121½	3	71	..	39½	5	19	1	13	..	9	..	8	..	5	..	4	1	3	..	13,353·5	184
45 „ 50	266	6	221	9	133	3	66	3	42	1	20	1	16	2	10	..	10	..	8	..	7	..	6	..	4	..	9,022	154
50 „ 55	157½	4	122	6	78	4	54	3	27½	2	15	1	5	..	4	1	3	..	2	..	1	..	1	..	1	..	5,509	134
55 „ 60	69½	3	44½	3	25	..	18	2	12	1	7	..	4	2	2	..	1	1	..	..	..	..	..	..	..	..	3,193	103
60 „ 65	28	1	20	3	12	..	7	1	4	..	2	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1,290	56
65 „ 70	7	..	6	..	4	1	2	..	1	..	1	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	378·5	23
70 „ 75	1	..	1	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	93	6
75 and upwards	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	17	2
	2,590	36	2,086½	49	1,323½	17	700	16	402½	10	217½	11	133	5	86	2	64	2	47	1	35	..	26	1	19	..	89,601	1,190

TABLE No. IV., *showing, in the Case of the Lives at Risk in Table III., the Expected Deaths by the English Life Table No. 2.*

Ages at Entry.	1st Year. (half.)	2nd.	3rd.	4th.	5th.	6th.	7th.	8th.	9th.	10th.	11th.	12th.	13th.
Under 20	1·343	2·421	1·851	1·450	1·196	1·026	·881	·790	·702	·568	·449	·341	·270
20 and under 25	5·096	8·606	6·803	5·516	4·714	3·932	3·237	2·714	2·292	2·004	1·653	1·374	1·054
25 „ 30	11·955	21·224	18·091	15·682	13·750	12·529	10·991	9·572	8·201	7·150	6·082	5·264	4·329
30 „ 35	15·649	28·351	24·775	21·381	19·009	16·887	15·149	13·298	11·687	10·238	8·869	7·736	6·482
35 „ 40	16·491	29·511	25·555	22·152	19·762	18·128	16·478	14·491	12·480	10·985	9·522	8·452	7·391
40 „ 45	15·237	28·071	24·440	21·884	19·402	17·404	15·634	13·958	12·249	10·883	9·251	7·889	6·715
45 „ 50	12·633	23·001	20·201	17·782	15·775	14·257	12·619	11·097	9·842	8·970	8·343	7·284	6·556
50 „ 55	9·645	17·341	15·379	13·496	12·046	11·137	10·314	9·201	8·046	7·337	6·712	6·107	5·083
55 „ 60	7·565	13·972	12·926	11·762	10·569	9·815	8·759	7·783	6·900	6·038	5·412	3·970	2·684
60 „ 65	4·924	8·984	7·367	6·508	5·970	5·215	4·867	4·562	3·702	3·243	3·022	2·224	1·723
65 „ 70	2·247	3·902	3·398	2·797	2·191	2·146	1·574	1·392	1·141	1·133	1·020	·752	·689
70 „ 75	·919	1·499	1·242	·957	1·032	·892	·719	·523	·440	·471	·322	·149	·160
75 and upwards	·399	·359	·232	·249	·268	·218	·160	·171	·183	..	..	..	..
	104·103	187·242	162·260	141·616	125·684	113·586	101·382	89·552	77·865	69·020	60·657	51·542	43·136

Ages at Entry.	14th.	15th.	16th.	17th.	18th.	19th.	20th.	21st.	22nd.	23rd.	24th.	TOTALS.
Under 20	·160	·076	·047	·048	·021	..	..	..	..	..	..	13·640
20 and under 25	·497	·228	·107	·060	·025	·013	·013	·013	·014	..	..	49·965
25 „ 30	2·699	1·419	·809	·523	·323	·245	·131	·095	·066	·019	..	151·149
30 „ 35	4·410	2·324	1·356	·774	·585	·356	·236	·178	·144	·086	·091	210·051
35 „ 40	5·037	2·736	1·691	·789	·593	·445	·452	·327	·300	·292	·227	224·287
40 „ 45	4·979	2·893	1·840	1·107	·582	·429	·320	·303	·203	·177	·145	215·995
45 „ 50	4·270	2·313	1·597	·825	·710	·485	·524	·460	·429	·391	·276	180·640
50 „ 55	3·506	2·575	1·462	·852	·296	·248	·205	·138	·072	·077	·083	141·358
55 „ 60	1·627	1·246	·907	·553	·353	·202	·112	..	..	..	..	113·155
60 „ 65	1·117	·703	·433	·243	..	..	..	..	..	..	..	64·807
65 „ 70	·500	·268	·149	·160	·171	..	..	..	..	..	..	25·630
70 „ 75	·171	..	..	..	..	..	..	..	..	..	..	9·496
75 and upwards	..	..	..	..	..	..	..	..	..	..	..	2·239
	28·973	16·781	10·398	5·934	3·659	2·423	1·993	1·514	1·228	1·042	·822	1402·412

I beg now to submit, to be substituted for similar statement in the middle division of Table VII. of my paper in No. XLVI., the following—

TABLE No. V., *showing, in reference to "Town Males," the Deaths Actual and Expected by English Life No. 2, during first six Years and thereafter; also the Percentage of the Deaths, Actual and Expected, on the Lives exposed to Risk, and of the Actual on the Expected.*

Years of Assurance.	Expected Deaths by E. L. No. 2.	Percentage of Expected on Lives at Risk.	Lives at Risk.	Actual Deaths.	Percentage of Actual on Lives at Risk.	Percentage of Actual on Expected Deaths, E. L. No. 2.
1st (half of)....	104·103	1·302	7,993·5	54	·675	51·871
2nd	187·242	1·344	13,931·5	142	1·019	75·837
3rd	162·260	1·393	11,641·5	145	1·245	89·362
4th	141·616	1·446	9,789·	117	1·195	82·617
5th	125·684	1·502	8,367·5	87	1·039	69·220
6th	113·586	1·561	7,272·	111	1·526	97·723
7th (half of) ..	50·691	1·621	3,126·5	50	1·599	98·637
First six years..	834·491	1·414	58,995·	656	1·111	78·611
Total after six..	567·921	1·855	30,606·	534	1·744	94·027

It thus appears, that while the Scottish Amicable male lives, non-hazardous, show still a higher percentage of excess of expected deaths during the whole experience and during the first six years, after the first six years the "male lives (town)" show a larger percentage. Thus:—

TABLE No. VI.

Deaths.	MALE LIVES (TOWN).			SCOTTISH AMICABLE MALES NON-HAZARDOUS.		
	First Six Years.	After First Six Years.	Total.	First Six Years.	After First Six Years.	Total.
Actual	656·	534·	1190·	322·	310·	632·
Expected by English Life No. 2	834·491	567·921	1402·412	461·334	315·659	776·993
Excess of Expected Deaths	178·491	33·921	212·412	139·334	5·659	144·993
Percentage of Excess on Actual	27·20	6·34	17·84	43·27	1·82	22·93
Percentage of Actual on Expected	78·61	94·02	84·85	69·79	98·20	81·34

I shall be very glad if it should be found that we may anticipate a less rate of mortality amongst select assured lives after six years than the English Life Table No. 2 indicates; but while the "male lives (town)," when thus dissected, do exhibit a more favourable view, yet if we take the Equitable "male lives (town),"

and Scottish Amicable male lives non-hazardous, together, the agreement is very close, and the probability is that there will not be found to be any material error in the assumption, especially for calculating the values of annuities.

The expected deaths after the first six years, by the English Life Table No. 2, are—

For "male lives (town)"	567·9
„ Scottish Amicable non-hazardous males .	315·6
„ Equitable	4,067·3
	4,950·8

While the actual deaths are—

"Male lives (town)"	534
Scottish Amicable non-hazardous males . .	310
Equitable, in one view	3,936
	4,780
Adding to the Equitable in another view, as explaiued in Number XLVI. of the *Magazine* .	160
	4,940

According to the principles on which Table VI., in former Number was calculated, a correction might also be made on Column No. 3 of that table (percentage of mortality, "town males," first six years of assurances). This would cause a small deduction from the percentages, and, to a very trifling extent, would affect the values of annuities for "town males" under Table VIII. in said paper. It is not, however, worth while making these alterations, especially as the experience founded on gives only calendar years, and not actual years of assurance. The rates of mortality, therefore, for first six years, in any view, can be only approximate, and other approximations might give higher percentages. It is hoped that it will now be apparent that it is very desirable to have experience data so framed as to exhibit the experience of actual years of assurance, and not merely calendar years, especially as many Offices, it is understood, do a much larger business at the close of the year. Even, however, if the business were regularly distributed over the year, it is not satisfactory to deduce the mortality for the first year of assurances, either by taking double the first calendar year or adding to the mortality of the first calendar year half of that for the second year, and so on. In later years such a mode could make little or no difference in results; but, independent of the marvellous difference represented to exist in the London Equitable Experience in

the first and second calendar years of entry (which may probably admit of some explanation), it is sufficiently obvious otherwise that the advancing rate of mortality in the early periods of assurance is so strongly marked as to require at least exact divisions of the years of the duration of assurances to enable the facts to be sufficiently accurately ascertained and exhibited.

Among assurance statistics I have not observed any probable estimate of discontinuances otherwise than by death in each year of the assurances.

The following is a

TABLE (No. VII.) *showing the Percentage of Discontinuances in various Years of Assurances according to the Experience of the "Male Lives (Town)" of the "Experience of Offices" and the Scottish Amicable Male Lives Non-hazardous—the Discontinuances being, in the case of the former, reckoned as Policies discontinued in each calendar year of assurance out of Policies at commencement of year (or at entry in the case of first calendar year), and, in the case of the Scottish Amicable, being reckoned as years of risk discontinued in each year of assurance out of number at risk for the year plus such years of risk discontinued; also Rough Estimates of the probable Discontinuances in each Year of Assurance out of* 100 *at commencement of each year, and out of* 100 *at entry.*

Year of Assurance.	Discontinuances out of 100 at commencement of each Year.			Roughly Estimated Discontinuances out of 100 at Entry.
	Scottish Amicable Male Lives Non-hazardous.	Town Males.	Roughly Estimated.	
1	1·3	·7	1	1
2	9·8	9·9	10	9·9
3	7·1	6·7	8	7
4	5·5	5·8	6	5
5	5·1	3·7	5	4
6	4·7	3·9	4	3
7	4·1	3·5	3	2
8	4·3	6·0	2·5	1·5
9	2·8	2·3	2	1·3
10	2·3	1·7	1·9	1·1
11	3·7	1·7	1·8	1·0
12	2·7	1·8	1·7	·9
13	1·7	1·4	1·6	·80
14	2·7	1·4	1·5	·74
15	2·6	1·7	1·4	·68
16	1·2	0·2	1·3	·62
17	2·2	0·4	1·2	·56
18	1·2	0·0	1·1	·51
19	1·	0·0	1	·46
20	1·6	3·0	1	·43
				42·50
21 and after	..	..	1	7·5
				50·

It will be seen from the preceding table that I estimate that about one quarter of the policies issued are discontinued otherwise than by death in the first five years, and about a like number after that, or one-half of the whole. This includes short period policies which have run their term, but these must be a very small proportion, and the number otherwise discontinued is obviously very large. No doubt the estimate will be high for the experience of some Offices, but taking in all Assurance Offices, I should not consider it an over estimate according to past and present experience. It is to be hoped that, both as an index of the general advance of provident habits and of greater appreciation of the advantages of life assurances, future experience may establish a smaller proportion of discontinuances.

Notes on the Early History of Tontines. By Frederick Hendriks, *Esq., Actuary to the Globe Insurance Company.*

In the third of my papers entitled "Contributions to the History of Insurance and of the Theory of Life Contingencies,"* after treating upon the earliest examples which, after a somewhat tedious search, I could find of tontine associations practically carried out—examples confined, as it will be recollected, to some comparatively small projects of the *Dutch* municipalities of Kampen, Middelburg, and Groningen, employed for raising capital sums, by the grant of tontine life annuities, in 1670 and 1671—the circumstance was mentioned, that the credit of the invention was properly accorded to the Italians, or more particularly to the Italian *Tonti*, whose name has been so effectually incorporated into several languages through the word "tontine."

Upon the occasion referred to (writing in 1852), I mentioned that it would be desirable to ascertain whether previous tontine annuity schemes—as, for instance, those instituted by Tonti, in France or elsewhere, and prior in date to 1670—included any tables assimilating to that of the Kampen tontine of the same year.

Although the special object of this note was not stated, it may be mentioned that it was as follows:—The sixth column of the table of the Kampen tontine showed an estimate of the number of persons who, from the age of 1 to 60 years, in each twelve years, and thenceforward for each year up to the age of 80 years, are assumed

* *Vide Assurance Magazine and Journal of the Institute of Actuaries*, vol. iii., pp. 93-120, and particularly § 29, p. 111, and Note, p. 116.

to remain still in existence. The projectors of the tontine stated (with reference to this column) that they are aware that anything like a precisely accurate calculation of the proportion dying out of a given number is not attainable, but "the assumed numbers had been framed upon a new elaborate scrutiny and observation, and there was no ground for assuming that the results to be experienced would range either below or above the anticipated number of survivors." Upon this I suggested in the context, that the idea of a *life table,* arranged in the form of the number surviving to a given age out of a specified number living at any age, *originated,* roughly though it may have been, in these tontine annuity schemes.

Although ten years have elapsed since this suggestion, such researches as have been made hitherto for the particulars of Tonti's scheme have not enabled us to discover them. If they do still exist, it is probably in a manuscript form in some of the French State archives. In the course, however, of these inquiries, details respecting the biography of Tonti have come under my notice which are not only interesting in themselves, as supplying information not obtainable from English sources, but as establishing the fact that it was not until 1689 that any tontine scheme was successfully carried out in France, although it would appear to have been projected before 1648.

The letters of Tonti to Colbert still remaining in the French archives were brought to light for the first time in France in 1852; but buried as they are in the vast mass of ancient administrative correspondence published by the Government, it will be useful to reproduce them in a more easily available form. I have, therefore (see Appendix A) translated them, and also the excellent prefatory remarks of the late M. Depping; and I am not without hope that they may serve to indicate to future writers, who, in biographical or politico-economical collections, have to refer to Tonti, that more may now be said of his career than the meagre notice, which is all that can be usually found respecting it, as to his place of birth having been Naples, and as to the tontine system having been named after him.

In a paper read to the Statistical Society in May, 1857,* I took occasion to point out that the writings of an old and important author on economical science, John Houghton, showed how early in England, upon the introduction of the tontine system, was the appreciation of the sketch of a table of mortality given by Graunt

* "On the Statistics of the British Land Tax Assessment," *vide* Journal of the Society, vol. xx.

in his well-known *Observations upon the Bills of Mortality*, the first edition of which appeared in 1662. Houghton was an apothecary in the city of London, a Fellow of the Royal Society and a contemporary of Halley, Evelyn, Pepys, and Sloane, who all had a high opinion of his talents and contributed to the serial publications which he issued, in twopenny and penny numbers, between Sept., 1681, and Sept., 1703. The mass of these publications appeared under the title of *Collections of Letters for the Improvement of Trade.* Such of them as had appeared in *folio* (omitting the advertisements, the wind and rain tables, the stock and share lists, the weekly prices of corn in various parts of the country, the prices in the provision and other markets, and a condensed notice of special news from the *London Gazette*) were republished, twenty-four years after the date of the last paper, by Richard Bradley, Cambridge Professor of Botany (3 vols. 8vo.; London, 1727). In this republication Bradley notices that he "had the greatest pleasure in the opportunity of restoring to the world Mr. Houghton's useful writings, which, in all probability, would never else have been again published; and" (he continues) "it is very likely that there are not, in all our English libraries, ten complete sets; for, as they were published in single papers, many of them were undoubtedly lost, and the few that are now left are esteemed as valuable as choice manuscripts."

Bradley was either unacquainted with, or else could not obtain, the papers which Houghton had published in a *quarto* form between 1681 and 1693. In one of these papers (No. 13, for Tuesday, Feb. 13, 1683), entitled "Some Considerations upon the Proposals approved of by the City of London for Subscriptions on Lives, wherein are some Observations and Conjectures upon the East India Company and Bankers," the tontine plan is so well described, and the statement it contains of Graunt's Table of Deaths, in decades of age, as applicable to determine the "reasonable" expectations of survivorships, is so important—preceding in date, as it does, the more learned mathematical researches of Dr. Halley on the Breslau Table—that it cannot fail to be interesting to the actuarial as well as the general readers of the *Assurance Magazine*.* It is with this view here reproduced from my printed copy. (See Appendix B.)

* The identification, by Houghton, of Petty's labours with Graunt's, where he observes that "Major Graunt, or rather that learned and ingenious virtuoso, Sir William Petty, in his admirable *Observations of the Bills of Mortality* of London, &c... This great man saith," &c., may appear to lend some colour to the early origin of the question which Mr. Hodge and Professor De Morgan discussed in this *Journal*, as to which of the two (Graunt or Petty) was the real author of the celebrated *Observations;* but I am inclined to the same conclusions as Professor De Morgan, that Graunt was *the* man, and that Sir William Petty may have very materially assisted him.

APPENDIX A.

The following notes and correspondence are extracted from *Correspondance Administrative sous le Règne de Louis XIV., entre le Cabinet du Roi, les Sécrétaires d'État, &c.* Recueillie et mise en ordre par G. B. Depping. (Paris; Imprimerie Impériale, 1852; 4to.) See volume iii., "Affaires de Finances, Commerce, Industrie:" Introduction.

M. Depping remarks upon its being evident from Tonti's letters to Colbert, that it was under the ministry of Cardinal Mazarin that Tonti came to Paris and offered his plan of a tontine to the Government. The Cardinal understood, or rather exaggerated to himself the advantages of this financial enterprise, and fancied he perceived in it millions of profit for the State. The tontine was adopted, and the inventor received a pension of 6,000 livres, which proves to what degree the Cardinal's mind had been struck by the importance of the financial ideas of his countryman. The latter, in his letters to Colbert, says that he received this pension from 1648 to 1660; that in the following years he received no more than trifling instalments. From that time his letters are filled with lamentations upon the discomfort in which he was living with a family of seventeen or even nineteen persons. He avows his miserable condition, expresses his fear of being cast into prison by his creditors, and implores the pity of the king and of the minister—conjuring the latter, by the name of Mazarin, their former common protector, to come to his help. These complaints are very earnestly expressed in the years 1663, 1664, and 1665—that is to say, at the very time when the royal edicts confirmed the establishment of the tontine as a State institution. He asks for Madame Colbert and the queen to interest themselves in his daughters '*grandes et bien faittes*,' as he says, in order that they may be admitted into convents. His embarrassments do not prevent him from meditating upon other projects useful to France. He proposes the establishment of a new India Company, which would procure for the State a fund of 40 to 50 millions; he advises plantations of mulberry trees upon all the great roads of the kingdom, so as to dispense the population from having recourse to foreign silks. Then one learns from his correspondence, that he had written, in 1664, a relation of the present conduct of the court of France; that the chancellor, Le Tellier, in the king's name, had prohibited him from publishing this work, which, nevertheless, in spite of himself, so it is asserted, a bookseller caused to be printed.*

We are ignorant of what occurred concerning him during the subsequent years; it is a question whether this work or some other indiscreet act drew down upon him disgrace at court. It is not in my power to answer. Tonti disappears from the scene of this world, and I confess that I have been extremely surprised to find a letter of his dated from the Bastille, 4th March, 1675, and conveying the information that he had been languishing for seven years in that prison with his two sons, and that his

* "Last day of August, 1665. I humbly supplicate your Excellency to obtain for me, from the king, the justice for which I have appealed to His Majesty, against a bookseller who has had the effrontery to cause to be printed the account of the present conduct of the court of France, which I composed last year, and which His Majesty had commanded me, for certain reasons, through M. Le Tellier, not to have printed. I hope for this favour through the goodness of your Excellency." (*Vol. Verts, C.*)

family, from whom he was cut off, was living in the profoundest misery. The unfortunate man does not ask for his liberty; he knows, perhaps, that he will not obtain it; he only implores Colbert, in the name of God, and by his veneration for the memory of Cardinal Mazarin, to continue to have pity upon him, and not to leave those belonging to him without help.*

From this period the traces of his existence vanish. The Bastille has presented more than one mystery of such a kind. It is only known that one of this Tonti's sons became a distinguished officer and signalised himself in the expeditions against the savages in America. It is strange that the historians of Louis XIV. and contemporary memoirs preserve silence respecting the fate of the unfortunate Italian, who probably had more ideas than experience, more dash (*plus d'abandon*) than prudence.

His plan of a tontine was, besides, conceived more in the interest of Government than of private persons investing in it. It was a voluntary loan which he desired to effect through its means, by offering to lenders the temptation of a large future interest. One is forced to believe that the public was not taken by it, and that it was from mistrust that the French—who, in general, at that period, were not at all used to risk their funds in speculations of chance—showed but little eagerness to enter into the tontine association. The first which Tonti proposed, in 1653, was not carried out; the second, which was brought before the public three years subsequently, under the name of the 'Royal Bank', and which concealed a lottery, but wherein the Parisians persisted in seeing another tontine, was, by virtue of royal letters patent, registered, and had a commencement of organisation, but was not supported by the investments of the public. Untired of invention, the Neapolitan economist would soon have carried out another project, that of the ecclesiastical tontine, to furnish funds with the view of extinguishing the debts of the clergy, which was again a loan. No one allowed himself to be taken with it, and as the poor inventor was languishing in prison, there was no further question of tontines. But in the sequel, when the embarrassments of the treasury obliged recourse to all kinds of expedients, the Count de Pontchartrain wished also to try and revive the tontine—that is to say, to procure funds for the State by means of the plan originally conceived by Tonti. In 1689, therefore, there apppeared a royal edict for the organisation of a tontine, in which each person, by depositing a sum of 300 livres, might assure, for his old age, a rent increasing with age. This, at least, was completely enough carried out to encourage the Government, some years after the first edict, to found a second tontine upon the same plan. If it did not have all the success hoped for from it, the two associations contributed at least to diminish a little the distress of the treasury.

The letters referred to by M. Depping are the six following :—

* "Bastille, 4th March, 1675. I return you a million thanks for the aid of 600 livres that your Excellency has obtained for me from the king, which have been used in procuring for me clothes and linen, as also in giving like articles to my two sons, who are confined in this place with me. I hope also that your Excellency will procure for me from His Majesty, when the good God shall inspire him to do so, the 1,600 livres which I owe to the people of this castle, who have furnished me, for seven years, with the things which were necessary for me and likewise for my sons; and nevertheless, Monseigneur, I implore you, in the name of God and by the memory of the late Monseigneur the Cardinal Mazarin, to continue to me your bounty for the subsistence of my daughter, who has the charge of the rest of my family, which is reduced to the last stage of distress." (*Vol. Verts, C.*)

(1) TONTI *to* COLBERT.

"*Paris*, 19*th January*, 1663.

"M. Le Tellier has this morning told me that the King had commanded him to return you my petition, similar to the one which is hereto annexed, which I presented to His Majesty a few days ago; and he has ordered me to see you thereon. You will recollect, if you please, Sir, that you had led me to hope for some relief if the matter depended upon you; and now that it is in your hands by order of His Majesty, I very humbly beseech you to do me the favour, when you speak to him about it, to represent to him the deplorable state to which I am reduced, with a numerous family of seventeen persons, by having sacrificed myself for his service, and by the great losses I have undergone through the same cause. And, if my past services are no longer to be considered, I entreat you to submit it to His Majesty that I am on the point of perishing in a prison, and of being denounced; and should this take place it would bring a notable harm to the tontine business, which the late Cardinal esteemed as a mine of gold for the King, and that there might be drawn from it every year several millions which would never be subject to redemption. I hope, by your goodness, and your zeal in the King's service, to receive from His Majesty, through your medium, a prompt help to solace me in my afflictions, and to remain for ever obliged to you for it."

(2) TONTI'S PETITION TO THE KING.

"Laurens Tonty humbly beseeches your Majesty to consider that he has been, and still is, obliged to go to much expense for the advancement and establishment of the tontine, delayed on account of the Chamber of Justice; and that he has received nothing from your Majesty for the years 1661 and 1662 of the pension of 6,000 livres a year which the late Cardinal Mazarin caused to be given him, by order of your Majesty, in consideration of the great services he had rendered to your Majesty, both within and without the kingdom. And petitioner finding himself burdened with a family of nineteen persons, and, amongst others, with five grown up and well favoured daughters, it is altogether impossible for him to be able to subsist any longer if your Majesty should not have the goodness to order the said pension of 6,000 livres to be paid up for the past years. M. du Plessis Guénégaud, Secretary of State, can testify to your Majesty that he has issued, by order of His Eminence, every year, down to the year 1660, the ordinances for payment of the current instalments of the said sum of 6,000 livres, which were paid to him by the treasurers of the savings fund. And will, in duty bound, pray God for your Majesty's health and prosperity."

(3) TONTI *to* COLBERT.

"1*st March*, 1663.

"I return you a million thanks for your goodness in causing the second edict for the tontine to be sealed; and I hope that, through your ministry, the King will soon see this business established, to draw from it the great advantages which may be expected immediately and prospectively. Following your advice, I have spoken to the King, supplicating him very humbly to have mercy upon me, by having continued to me the help which the late Cardinal accorded me, as much in consideration of the services I had rendered to the State, and the loss of property I have endured in

sacrificing myself for the crown, as of the judgment which His Eminence had formed upon the tontine affair as a hidden treasure in this kingdom. But I hope everything from your protection; and as you have graciously told me that this affair (the tontine) is a good one, I conjure you, in God's name, and by the memory of His Eminence, to inspire the King with the same sentiments respecting it as you have; and, at the same time, to do me the service of representing to him the trouble in which I am, with a family of nineteen persons, as also that I am continually forced to be at expense and care in perfecting this affair, for which I am not in a competent condition without the help of His Majesty, and which I entreat you, Sir, very humbly, to procure for me, and shall for ever be obliged."

(4) TONTI *to* COLBERT.

"19*th October*, 1663.

"The Marshal de la Meilleraie had decided upon the tontine affair as extremely good and very advantageous for the province of Brittany, but considered that you could not wait for the issue of the affair, seeing that the States would have to pay every month a portion of the money for which they were bound for the King's buildings, and that another fund would have to be created for the tontine, which is impossible for them at present, on account of the engagements which the said province has been obliged to undertake, finding the whole of its means alienated for four or five years. That which consoles me for not having been able to succeed in that country is the order which the King has caused to be conveyed to me through M. de Lionne, not to send this (tontine) affair to Rome, where it would have infallibly succeeded, to the prejudice of His Majesty; and, if it had been put into operation in Brittany, would by this time be published throughout the country.

"I took the liberty, within the last few days, to give you occasion to beg Madame, your wife, to be good enough to contribute, by her interest and endeavours, to give two of my daughters as spouses to Jesus Christ, and they will pray all their lives for the health and prosperity of all your family.

"I have submitted a petition to the King, humbly begging him to consider, that for three years I have only received 3,000 livres of the pension of 6,000 livres a year which His Majesty had caused to be paid me from the year 1649 down to 1660, in consideration of my services; and as I am pursued by my creditors, and am bound to give honourable subsistence to my family of seventeen persons, according to my position, I have had recourse to His Majesty to receive of his goodness the wherewithal to remedy my present necessities. I very humbly entreat you to support it with your protection, and to continue to me your favours, which will secure my lasting obligation."

(Accompanying this letter is a note addressed by Tonti in favour of his daughters, "*grandes et bien faittes*," for the placing of whom in a convent he wishes that the Queen would interest herself.)

After the lapse of a year, Tonti was again driven to solicit payment of his pension. He wrote to Colbert on the 27th November, 1664:—"You will remove your favouring protection from a family of condition which is very numerous, and for whom it is impossible to find further means of

subsistence, as it is on the point of seeing me perish in prison, if your Excellency does not procure it some prompt assistance from His Majesty out of what is due to me for the past, and by the restoration of the pension which has been paid to me from the year 1648 down to 1660. I conjure this most pressingly, by the glorious memory of his late lordship the Cardinal Mazarin, by your own individual generosity, and by the respectful submission with which I am," &c.

(5) TONTI *to* COLBERT.

"It has occurred to me, that, after the approbation with which your Excellency has honoured the tontine, and after the evidences which you have given of wishing for its establishment, your Excellency would not disapprove of the liberty which I take to disclose to you an expedient which has come into my mind, and which I submit to your judgment and good pleasure in the contingency of the redemption of the rents of the Hotel de Ville. The majority of those who will have received their money will experience difficulty in investing it safely and usefully—being possessed, perhaps, of that alone for their subsistence—and will be rejoiced to place it advantageously. To which end it has seemed to me that the establishment of the tontine will be most appropriate, by giving, through His Majesty, to those who voluntarily consent to it, a quarter more in places in the tontine for the whole of their redemption, or for a part of it, of that which it shall be fixed to give them in ready money at their own choice and liberty. Wherein the advantage would be reciprocal, inasmuch as, instead of a small fixed sum which the proprietors of the said rents accepting the said places would receive, after many difficulties raised by the officials to delay the payment, they would have their revenue at 7·14 per cent. (*au denier* 14), and would see it increase every year. And in order to compensate for the loss of their funds in case of death, they would put themselves into a position, by this voluntary acceptance of places for a whole or part of their redemption money, to enjoy, previous to their death, sums to the extent of 100,000 livres of revenue, which would exceed not only the principal, but also the rent of their money in some traffic or legitimate trade in which they might employ it; and the great advantage to His Majesty would be, that, without opening his purse, he would inherit the revenues of each class by the death of the last survivor in it, and would thereby find himself relieved from payment of the interest of the places without redeeming the funds.

"It is for your prudence, my Lord, to decide this, and for the perfect knowledge which your Excellency has of the equity or possibility of things and of the proper time to put them into practice; the introduction only of the subject have I thought it open to me to make with the submission which is due," &c. &c.

(6) TONTI *to* COLBERT.

"22*nd May*, 1666.

. . . . "I believe there is an easy means whereby to procure fresh funds of 40 to 50 millions. It should be managed so that all persons in the kingdom—nobles, officers, and others—will ask to subscribe, and with joy, even when they might be sure to lose their investment, by reason of the great advantages they would otherwise derive. If the whole of this great fund cannot be at once employed in commerce, the King might make

use of a part to redeem his domain, and for the execution of other designs. This might be done without its being known; and should it be known, no complaint against it would be brought forward, as interest would be paid to the Society.* By the same means everyone would speak well of the Society, and would procure its preservation and augmentation, instead of as at present, everyone speaking ill of it. Officers particularly complain that they were constrained to enter it—that it is a trap to catch the money of those who have invested in it. To shut the mouths of the dissatisfied, if your Excellency judge it to be expedient, after this great fund shall have been created, one might make an offer to all those who had already entered the Society, to return them their money if anyone regretted having done so. Not a single person would withdraw, because of the great advantages he would derive through my expedient. If the proposition be agreeable to your Excellency, I will also present the methods of showing the facility with which it may be carried out."

Appendix B.

A Collection of Letters for the Improvement of Husbandry and Trade.

Numb. 13.—Tuesday, February 13, 168⅔.

The contents:—Some Considerations upon the Proposals approved on by the City of London for Subscriptions upon Lives, wherein are some Observations and Conjectures upon the East India Company and Bankers.

Sir,—In my last I gave you an account of the great undertaking, to promote trade by a bank, approved on by the City of London. Now I must give you an account of another great affair, carryed on by the same persons, and 'tis printed in a large sheet by them as followeth, viz.:—

"Proposals made and approved by the City of London for a yearly increase of wealth, by subscriptions, to advance money at interest, for lives, of whatsoever age or sex, under ten several ranks or classes; which subscriptions will produce great advantage to the survivours, as is particularly instanced in the schemes and paragraphs following, viz.:—

	The particular Summs to be subscribed.	The Number of Persons requisite for each Rank.	Total Summe to be Subscribed by each Rank.	The Yearly Interest thereof.
1	5	2,000	10,000	600
2	10	1,000	10,000	600
3	20	500	10,000	600
4	25	400	10,000	600
5	50	200	10,000	600
6	100	100	10,000	600
7	200	50	10,000	600
8	400	25	10,000	600
9	500	20	10,000	600
10	1,000	10	10,000	600

* M. Depping states that this has reference to the organisation of a grand India Company, a project upon which Tonti does not give further explanation.

The Subscribers' Names, Qualities, Sexes, and Places of Habitation.	1. Rank of Persons under 7.	2nd. Between 7 and 14.	3rd. Between 14 and 21.	4th. Between 21 and 28.	5th. Between 28 and 35.	6th. Between 35 and 42.	7th. Between 42 and 49.	8th. Between 49 and 56.	9th. Between 56 and 63.	10th. Between 63 and 70.
A. B., of Cheapside, London, Goldsmith, for his son John....	5									
The same, for C. D., of Cornhill, Gent.	..	..	..	5						
E. F., of Paternoster Row, Mercer, for himself	..	..	..	..	..	..	..	5		

"By which schemes it appears there is £10,000 to be subscribed on each rank or class of summs and persons, the interest of which comes to £600 per annum. But no subscriber shall be held obliged to pay his money until the said summ of £10,000 be fully underwritten—(that is to say) if he have subscribed £5, until £1,999 other persons of the same rank or class (as to age) have each of them subscribed £5 to make up the same £10,000; or, if he have subscribed £50, until 199 other persons of the same rank or class (as to age) have each of them subscribed £50.

"Every person for whom a subscription shall be made, shall, from the time his money is paid, receive interest after the rate of six pounds per cent. per annum; and as any of those that are in the same rank dye, the survivours of that rank shall receive the interest money that should have been paid to the deceased, equally divided amongst them: (that is to say) if one of the ten subscribers for £1,000 each man, do dye, the other nine that shall survive shall receive, besides the interest for their own £1,000 (each) subscribed (which is £60 per annum), their equal share of that £60 which would have accrued to the deceased, and so of the rest.

"Every subscriber, when he comes to subscribe, shall declare the age or ages of the persons for whom he doth subscribe—(viz.) whether they be under the age of 7 or above 7, and under the age of 14 or above 14, and under the age of 21 or above 21; and so of the rest of the said ranks. And if any person underwriting, either for himself or any other, shall declare and subscribe the person for whom such subscriptions shall be made to be of any other age than by the rules or instances aforementioned are allowable to pass in such rank or class as he shall underwrite for, such subscriptions, and the moneys thereupon paid, shall be forfeited, and go to the rest of that rank.

"If any person shall underwrite the first five summs above mentioned, which amounts to but £110, and enter himself, or those for whom he underwrites, according to their respective ages, in five several ranks or classes, he hath thereby a possibility of receiving five times £600, which is £3,000 per annum, during his life, if he survive the rest. And so if any shall underwrite (in like manner) all the ten summs first abovementioned, which amounts to £2,310, he hath like possibility of receiving ten times £600 per annum during his life, if he survive the rest.

"Every person, when he comes to demand any interest money, must bring certificate from the trustees of the respective rank or class in which

the subscription was made, that the person upon whose life the money is demanded was alive at the time when the interest by him demanded became due; but if no demand be made of any person's interest by the space of three years successively, such persons shall be held as dead, and his interest for the said three years, and until he shall afterwards appear to make demand, shall be lost to him, and divided among the rest of the subscribers; which said trustees shall, for the first year, be nominated and chosen by the major part of the subscribers in each rank respectively, as soon as the said rank or class shall be filled up, and afterwards annually.

"If any person desires to transfer his interest, it will sell every year for more money than it would have done the year before; for, the more persons dye, the greater will the income be to the survivors; and any man may transfer his interest to whom he pleases.

"This fund, for the security of the payment of the said interest, shall be settled to the satisfaction of the subscribers, as shall be advised by council learned in the law."

But to these proposals I hear two great objections—

1st. That the subscribers shall have but common interest, and loose the principal; which is not a pennyworth for a penny.

2ndly. That people may club together, and leave to the longest liver interest and principal; or it may be again divided among the heirs, or the heirs may come in afresh upon interest, and it may be a fund for such interest *ad infinitum*.

These objections I'll consider with all fairness imaginable; and first, of the first.

'Tis true the subscribers shall have no more but what the law calls common interest—viz., 6 per cent; but whether the currant interest of the town and country is like to be such, is to be enquired into; and so much as the interest shall be less worth, so much will the proposals appear better; and if my foresight fails me not, within a while interest will not be so much worth. My reasons take as follow:—

Within this two years I went myself to the East India Company, in behalf of a friend, to offer them some money at 3 per cent.; and though I made one of the officers my friend, to entreat for me, yet it would not be accepted. And 'tis notorious that, about that time, abundance of people did lend them at that rate; the reason, I suppose, was, because they could not get more with security to their satisfaction. Nay, to others 'twas lent at 4, 4½, and 5, currantly; and although now 'tis risen to 6, yet when the cause of its rise shall be removed, it must of necessity again fall (for it is out of the power of laws to ascertain interest, as is apparent by these instances, and the high rates the King and others have given when their extraordinary occasions have forc't it, besides procuration, continuation, &c.), and the causes of its rise, according to my best observation, have been as follows:—

Besides the great quantities of money carryed out by the Turky Company, and to several other places in Europe, according to common custom, several interlopers for India provided themselves with a great quantity. The East India Company hoping, by a very great trade, to prejudice these interlopers, provided much more than ever they did before. These extraordinary occasions, unless the quantities were proportionably increast,

could do no less than raise interest (every one raising their commodity according to the eagerness and multiplicity of good chapmen); but that which made it more than ordinarily break out was, that some persons that were not pleased with Sir John More's government in his mayoralty, thought to prejudice him by draining Ben. Hinton, his intimate, a goldsmith and banker in Lombard Street. This was told to me about a fortnight before the first bankers of this late storm—viz., Mr. Addis and his partners—went off; and this is confirmed to me by a considerable man in this city, who tells me that Dissenters say the bank will incourage trade too much to be countenanc'd at this time. When money was thus drawn from Mr. Hinton, almost everybody (although 'tis probable the most part knew not why) thought it best to secure their own, and ran with open mouth upon all the bankers for money, thinking it better to let it lye dead a while in their chests, than to run a hazard of trusting such, who, for ought they knew, might do as Mr. Addis and some others near him had done. To joyn with this, some Dissenters being excommunicated, and a discourse that all the rest that would not comply should, made, I suppose, must folk willing to have their money out of such hands. Also the many rich interlopers that went and were designed to go to India, together with all the jealousies imaginable raised by them and their friends upon the Company, made a great many of the fearful members of the Company eagerly sell their stocks; and, perhaps, some of their designing ones too, that they might afterwards have opportunity of buying again cheaper.

This, when the Company had most need of money to set out their numerous fleet—upwards of thirty sail—made their creditors run very earnestly on them also for the money they had lent them, which put them to such straits, that, instead of 3, they were willing to give 5 or 6 per cent., and some say promises of good turns into the bargain. But all would not do, their auctions fell from 365 to 245, and they were forc't to put a stop to payments for three months; and, in the meanwhile, have appointed a sale, and expect several rich ships home, with which they question not to give a stop to all reasonable complaints.

This, all laid together, I take to be the reason why money is at this high rate; but if my conjectures are false, I beg pardon, and I wish some more knowing would give us the true causes. But if I am not mistaken, then it will follow that when the hoarders are weary of keeping up their money, when what was exported last year to all parts shall return with a duck in its mouth, when the East India auctions shall again rise, when the fright shall be over (as usually in these cases it is after a little while), when we shall come to have less disputes and be more united about religion—all which I hope shortly to see—then you'l find money as cheap, if not cheaper, than usually, and the East India Company offered again more money than they have occasion for; and I verily believe they may, if they will, be one of the chiefest funds in the nation; for although they owe a great deal of money, yet 'tis visible that they have a far greater stock; and 'tis also plain that 'tis their interest to keep up their credit, although it were by lending their own private cash, for otherwise they loose more by the fall of their auctions then all their debts come to; as lately 'twas said, their debts were about £800,000, and their principal stock is about £750,000, and every £100 fell in their auctions, as above, from 365 to 245, which is 120 per cent., amounting to £900,000.

This money, or so much of it as will make currant payment, I perswade myself they will raise by themselves and friends, unless they find some better expedient, although, perhaps, forbearance of a dividend or two may do the business. If so, then, seeing the bankers are single and their stocks not so visible, and some or other of them drop off, what should hinder but (this Company appearing thus staunch) most folk should run their money in here cheaper than other places by 1 or 2 per cent. (except in the Guinia Company, which I take to be as safe as this). And as for dangers from interlopers, I see no great reason to fear, because the Guinia Company, in spite of as many interlopers as will go, are in a thriving condition. And this India Company may, with a less gain per cent. in many more hundreds, get more money and more dishearten their adversaries. There is no necessity for a double trade, to have double fortifications, double agents or factories, neither will their need to the great ones in India double presents, nor altogether be among themselves double petty charges. Much more I could say on this subject, but I don't think it needful here; my drift is mainly to show that if money should come again to 3 per cent., then the allowance of 6 will be equivalent to 12 when interest is at 6.

The second part of the first objection is, that the principal will be lost.

'Tis true it will be so, but who is it will find it? Why, 'tis the City of London, from whom most of the money expected to be subscribed hath been gotten; for if they are citizens they give it to their own body and for their own use. 'Tis probable their children may be the orphans to reap the benefit of it. But suppose it should be spent in triumphs, Lord Mayor's shows, publick buildings, festivals: is it not such like that inables us now to live so well, and makes us the renown of the whole earth? I am strongly perswaded that a stately Lord Mayor's shew makes London, from strangers that flock to see, get more money, six times over, than ever the charge of the pageantry came to; and if they had stock to enlarge their glory, I am sure a proportional expense will follow it. O what crouds flockt hither to see the glory of the King's return and coronation.

But if the subscribers should not be citizens, yet it is likely it will turn to their children's advantage; for London is the means of preferment to most of the country's progenie. And I question not, but when the city shall reap these advantages, 'twill be a good argument for preferment to places for the heirs of such subscribers.

To the second objection—viz., that the people may club, &c.

I confess, in theory, all this is true; but 'tis next to impossible to believe that e're it should be practised; for who, without some consideration of loaves for his pain, will gather this club together? or will they *more fungorum*, as mushrooms, all start up together in a night to throw in their dust? But if they should, who should find out a security, search the title, or be council learned in the law, to settle and secure this fund? Or, if paid for, will not the charge of doing these things, when taken out of these little combinations, reduce this profit to a less than what is here proposed? I doubt it will. However, till that be tryed, this is the better; and everyone hath liberty to please himself. I believe I shall never be of that club; whatever I shall be of the other, I can't yet say. If the first part of this objection won't take, the rest ne're can, as being dependants on it.

Hoping that what's here said may be some answer to the objections—or, at leastwise, make the prejudice of subscribing appear not altogether so great as at first it might be thought for—I'll strive now to show what in likelihood will be the advantages of these subscriptions.

'Tis told you 6 per cent.—with the whole, £600 per annum—to be divided among the survivours, even to the last man. Nobody subscribing for himself that looses here, but he that dyes; and rather than he shall want money in the next world, he, if he leads a good life here, shall have a note to St. Peter to turn the key for nothing. But for those that shall live long here, let's see what in likelihood shall be their profit.

Major Graunt, or rather that learned and ingenious virtuoso Sir William Petty, in his admirable *Observations of the Bills of Mortality* of London—a book useful to a multitude of purposes, and a pattern for many other great designs; this great man, I say, in p. 14 of his second edition, saith, "That about one-third of all that were ever quick dye under five years old, and about 36 per cent. under six."

If so, 'tis to be supposed that those of the first rank that shall live but seven years after subscription shall receive upwards of 9 per cent.

But in p. 56, "That three dye yearly out of eleven families, of each eight persons—*i. e.*, eighty-eight; if it were ninety, 'twould be one in thirty. And at this rate, whosoever lives thirty year in a place may have no neighbour that was cotemporary with him in his first year. Therefore, they of the said rank may, in thirty years, reasonably expect almost £200 per annum a piece.

In p. 58 he saith, Of one hundred there dies within the first six years 36; the next ten years,

Or decad	24
The second decad	15
The third	9
The fourth	6
The next	4
Next	3
Next	2
Next	1

From whence it follows, that, of the said 100 conceived, there remain alive

At six years end	64
At sixteen	40
At twenty-six	25
At thirty-six	16
At forty-six	10
At fifty-six	6
At sixty	3
At seventy-six	1
At eighty	0

In p. 65 he saith, In the country about one in fifty dye yearly, but in London about one in thirty; and that London is not so healthful now as heretofore. Wherefore it may be advantageous to subscribe on countrey lives rather than city ones.

If what I have already said shall appear reasonable, then it may be worth while to consider what people in likelihood it may be fittest for.

I do suppose it very proper for all landed men to put in £5 a piece for their younger children, for if they die quickly the estate will be free to the heir; if not, 'twill be considerable, and the estate shall not need to be clog'd for maintenance.

'Twill be proper for all tradesmen who live gentilely from hand to mouth, but never provide much aforehand; this way their children may have portions, and themselves be well kept in their old age.

'Twill make all sorts of old people be made much on, because the longer they live the more they'l have.

Merchants, several other tradesmen, and gamesters, that live by hazard, may sometimes, at extraordinary hits, put something in here to keep in cases of extraordinary losses.

Poor servants, as soon as they get £5, may by it take care for old age.

Friends may put in for their she relations who are ill marryed, whereby they shall never want; and 'twill be a means for forcing kindness from their husbands.

'Twill be better for wives than joynters, and husbands may imploy a great deal of the money that should buy land.

If families grow so low, that there is a necessity for selling of joyntures, some of the money may be secured here, and the widdow fare ne're the worse.

If every University man, when he comes to preferment of £100 per annum, would put in £5, and so for every £100, to have the interest go towards the library of his Colledge, 'twould, in a while, make them very great; and 'twould do the like for Sion Colledge, if each such London minister would do so.

And if twenty good folks would give £5 a piece, for ought I know it might raise such a Colledge as Mr. Abraham Cowley speaks on in his *Discourse of Agriculture for the Improvement of Husbandry.*

Multitudes of other conveniences I could enumerate, but they that will consider these may find enough more that will be agreeable to their own circumstances; and there is a book signed by Mr. Wagstaff, the City Town Clerk, which book is entituled *Proposals for Increase of Wealth by Subscriptions.* That will show you divers other instances.

Whatever is said of £5 will serve for any other summe.

Sir, if these reasons shall be undenyable, I pray incourage this design; if not, persuade somebody to show the contrary. But however, pardon me, who, unless I do subscribe, am like no ways to be concerned in it, except in good wishes for that city from which I have had my well-being.

Farewell, &c.

On the Construction and Use of Commutation Tables for Calculating the Values of Benefits depending on Life Contingencies. By PETER GRAY, *Esq.*, F.R.A.S.*

(Concluded from page 108.)

WE have now to give some examples of compound benefits, which are those consisting of two or more simple benefits; but the combinations which may be formed of these being obviously very numerous, it would be beside our present purpose to attempt giving a complete list of them. Our object will be, in selecting a few of them for illustration, to indicate the method of dealing with the more complicated cases, and also to prepare the way for the most general application of the Commutation Tables, which application will form the subject of the concluding portion of this paper. A very complete list of the formulæ for the more elementary of these benefits is contained in Professor De Morgan's first paper on the subject; and as it is hoped that little difficulty will be experienced with these, after the illustrations to which our space limits us, we shall not scruple, as we have occasion, in the solution of any of the problems with which the present paper will be occupied, to refer to any of the learned gentleman's formulæ which we may not have deduced for ourselves. Our references will be made in the following manner, which is rendered necessary in consequence of the formulæ not forming one consecutive series. Formula 10, on page 16, for example, will be denoted thus, [16, 10]; formula 7_1, on page 18, thus, [18, 7_2]; and so on.*

As we are no longer to confine ourselves to benefits whose amount is £1, we again point attention to a remark made on page 99, to the effect that, when we have the present value of a benefit of £1, that of a like benefit of any other amount will be found by multiplying the first-named present value by the number of pounds in the amount in question. As it is convenient to have distinctive

* Extracted from the *Mechanics' Magazine* for 1842.

† It may be of use here to point out a few typographical errors in Professor De Morgan's papers, which might otherwise embarrass the student:—

First paper,	page 11,	line 22,	for $(A+\overline{n-1}h)$, read $(A+\overline{n-1}H)$.
"	" 16,	" 29,	for $(a+(n-1)h$, read $a\pm(n-1)h$.
"	" 17,	" last,	for SC_{x-1}, read SC_{x+h-1}.
"	" 21,	" 30,	for M_x, read R_x.
Second paper,	" 2,	" 25,	for $(1-v)N_{x,y}$, read $(1-v)N_{x-1,y-1}$.

Also, the terminating braces are omitted in the expressions [18, 11] and [19, 15_2] of the first paper.

symbols to represent the amounts of benefits of different kinds, we make use of the following for this purpose:—

s, The amount of an endowment.
a, The annual rent of an annuity.
h, The annual increment or decrement of a variable annuity.
S, The amount of an endowment assurance.
A, The amount of an assurance.
H, The annual increment or decrement of a variable assurance.

Perhaps we may require a few others. If so, we shall explain them as they are introduced.

Also, since the expression for the present value of a compound benefit is the sum of the expressions for the present values of the simple benefits of which it is composed, and since these are fractions having for their denominator D_x, it will likewise *generally* be a fraction having the same denominator. We shall, therefore, to economise space, usually omit this common denominator; but it must be carefully remembered that the expressions are incomplete without it. We have said that the expression for the present value of a compound benefit is *generally* of the form alluded to. The exception is (De Morgan, I., pp. 14, 15), when a part of the benefit depends on the unknown item of payment. In this case the expression takes another form. When it does so, it will be exhibited without abbreviation.

In what follows we shall no longer adhere to the formality of problems. For after reference, however, we shall number the expressions we deduce with Roman numerals, in continuation of the number at which we have arrived in the previous problems.

Referring to remark 4, on page 178, we further premise, that, benefits being divisible into the two classes of annuity benefits and assurance benefits, if we deduce the expression for a benefit belonging to one of these classes, it will obviously be unnecessary to do so for the corresponding benefit belonging to the other class, since the relation indicated in the remark quoted *always subsists.*

We proceed now to the more legitimate subject of this portion of our paper.

The increasing benefits of which we have hitherto spoken are those in which the annual increase is equal to the first payment. But the Commutation Tables can also be applied to finding the value of increasing benefits, in which the annual increase is in no way dependent on the first payment; and also of decreasing benefits, with the like latitude as to the magnitude of the decrease.

Thus, a life annuity whose successive payments are to be £a, £$(a+h)$, £$(a+2h)$, £$(a+3h)$, &c., may be decomposed into the following annuities, viz.—a life annuity of £a, and an annually increasing annuity, to be entered upon one year hence, of £h, £$2h$, £$3h$, &c. The present value of the first is (Prob. II.), $a\mathrm{N}_x$; and of the second (Prob. VII.), $h\mathrm{S}_{x+1}$. The present value of the compound benefit, therefore, is $a\mathrm{N}_x+h\mathrm{S}_{x+1}$. In like manner it may be shown that the present value of a life annuity, whose successive payments are to be £a, £$(a-h)$, £$(a-2h)$, &c., is $a\mathrm{N}_x-h\mathrm{S}_{x+1}$. The following formula will, therefore, include both cases, the upper sign having reference to the increasing, and the lower to the decreasing benefit:—

$$a\mathrm{N}_x \pm h\mathrm{S}_{x+1} \quad . \quad . \quad . \quad \text{(XIX.)}$$

By the remark on page 178, already referred to, the formula for the corresponding assurance benefits will be

$$\mathrm{AM}_x \pm \mathrm{HR}_{x+1} \quad . \quad . \quad . \quad \text{(XIX.)}$$

According to what has been said above, we shall not usually give the formulæ for the two classes of benefits, since, as we have seen, the formulæ for the one class are so readily derived from those for the other.

If, in (XIX.), $a=h$ for the increasing benefit—that is, if the annual increase be equal to the first payment—the formula becomes

$$a\mathrm{N}_x+a\mathrm{S}_{x+1}=a(\mathrm{N}_x+\mathrm{S}_{x+1})=a\mathrm{S}_x,$$

by (10), which agrees with (VI.), as it ought to do.

While the above formula expresses, for every value of a and h, the true values of the benefit, yet it must be observed, that, in the case of the decreasing benefit, h may be taken so large, that the annuitant (we confine our remark to the annuity, although it is equally applicable to the assurance benefit), if he live long enough, will have to pay instead of to receive. Thus, if a person aged 30 enter upon a decreasing life annuity, whose successive payments are to be £10, £9, £8, &c. (that is, $a=10$, $h=1$), it is evident that the 10th payment will be £1, and the 11th 0. And, since the annuity is for the whole life, the decrease still goes on, so that the 12th, 13th, 14th, &c., payments will be −£1, −£2, −£3, &c.; that is, the annuitant, instead of having anything to receive, will have these sums to pay.

It may be, also, that the present value of the payments to be thus made by the annuitant will exceed that of the payments he

will have previously received. This is indicated in the application of the formula to any particular case, by its numerical value in that case becoming negative, which will evidently be when $h\mathrm{S}_{x+1}$ is greater than $a\mathrm{N}_x$. A negative value presented by the formula indicates that the *purchase-money* for the benefit must be paid by the *seller*.

To avoid the inconvenience of the payments becoming negative, h must never be taken larger than $\frac{a}{n}$, n being any number not less than the number of years during which the annuity is to last. In the case of an annuity for the whole life, the least value of n will be the difference between the age at which the annuity is entered upon and the oldest age in the table, when the last payment that can possibly be received will be £h.

If, in (XIX.), we write $x+n$ for x, we are furnished, as in the case of the simple benefits, with the expression for the same benefit to be entered upon n years hence. This expression is

$$a\mathrm{N}_{x+n} \pm h\mathrm{S}_{x+n+1} \quad . \quad . \quad . \quad (\mathrm{XX.})$$

It may also easily be deduced by decomposing the compound benefit.

The expression for the same benefit to last n years is deduced in the following manner:—This modification of the benefit consists of an annuity of £a, to be entered upon immediately and to last n years, and of an increasing annuity of £h, £$2h$, £$3h$, &c., to be entered upon 1 year hence and to last $n-1$ years, and which is to be either added to or subtracted from the other, according as the benefit whose value is sought is an increasing or a decreasing one. The present value of the first portion is (IV.), $a(\mathrm{N}_x-\mathrm{N}_{x+n})$; and that of the second portion is found as follows:—the present value of an increasing annuity of £h, £$2h$, £$3h$, &c., to be entered upon in k years and to last n years, is, by the table on page 177, $h(\mathrm{S}_{x+k}-\mathrm{S}_{x+k+n}-n\mathrm{N}_{x+k+n})$. If, therefore, in this expression, we substitute 1 for k, and $n-1$ for n, we shall adapt it to our present purpose. Making these substitutions, the expression becomes $h(\mathrm{S}_{x+1}-\mathrm{S}_{x+n}-(n-1)\mathrm{N}_{x+n})$. But $(n-1)\mathrm{N}_{x+n}=n\mathrm{N}_{x+n}-\mathrm{N}_{x+n}$; hence the expression becomes $h(\mathrm{S}_{x+1}+\mathrm{N}_{x+n}-\mathrm{S}_{x+n}-n\mathrm{N}_{x+n})$. But, by (10), $\mathrm{N}_{x+n}-\mathrm{S}_{x+n}=-\mathrm{S}_{x+n+1}$. Therefore the expression becomes, finally, $h(\mathrm{S}_{x+1}-\mathrm{S}_{x+n+1}-n\mathrm{N}_{x+n})$. And if to this we connect by the proper sign the expression for the present value of the first portion of the benefit, we have, as the expression sought—

$$a(\mathrm{N}_x-\mathrm{N}_{x+n}) \pm h(\mathrm{S}_{x+1}-\mathrm{S}_{x+n+1}-n\mathrm{N}_{x+n}) \quad . \quad . \quad (\mathrm{XXI.})$$

This expression may also be deduced in a somewhat different manner, which, as our object is illustration, we likewise insert. The benefit under consideration evidently admits of decomposition into the two following simpler benefits, viz. (confining ourselves, for perspicuity, to the increasing benefit), a uniform annuity of £$(a-h)$ for n years, and an increasing annuity of £h, £$2h$, &c., also for n years, and both to be entered upon immediately. The present value of the first is, (IV.), $(a-h)(N_x-N_{x+n})$; and of the second, (VIII.), $h(S_x-S_{x+n}-nN_{x+n})$. Adding these expressions, we have—

$$(a-h)(N_x-N_{x+n})+h(S_x-S_{x+n}-nN_{x+n})=$$

$$a(N_x-N_{x+n})-h(N_x-N_{x+n})+h(S_x-S_{x+n}-nN_{x+n})=$$

$$a(N_x-N_{x+n})+h(S_x-N_x-S_{x+n}+N_{x+n}-nN_{x+n}).$$

Now, by (10),

$$S_x-N_x=S_{x+1}, \text{ and } S_{x+n}+N_{x+n}=S_{x+n+1};$$

hence the expression becomes, as before—

$$a(N_x-N_{x+n})+h(S_{x+1}-S_{x+n+1}-nN_{x+n}).$$

It will be seen, on comparing this expression with (XIX.), that it does not follow the same law as the simple benefits, in passing from the expression for the value of a benefit to last the whole life to that for the value of the same benefit to last n years. Did the law referred to hold here, the signature of N, in the coefficient of h, would be $(x+n+1)$. (See De Morgan, I., page 21.)

We shall not seek to deduce here any more of Professor De Morgan's formulæ. We leave the others as a most improving exercise for the student, and pass on to the consideration of a few miscellaneous benefits.

£A are to be received by (x), or his representatives, at the end of n years, if he be then alive, or at the end of the year in which he dies, if that event take place before the expiry of the n years. Required the present value of the benefit. This benefit is evidently equivalent to an endowment of £A payable in n years, and a temporary assurance of the same amount, to last n years. Its present value, therefore, is, by (I.) and (XIII.),

$$A(D_x+M_x-M_{x+n}) \quad . \quad . \quad . \quad \text{(XXII.)}$$

This is a benefit of not unfrequent occurrence in practice. Several of the Companies publish tables of the equivalent annual premiums, the method of finding which will be shown hereafter.

Required the present value of a life assurance of £A on (x), with which the sum paid is to be returned.

If we call the present value of this benefit P, then £(A+P) will be the sum to be received at death; the benefit, therefore, will be a life assurance of £(A+P), the present value of which is (using the denominator in this case)—

$$\frac{(A+P)M_x}{D_x}.$$

And, by condition, we have

$$P=\frac{(A+P)M_x}{D_x} \therefore PD_x=AM_x+PM_x.$$

Whence,

$$P=\frac{AM_x}{D_x-M_x} \quad . \quad . \quad . \quad \text{(XXIII.)}$$

If, at death, along with the sum assured, the sum paid is to be returned, with simple interest upon it from the date of payment, the sum to be received, if death take place in the first year, will be $A+(1+r)P$; if in the second, $A+(1+2r)P$; if in the third, $A+(1+3r)P$, &c.; r denoting the interest of £1 for a year. The benefit, therefore, is a life assurance of $A+(1+r)P$, increasing annually by rP; and the expression for the present value will be, by (XIX.),

$$\frac{\{A+(1+r)P\}M_x+rPR_{x+1}}{D_x}.$$

Equating this to P, as in the last case, we should find—

$$P=\frac{AM_x}{D_x-(1+r)M_x-rR_{x+1}} \quad . \quad . \quad \text{(XXIV.)}$$

This and the preceding case illustrate a remark previously made, that when a portion of the benefit depends on the unknown (that is, unknown till the equation is solved) item of payment, the denominator of the expression for the present value is no longer restricted to D_x. These and such like cases, however, belong more properly to that portion of the subject which will be treated in the remainder of this paper.

The annual rent of an annuity upon (x) is to be a for the first k years, b for the next m years, c for the next n years, and d for the remainder of life. Required its present value.

Column S will not serve our turn in this case, since that column is applicable only when the variation in the payments is annual, and equable throughout the whole duration of the annuity.

The problem, as proposed, has no such limitations; and we must, therefore, find the present values of the separate portions of the annuity, and add them together for the whole present value required.

The annuity consists of four portions, the present value of the first of which is, by (IV.),

$$a(N_x - N_{x+k});$$

of the second, by (V.),

$$b(N_{x+k} - N_{x+k+m});$$

of the third, by (V.),

$$c(N_{x+k+m} - N_{x+k+m+n});$$

and of the fourth, by (III.),

$$d(N_{x+k+m+n}).$$

And the sum of these (divided, as usual, by D_x), is the present value required. This sum is

$$a(N_x - N_{x+k}) + b(N_{x+k} - N_{x+k+m}) + c(N_{x+k+m} - N_{x+k+m+n}) + dN_{x+k+m+n} =$$

$$aN_x + (b-a)N_{x+k} + (c-b)N_{x+k+m} + (d-c)N_{x+k+m+n} \quad . \quad \text{(XXV.)}$$

This expression shows, as will likewise readily appear from other considerations, that the benefit proposed admits of decomposition into the four following portions, viz., a life annuity of £a, and three deferred annuities of £$(b-a)$, £$(c-b)$, and £$(d-c)$, to be entered upon respectively at intervals of k, m, and n years. Also, that if b, c, and d be respectively greater than a, b, and c, the payments will increase at the expiry of each term, and *vice versâ*. If we adopt the former supposition, and suppose also $b-a=c-b=d-c=h$ (that is, that the annual rents of the deferred annuities are equal), the expression will become—

$$aN_x + h(N_{x+k} + N_{x+k+m} + N_{x+k+m+n}) \quad . \quad . \quad \text{(XXVI.)}$$

And if we further suppose $k=m=n=1$ (that is, that this uniform increase takes place at intervals of one year), it will become—

$$aN_x + h(N_{x+1} + N_{x+2} + N_{x+3});$$

which, by (6), is equal to

$$aN_x + h(S_{x+1} - S_{x+4}).$$

And this is the expression we should have derived for the benefit,

subject to these restrictions, and for the particular case in which $n=4$, from the formula [16, 12].

We have been occupied hitherto with the consideration of the present values, or single premiums, equivalent to benefits of various kinds. It is seldom, however, that benefits are paid for by a single premium. The more usual mode is by annual premiums, which may be either uniform or variable, and payable either during the whole life, or only for a specified number of years. It is necessary, therefore, now to show how the amounts of premiums, payable as described, may be found. The method of doing so constitutes the most general application of the Commutation Tables; and it is this application which most strikingly displays the great power of the method of computation we are endeavouring to illustrate, and its vast superiority to all other methods heretofore in use. It will be seen, as we proceed, that the previous portions of this paper have been but preliminary to that which forms the subject of our present remarks.

The principle upon which the application of the tables that we are now to describe depends, is so simple, that some may be disposed to award but a small portion of merit to the person by whom it was first pointed out. But as we are of those who believe that the merit of a contrivance is to be estimated in proportion to the utility of the object it has in view, and the simplicity and efficiency of the means by which that object is attained; and as the possession of those qualities, in an eminent degree, by the contrivance in question, cannot be gainsaid, we take a very different view of the matter, and presume to think the contrivance one of very great merit.

It is to Professor De Morgan that we are indebted for the contrivance we have alluded to. Others before him had, doubtless, shown how many of the problems might be solved by the Commutation Tables; but it was reserved for him to devise a general method equally applicable to all, and to exhibit an equation which should include in it almost every case, as regards both benefits and premiums, that can be proposed.*

We now proceed to explain the principle of this contrivance. The value of an annual premium whose continuance depends upon the continued existence of a given life, is evidently as legitimate a

* Mr. Davies' work having been long out of print, we have not been able to procure even a sight of it. But we have no reason to believe that it contains anything akin to the contrivance we have referred to. With the other works named on pp. 84, 85, as treating on the use of the Commutation Tables, we possess a competent acquaintance; and, certainly, in none of them is the least trace of this contrivance to be found.

Example 3.—An endowment of £s, to be received in n years, provided (x) be then alive, is to be paid for by an annual premium, π, to last l years. Required π.

Here the benefit side is, by (I.), sD_{x+n}; and the payment side is, by (IV.), $\pi(N_{x-1}-N_{x+l-1})$.

$$\therefore \pi(N_{x-1}-N_{x+l-1})=sD_{x+n};$$

whence,

$$\pi=\frac{sD_{x+n}}{N_{x-1}-N_{x+l-1}}.$$

If $s=$£100, $x=23$, $n=20$, and $l=10$, we have

$$\pi=\frac{100D_{43}}{N_{22}-N_{32}}=\frac{114562{\cdot}6}{58976{\cdot}15-32946{\cdot}10}=\frac{114562{\cdot}6}{26030{\cdot}05}=4{\cdot}4012=£4.\ 8s.$$

If the premium be payable till the endowment becomes due, we have only in the formula to substitute n for l; and, to apply it to the particular case just solved, we have

$$\pi=\frac{100D_{43}}{N_{22}-N_{42}}=\frac{114562{\cdot}57}{58976{\cdot}15-17483{\cdot}64}=\frac{114562{\cdot}57}{41492{\cdot}51}=2{\cdot}7610$$
$$=£2.\ 15s.\ 3d.;$$

a smaller premium, since, commencing at the same time, it continues longer.

Example 4.—£A, to be received n years hence if (x) be then alive, or at the end of the year of death if that event take place before the expiry of n years, is to be paid for by an annual premium, π, to continue until the receipt of the benefit is determined. Required π.

Here the benefit side is, by (I.) and (XIII.), $A(D_{x+n}+M_x-M_{x+n})$; and the payment side is, by (IV.), $\pi(N_{x-1}-N_{x+n-1})$.

$$\therefore \pi(N_{x-1}-N_{x+n-1})=A(D_{x+n}+M_x-M_{x+n});$$

whence,

$$\pi=\frac{A(D_{x+n}+M_x-M_{x+n})}{N_{x-1}-N_{x+n-1}}.$$

If A=£100, $x=30$, and $n=30$ (that is, if the benefit be receivable at 60, or at death, if before 60), we have

$$\pi=\frac{100(D_{60}+M_{30}-M_{60})}{N_{29}-N_{59}}=\frac{41857{\cdot}53+74295{\cdot}67-24598{\cdot}20}{39382{\cdot}08-4487{\cdot}42}=$$
$$\frac{91555{\cdot}00}{34894{\cdot}66}=2{\cdot}6238=£2.\ 12s.\ 6d.$$

Example 5.—Required the annual premium, π, payable till death, for an assurance of £A on (x), with which the whole of the premiums paid are to be returned.

Here the sum to be received if death take place in the first year is $A+\pi$; if in the second, $A+2\pi$; and so on. Therefore, the benefit side is, by (XIX.),

$$(A+\pi)M_x+\pi R_{x+1}=AM_x+\pi R_x, \text{ by (10)};$$

and the payment side is, by (II.), πN_{x-1}.

$$\therefore \pi N_{x-1}=AM_x+\pi R_x;$$

whence we obtain

$$\pi=\frac{AM_x}{N_{x-1}-R_x}.$$

If $A=£100$, and $x=30$, we have

$$\pi=\frac{100M_{30}}{N_{29}-R_{30}}=\frac{74295\cdot67}{39382\cdot08-17127\cdot42}=\frac{74295\cdot67}{22254\cdot65}=3\cdot3338$$
$$=£3.\ 6s.\ 8d.*$$

Example 6.—An assurance of £A, for n years, on (x), is to be paid for by an annual premium, π, also to last n years. Required π.

Here the benefit side is, by (XIII.), $A(M_x-M_{x+n})$; and the payment side, by (IV.), $\pi(N_{x-1}-N_{x+n-1})$.

$$\therefore \pi(N_{x-1}-N_{x+n-1})=A(M_x-M_{x+n});$$

whence

$$\pi=\frac{A(M_x-M_{x+n})}{N_{x-1}-N_{x+n-1}}.$$

This expression assumes a very convenient form, if, instead of column M, we use its value in terms of N. Thus, since, by (13),

$$M_x=vN_{x-1}-N_x, \text{ and } M_{x+n}=vN_{x+n-1}-N_{x+n},$$

we have

$$\pi=\frac{A\{v(N_{x-1}-N_{x+n-1})-N_x+N_{x+n}\}}{N_{x-1}-N_{x+n-1}}=A\left(v-\frac{N_x-N_{x+n}}{N_{x-1}-N_{x+n-1}}\right).$$

* The Commutation Table, as it stands, does not enable us, conveniently, to find the amount of annual premium equivalent to a benefit which consists partly of a return of all the premiums paid, with simple interest upon them from the date of payment, the incremental portion being in this case of the form m, $3m$, $6m$, $10m$, &c. The addition of another column, formed from R as R is formed from M, would, however, afford the means of doing so. We might, obviously, add as many columns as we pleased in this way. Their properties would be such that, calling the column formed from R the *first*, the division of any number in the nth column, by the corresponding number in D, would give the present value of an assurance whose payments should be the series of figurate numbers of the nth order; and the remark may be extended, *mutatis mutandis*, to the annuity columns. But such properties being more curious than useful, we do not insist upon them. (See De Morgan, I., p. 23.)

If $A = £100$, $x = 30$, and $n = 10$, the solution by the first of these formulæ will be

$$\pi = \frac{100(M_{30} - M_{40})}{N_{29} - N_{39}} = \frac{100(742{\cdot}9567 - 522{\cdot}6503)}{39382{\cdot}08 - 21306{\cdot}62} = \frac{23030{\cdot}64}{18075{\cdot}46} =$$

$$1{\cdot}2188 = £1.\ 4s.\ 5d.$$

The solution by the second formula will be

$$\pi = 100\left(v - \frac{N_{30} - N_{40}}{N_{29} - N_{39}}\right) = 100\left({\cdot}961539 - \frac{17159{\cdot}95}{18075{\cdot}46}\right) =$$

$$100({\cdot}961539 - {\cdot}949351) = 1{\cdot}2188,$$

as before.

Example 7.—An annuity of £a, deferred for n years, on (x), is to be paid for by a uniformly decreasing annual premium, to be extinguished when the annuity is entered upon. Required π, the first year's premium.

Here the benefit side is, by (III.), aN_{x+n}; and, since the premium is to be extinguished in n years, it will make $n+1$ payments. In order, therefore, that the $(n+2)$th payment, which would be due when the first payment of the annuity is receivable, may be 0, the annual decrease must be $\frac{\pi}{n+1}$. The payment side consequently is, by [20, 6],

$$\pi\{N_{x-1} - \frac{1}{n+1}(S_x - S_{x+n+1})\}.$$

Hence,

$$\pi\{N_{x-1} - \frac{1}{n+1}(S_x - S_{x+n+1})\} = aN_{x+n}.$$

$$\therefore \pi = \frac{aN_{x+n}}{N_{x-1} - \frac{1}{n+1}(S_x - S_{x+n+1})}.$$

If $a = 100$, $x = 40$, and $n = 20$, we have

$$\pi = \frac{100N_{60}}{N_{39} - (S_{40} - S_{61}) \div 21} = \frac{406884{\cdot}83}{21306{\cdot}62 - (252533{\cdot}78 - 29648{\cdot}60) \div 21}$$

$$= \frac{406884{\cdot}83}{21306{\cdot}62 - 10613{\cdot}06} = \frac{406884{\cdot}83}{10693{\cdot}56} = 38{\cdot}0495 = £38.\ 1s.$$

Example 8.—An increasing life assurance of £A, £(A + H), £(A + 2H), &c., on (x), is to be paid for by a premium which is to

be π for the first n years, $\frac{2}{3}\pi$ for the next n years, and $\frac{1}{3}\pi$ for the remainder of life. Required π.

The benefit side is, by (XIX.), $AM_x + HR_{x+1}$; and the payment side is—

For the first n years, $\pi(N_{x-1} - N_{x+n-1})$;
" second, $\frac{2}{3}\pi(N_{x+n-1} - N_{x+2n-1})$;
Remainder of life, $\frac{1}{3}\pi N_{x+2n-1}$.

Therefore, adding these for the whole payment side, we have

$$\pi\{N_{x-1} - \tfrac{1}{3}(N_{x+n-1} + N_{x+2n-1})\} = AM_x + HR_{x+1}.$$

$$\therefore \pi = \frac{AM_x + HR_{x+1}}{N_{x-1} - \frac{1}{3}(N_{x+n-1} + N_{x+2n-1})}.$$

If $A = £100$, $H = £5$, $x = 30$, and $n = 7$, the formula becomes

$$\pi = \frac{100M_{30} + 5R_{31}}{N_{29} - \frac{1}{3}(N_{36} + N_{43})}.$$

And, taking the numbers from the table, we should find

$$\pi = 6{\cdot}1644 = £6.\ 3s.\ 3d.$$

This mode of paying premium, although not unusual in practice, is not included in Professor De Morgan's general problem (I., pp. 10, 11). We subjoin the general expression for the payment side of the equation when a benefit is to be paid for in this manner—that is, by a premium remaining constant during one or more terms of years, but varying, either by increase or decrease, at the end of each term. Let p, q, and r, denote any numbers whatsoever, whether whole or fractional; also, let l, m, and n, denote any terms of years whatsoever; then the payment side of the equation, for a premium which is to be π during the first l years, $p\pi$ during the following m years, $q\pi$ during the following n years, and $r\pi$ for the remainder of life, will be

$$\pi\{N_{x-1} + (p-1)N_{x+l-1} + (q-p)N_{x+l+m-1} + (r-q)N_{x+l+m+n-1}\}.$$

From this expression it appears, that, if p be less than unity, q less than p, and r less than q, the premium will decrease at the end of each term, and *vice versâ*. To apply this expression to the last example, let $x = 30$, $p = \frac{2}{3}$, $r = \frac{1}{3}$, $l = m = 7$; and, since there

are only two *definite* periods, during which the payments are uniform, $n=0$ and $q=0$. Hence the expression becomes

$$\pi(N_{29}-\tfrac{1}{3}N_{36}-\tfrac{2}{3}N_{43}+\tfrac{1}{3}N_{43})=\pi\{N_{29}-\tfrac{1}{3}(N_{36}+N_{43})\}.$$

Example 9.—A life assurance of £A on (x) is to be paid for by a sum in hand, σ, and an annual premium, π. Required π.

Here the benefit side is AM_x, and the payment side is $\sigma D_x+\pi N_{x-1}$.

$$\therefore\ \sigma D_x+\pi N_{x-1}=AM_x;$$

whence,

$$\pi=\frac{AM_x-\sigma D_x}{N_{x-1}}.$$

If $A=£1{,}000$, $x=40$, and $\sigma=£100$, we should find

$$\pi=18{\cdot}2308=£18.\ 4s.\ 7d.$$

Example 10.—A person now aged x years effected an assurance on his life of £A, n years ago, at an annual premium of £π, of which a payment is just due. He now wishes to dispose of his interest in the same, the purchaser to take on himself the payment of the future premiums, and to receive the sum assured when it becomes due by the death of (x). Required σ, the sum to be paid to (x) for his interest in the policy.

The sum to be paid to (x)—that is, the present value of the policy—is evidently the difference between the present value of the assurance and that of the future premiums.

$$\therefore\ \sigma D_x=AM_x-\pi N_{x-1};$$

whence,

$$\sigma=\frac{AM_x-\pi N_{x-1}}{D_x}.$$

If $A=£100$, and $x=40$, then, if the assurance was effected ten years ago, and the premium calculated by the same table as is made use of for the valuation of the policy,* we should have

$$\pi=\frac{AM_{30}}{N_{29}}=1{\cdot}8864.$$

Hence we find

$$\sigma=8{\cdot}9926=£8.\ 19s.\ 10d.$$

Here we must stop. The object of the writer, in commencing the present paper, was, as stated in the outset, to furnish an easy

* This is by no means necessarily the case. The *actual* premium must always be used.

introduction to the papers on the same subject by Professor De Morgan, in the *Companion to the Almanack;* as he had found, in the course of his experience, that persons even who had paid some attention to the subject were at a loss to comprehend the scope of the papers alluded to. Whether or not he has in any degree succeeded in his object, is for those to say who may have honoured his lucubrations with an attentive perusal. He is himself sensible of many deficiencies in them, some of which, perhaps, if the opportunity were afforded him, he might be able to amend. Such as they are, however, they are now before the public; and he trusts that, as the work of an amateur, they will be viewed with indulgence. In particular, he has to bespeak the forbearance of the learned individual, in the desire to render whose writings on the subject more accessible than heretofore, they have originated. It is only now, in taking leave of the subject, that he perceives, in its full force, the presumption of which he fears he has been guilty. But as he thinks it may be allowed that his object has been praiseworthy, he trusts this will be received as an atonement for deficiencies in other respects. In the preparation of the article the writer has made free use of Professor De Morgan's papers, as indeed he could hardly fail to do, since to them and to Mr. Jones's work he owes all the information he possesses on the subject. Notwithstanding this, however, and the circumstance that his paper considerably exceeds in length those of the learned Professor, the vein from which the materials have been taken is very far indeed from being exhausted. It will amply repay a search on the part of those whom interest or inclination may lead to cultivate this branch of science.

Erratum.—Page 104, line 2 from top, *dele* "or n into $k+n$." The writer is not aware of any other error affecting any of the formulæ, nor has he found an error in the table as printed.

NOTES AND QUERIES.

Mr. Robert Christie, in a letter to Mr. Samuel Brown, says:—"It has occurred to me that it may contribute to the object in view" (that is, to the collection of materials for framing a table of mortality based on the experience of the Offices) "if the extent of life assurance actually transacted in the United Kingdom were ascertained.

"With this view, having applied to each of the native Scottish Life

Offices* for information, I am now enabled to state the present aggregate of their annual income, their realised fund, and the amount assured. These particulars, carefully excluding fire insurance and all income derived from it, are the following:—

Annual income from premiums, and from realised or accumulated funds	£2,200,933
Realised or accumulated funds	12,807,057
Amount presently assured, including bonuses	54,692,877

"I hope you may be able to obtain similar information applicable to the London and other Offices."

The Public Debt due to the Bank.

			£	s.	d.
1694	Original subscription lent to Government at 8 per cent. per annum		1,200,000	0	0
1708	Advanced to Government without interest		400,000	0	0
„	Exchequer Bills cancelled		1,775,027	17	10
1717	Ditto ditto		2,000,000	0	0
1722	Advanced to pay off South Sea stockholders		4,000,000	0	0
			9,375,027	17	10
1727-8	Deduct amount paid from the Sinking Fund		1,775,027	17	10
			7,600,000	0	0
1728	Advanced on the security of taxes		1,750,000	0	0
„	Ditto upon lottery		1,250,000	0	0
			10,600,000	0	0
„	Paid from Sinking Fund	£500,000			
1738	Ditto ditto	1,000,000			
			1,500,000	0	0
			9,100,000	0	0
1742	Advanced without interest		1,600,000	0	0
1746	Exchequer Bills cancelled		986,800	0	0
1816	Advanced at 3 per cent. interest		3,000,000	0	0
			14,686,800	0	0

* Scottish Life Offices above referred to.

Scottish Widows' Fund and Life Assurance Society	Instituted	1815	Scottish Equitable	Instituted	1831
Edinburgh Life Company	„	1823	Caledonian (Life Branch)	„	1833
North British (Life Branch)	„	1823	Northern	„	1836
Scottish Union	„	1824	Scottish Provident	„	1837
Standard	„	1825	Life Association of Scotland	„	1838
Scottish Provincial	„	1825	City of Glasgow	„	1838
Scottish Amicable	„	1826	Scottish National	„	1841
			Colonial	„	1846

		£	s.	d.
	Brought forward . . .	14,686,800	0	0
1835	By the transfer of £4,080,000 Reduced Three per Cent. Annuities, equal to one-fourth of the debt, in accordance with the 9th section of the Charter Act of 1833 (3 & 4 Will. IV., cap. 98)	3,671,700	0	0
	Total debt . . .	£11,015,100	0	0

The sum of £2,984,900 has since been added to make up the £14,000,000; and the remaining £650,000 have been added under the Act of 1844, which empowers the Bank to issue equal to *two-thirds* of the country issues withdrawn from circulation.—*Money Market Review.*

Statistics of Human Life.—The total number of human beings on earth is now computed in round numbers at 1,000,000,000. They speak 3,064 now known tongues, and in which upwards of 1,100 religions or creeds are preached. The average age of life is 33⅓ years. One-fourth of the born die before they reach the age of 7 years, and the half before the 17th year. Out of 100 persons only 6 reach the age of 60 years and upwards, while only 1 in 1,000 reaches the age of 100 years. Out of 500 only 1 attains 80 years. Out of the thousand million living persons 330,000,000 die annually, 91,000 daily, 3,730 every hour, 60 every minute, consequently 1 every second. The loss is, however balanced by the gain in new births. Tall men are supposed to live longer than short ones. Women are generally stronger than men until their 50th year, afterwards less so. Marriages are in proportion to single life (bachelors and spinsters) as 100 : 75. Both births and deaths are more frequent in the night than in the day. One-fourth of men are capable of bearing arms, but not 1 out of 1,000 is by nature inclined for the profession. The more civilised a country is, the more full of vigour, life, and health are the people. The notion that education enfeebles and degenerates the human frame is not borne out by fact.—*Weekly Paper.*

CORRESPONDENCE.

MR. WOOLHOUSE'S RECENT PAPER.

To the Editor of the Assurance Magazine.

My dear Sir,—Mr. Woolhouse is quite accurate in his investigation (*ante*, p. 128), but whether he accurately interprets my meaning of 1839, is more than I can now tell. When I say that it is not necessary that the progression should be "precisely" that of Mr. Gompertz, there seems to be an indisposition to affirm that it can very widely depart. When I then proceed to give a very general equation, there seems to be an implication that there is something like an extensive form of solution. I suppose that

I intended to announce the general equation, without taking the trouble to investigate any possible limitations, but without denying that limitations may exist. This is the conclusion about my own meaning to which I came three years ago, when I inverted the question (vol. viii., p. 181) and reduced it to the solution of a functional equation. Mr. Woolhouse cannot have been aware* of this second paper of mine, in which is "required the law of mortality under which the table of two lives follows the same law as the table of one life." The demonstrated conclusion is that nothing but Gompertz's law will do. If anyone should happen to know—as was the fact—that I examined the proof of your reprint of my old paper, he will think that I ought to have made an allusion to the recent paper of a more direct character than yours. I am very glad I did not; the consequence has been that we have Mr. Woolhouse's simple algebraical treatment of the subject, which is quite within the grasp of an elementary student.

One thing, however, is wanted: Mr. Woolhouse's solution of the functional equation is good for integer differences of n, but the law may vary in an infinite number of ways during the parts of a year. All that is needed is to notice that the *unit of time* may be any whatsoever. The same caution applies to many cases in which functional equations are employed.

In this last remark I am supposing that Mr. Woolhouse, as is usual, contemplates such a *curve* of mortality as can be laid down by the help of the numbers living at the end of each year, and by the usual principles of interpolation. These principles contain the assumption that there are no inequalities whose cycle is precisely equal to the interval of time by which values are separated. When the moon's right ascension, obtained from theory for a succession of noons and midnights, is thence obtained by interpolation at all the intermediate hours, it is assumed that there are no perceptible inequalities which have a cycle of twelve hours of mean time; and the assumption being true, the method answers. It is customary to overlook any inequalities of mortality which run their course within a year, but this neglect will not endure for ever. The changes of season have a much more sensible effect on health and life than the rotation of the earth has on the moon's motion. The time may come when the trigonometrical considerations which enter into the *complete* solution of the functional equation may be called into use; and the number living at the age x may be a function of the sun's right ascension, and perhaps of the moon's right ascension also. In the mean time the limited solution of the functional equation, when the true grounds of its practical sufficiency are pointed out, may be made a useful lesson to those who are beginning the subject.

I am, yours very truly,

April 24th, 1862. A. DE MORGAN.

ANOTHER DEMONSTRATION OF THE EXPRESSIONS FOR THE VALUE OF SINGLE AND ANNUAL PREMIUMS.

To the Editor of the Assurance Magazine.

SIR,—Can you make room for yet another determination of the single and annual premiums for assurance?

* Mr. Woolhouse was not aware of it when he wrote.—ED. *A. M.*

An assurance of £1 is to be effected on (x), for which the single premium is A_x. This premium is not to be paid now, however, but is to remain a debt upon the policy till the latter becomes a claim; and an equitable consideration is to be paid annually, during the life of (x), for its forbearance. Now, the consideration for forbearance of £1 for a year being $1-v$, that for forbearance of A_x for the same time will be $(1-v)A_x$; and this, therefore, will be the annual payment to be made during the life of (x).

Hence the total consideration to be made for the assurance of £1 will be, first, a payment at the end of the year of death of A_x, the sum forborne, the present value of which is $A_x \times A_x$, or A_x^2; and, second, an annual payment of $(1-v)A_x$, the present value of which is $(1-v)a'_x A_x$, where a'_x denotes the present value of an annuity due of £1 on (x).

Therefore, equating present values,

$$A_x = A_x^2 + (1-v)a'_x A_x;$$

whence, dividing by A_x and transposing,

$$A_x = 1-(1-v)a'_x.$$

Since the claims arising by the death of (x) are satisfied by a payment from the Office of $1-A_x$, it follows that the foregoing transaction resolves itself into an assurance of $1-A_x$, paid for by the annual premium $(1-v)A_x$. Hence, if π_x denote the annual premium for an assurance of £1, we have

$$1-A_x : 1 :: (1-v)A_x : \pi_x$$

$$\therefore \pi_x = \frac{(1-v)A_x}{1-A_x},$$

a known form, which reduces to the more usual form by substitution for A_x in terms of a'_x.

I am, Sir,

Your most obedient servant,

P. GRAY.

7, *St. Paul's Villas, Camden Town,*
4th June, 1862.

INSTITUTE OF ACTUARIES.

PROCEEDINGS OF THE INSTITUTE.

Third Ordinary Meeting, Session 1861-62.—*Monday, 27th January,* 1862.

The President in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced various donations to the library.

The undermentioned gentlemen, duly nominated at the last ordinary meeting, were elected members of the Institute:—

Fellow—Robert Thomson, Esq.

Official Associate—J. L. Pilkington, Esq., F.S.S.

Associates.

Mr. M. P. Christie, B.A.	Mr. F. J. C. Taylor.
„ E. Justican.	„ Fred. Terry.
„ W. E. S. Macdonald.	„ Chas. Woolhouse.

Mr. James Terry read a paper "On the tendency of some systems of distribution of surplus to defeat the object of life assurance."

Thanks were voted to Mr. Terry, and the meeting adjourned to 24th February, 1862.

Fourth Ordinary Meeting, Session 1861-62.—*Monday, 24th February*, 1862.

The President in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced various donations to the library.

Mr. Charles Weeks, duly nominated at the last ordinary meeting, was unanimously elected an Associate of the Institute.

Mr. Bailey read a paper "On the principles on which the funds of Life Assurance Societies should be invested."

Thanks were voted to Mr. Bailey, and the meeting adjourned to Monday, 24th March, 1862.

Fifth Ordinary Meeting, Session 1861-62.—*Monday, 24th March*, 1862.

The President in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced various donations to the library.

Mr. William D. Biden, duly nominated at the last ordinary meeting, was unanimously elected an Associate of the Institute.

Mr. Archibald Day read a paper "On the statistics of first and subsequent marriages among the families of the peerage, considered specially with reference to the calculation of premiums for assurances against issue."

Thanks having been voted to Mr. Day, the meeting adjourned to Monday, 28th April, 1862.

Sixth Ordinary Meeting, Session 1861-62.—*Monday, 28th April*, 1862.

The President in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced various donations to the library.

Mr. William R. Row, B.A., duly nominated at the last ordinary meeting, was unanimously elected an Official Associate of the Institute.

Mr. Newmarch delivered an address "On the probable future of the rate of interest in this country."

Thanks having been voted to Mr. Newmarch, the meeting adjourned to Monday, 24th November, 1862.

THE

ASSURANCE MAGAZINE,

AND

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On the Forms under which Barrett's Method is presented, and on Changes of Words and Symbols. By PROFESSOR DE MORGAN.

THE mode of using Barrett's method has now something of farrago about it: a little discussion may be useful, were it only to awaken attention to the confusion arising from changes made without adherence to the recognised laws of perspicuity.

The farrago to which I have alluded arises out of a struggle between the *annuity*, as commonly understood, and what I shall continue to call the *annuity due*. According to the technical meaning of the word *annuity*, the payment begins to *grow* at the moment of the grant: thus, a yearly annuity for 12 years, deferred 20 years, begins action at the end of 20 years, and grows the first payment in the 21st year. I shall vary the phrase used in my "Essay on Probabilities" (*Cab. Cycl.*) by allowing the words "*annuity due* for x years" to signify an annuity of x *payments*, the first immediately. Thus an annuity for 12 years, now granted, will in a year become an annuity *due*, also for 12 years.

The value of a premium, at the moment of contract, is always that of an *annuity due*, for which *premium* might be made the proper technical term, if people chose: thus an annuity due of seven years might be called a premium of seven years. But the

word *premium* is so firmly fixed to the notion of *something bought with it*, that the proposal might be distasteful as a substitute for the term *annuity*, which is not necessarily the price of anything.

Many writers speak of an annuity as constituted by a number of yearly payments; and would therefore say that a premium is a kind of annuity. Ambiguities arise from using a word which implies a portion of time, without reference to the usual, and therefore proper, dependence of the meaning upon the commencement and termination of the period. Thus a *year* and a *century* are often spoken of as portions of time merely, as in "the year ending June 5," and "the century 1762–1861," which form parts of two years and of two centuries, in the more common meanings of the words. There is not much to find fault with here; but when we deal with the word *annuity* in the same way, we may easily be led into such errors as writing 9 for 8, or 8 for 9, which it is one great business of an actuary's life to avoid. Making the word annuity mean the two different things is dangerous, because the two things are so alike: had they been very different, no mischief would have arisen. If chain armour, mail made of interlaced rings—*annus* meaning really a *ring*—had been called *annuity*, the actuary would not have been injured by *this* double meaning of the word. Slight changes of meaning are dangerous for the very reasons which prevent any comparison from being satirical except when gross exaggeration is used. Some time ago the Admiralty encouraged the introduction into the navy of the word *declination*, to signify the *dip* of the magnet—the seamen having declination already in common astronomical use. My friend the late Lieut. Raper, a man far too high in scientific navigation to approve of such abuse of language, wrote a satire on the change, in which he recommended the Lords to try how inverting the names of the fore-topsail and the jolly boat would work. He thought it might be useful, if the seamen were to be trained in ambiguous terms, to accustom them to bold innovations—so that they might learn not to be surprised at hearing that a ship coming into harbour under close-reefed jolly boat sent the fore-topsail on shore with the purser's bags for fresh beef. But the satire was lessened by the very obvious consideration that the men might get used to this change in a couple of days. But let the foremast and mainmast change names, and it would be a very different thing. It is the greatest aggravation, and therefore the farthest from excuse—of trifling with names—or with symbols, which *are* names—that the ambiguity of language is applied to things which are very much

alike. Anything* but this: call Epsom salts and laudanum by one name who likes; but not Epsom salts and oxalic acid. There would be little danger of anyone taking the *wrong* Epsom salts under the first nomenclature; but every wise man would shun both under the second.

The first innovator in this matter was, I believe, Mr. Sang, whose admirable volumes (1841, 1859), or rather whose first volume, has every annuity in it of the *due* or *premium* kind. The second volume has, for whole-life annuities, both kinds, distinctly separated as "Annuity first payment now," "Annuity first payment one year hence." I am not criticising the plan of the first volume, but only the misuse of the word *annuity*. The short or temporary annuity is wanted in most cases—by me, ninety-nine times out of a hundred—for calculation of a premium. Mr. Sang's plan, for a work having the bulk of its annuities temporary, is convenient; but the use of the word *annuity* is a blemish, and gives risk of error. Those who are going to make frequent use of the book are in no danger; they will easily bear in mind that the annuity is in all cases *due*—that every 10 years' annuity, for instance, is £1 now, and £1 at the end of each of nine years to come. But a casual user of the volume does not expect, and cannot reasonably be expected to expect, that he should be obliged to examine narrowly into all the technical terms he finds, to see whether his author adheres to their usual meanings. Such a person might chance to hear that the work in question contains annuities payable up to 50, 55, &c., and he finds a page of "*Annuities* of which the *payment* is to cease on the attainment of the undermentioned ages." He wants an annuity for age 20, ceasing at 70, and he finds 21·965. The last payment is made at the age 69, and the first now. This is not what was wanted, and is not what the words usually mean. According to common language, technical or not, when a payment is to "cease at the age 70," the last payment is made at that age.

The two systems might be distinguished as the *initial* and the *terminal* systems. The same moment of time is both the end of one year and the beginning of another. The ordinary annuity has its moment of payment reckoned as the end of a year; the annuity *due*, or premium, has its moment of payment reckoned as the

* The record of stories which have taken rise in resemblance of surnames would be a curious book. When looking after the history of Barrett, I had to inquire into a rumour of his having been assisted in his calculations by Griffith Davies. In this there was no truth whatever; but Mr. Davies remembered teaching Mr. *Berry*, who was in the Hope Office with Barrett, and whom he did assist in official calculations after Barrett had left.

beginning of a year. When logarithms are given, as in Mr. Sang's work, the scale is, I think, decidedly turned in favour of the initial, or annuity-due, system. But, were it otherwise, putting the casual user out of the question and speaking only of one who wants it more frequently, he must be a Sybarite, a man whose rest is broken by a crumbled rose-leaf, who would take serious exception to a variation so easily remembered and allowed for, and so permanently adhered to. These volumes are a Carlisle three-per-cent. paradise.

I now come to Barrett's tables; the D and N tables, commutation tables, or whatever they are at last to be called. I should like to retain Barrett's name, but the alterations in the form of the *tables* render this impracticable. The *method*, however, ought to remain attached to Barrett's name. The original form of the tables, as given by him, is *initial*. The columns are A, B, C, in black letter. The number living at the age x being l_x, r the amount of £1 in one year, and n the oldest age of the tables, Barrett's forms are

$$A_x = l_x r^{n-x},\ B_x = A_x + A_{x+1} + \ldots \qquad C_x = B_x + B_{x+1} + \ldots$$

Hence $B_x : A_x$ is the annuity due, or the premium, and $B_{x+1} : A_x$ is the common annuity.

Griffith Davies, in his tract of 1825, gave his own alteration into terminal forms. This work, we are told, was put together in a hurry, out of larger materials, on the preliminary announcement of Mr. Babbage's* work, which appeared in 1826. The columns are D, N, S, M, R. The column C, used in calculating M, but of little independent use, was introduced into the statement of results by myself.

This D and N system is terminal; and further, D_x is $l_x r^{-x}$ or $l_x v^x$. We have $N_x = D_{x+1} + \ldots$, and $S_x = N_x + \ldots$; so that the common annuities are N_x and S_x divided by D_x, the annuities *due* requiring N_{x-1} and S_{x-1}.

Which of these two systems is the best? By mathematical analogy no doubt the initial system would be preferred. The same reasons, well known to students of the calculus of differences, which make it imperative to denote $D_0 + \ldots + D_x$ by ΣD_{x+1}, not ΣD_x, would also require that $D_x + D_{x+1} + \ldots$, and not $D_{x+1} + \ldots$,

* William Morgan, in his *Rise and Progress of the Equitable Society*, 1828, made some remarks on Mr. Babbage's account of the Equitable Society. Francis Baily criticised these remarks in the *Times*, June 26, 1828; and Morgan replied July 1, 1828. These letters ought to be preserved, and I have accordingly transmitted my copies to the Editor of this Journal, for publication.

should be denoted by SD_x, or whatever symbol of summation should be used. Again, for the computer's convenience, I should also be inclined to prefer this initial system, on the principle that in life assurance, which much exceeds annuity business in amount, the annuity due, or premium, is of more importance than the simple annuity. But I am now speaking of the creation of a system. Whether it would be worth while to disturb a system in use* for the difference between the two, introduces other considerations. I discussed this question with myself when I wrote my papers on the subject: and my conclusion was that it was *not* worth while; or, rather, that the inconvenience of conflicting systems would outweigh the slight theoretical and practical advantage both.

Dr. Farr was the introducer of the initial system into the commutation tables. His announcement of the change is made in the following words (*Sixth Report of the Registrar-General*, 1845, p. 593):—

"This derangement [Davies's alteration of Barrett's arrangement] of the simple relations of the respective columns for the supposed and not real convenience of the computer, always appeared to me the reverse of an improvement; and upon finding that the columns y and $S.y$ in Duvillard's table, and the original columns A and B in Barrett's table, stand in the same relation to each other as they now stand in the English tables, I adopted that arrangement without hesitation."

The justification of the change is twofold. First, Dr. Farr thought that the established system might be changed for the better; secondly, he found authority in Duvillard's table, which is hardly known, and in Barrett's table, which was known only by specimen, Davies's form being the only one ever *used*. The question of convenience or inconvenience, arising out of the simultaneous currency of conflicting systems, appears to have been thought not worth consideration. But the change, *when properly made*, is so easily used, that these remarks may pass only as warning to future writers not to vary existing methods without at least the attempt to take into account the question of change *as change*, and to justify themselves on this point. Such justification may be possible; but it is imperatively required.

The change was *not* properly made. Griffith Davies, with the tact of a mathematician, abandoned Barrett's *letters* when he abandoned Barrett's *use* of them. Dr. Farr retained N and S, altering the sense in which they are used. This is equivalent to

* We are reminded of the Greek philosopher, who maintained that life and death are all one. "Why then do you not kill yourself?" asked an opponent. "Because it *is* all one," replied the sage."

the alteration—and, worst of all, the slight alteration—of the meaning of a *name*. On this point I hold it unnecessary to say another word.

Were I now to write a work on the subject, I should use the letter I to distinguish the initial system; the simple letter, as of old, denoting the terminal system. I should have a new type cut, an N with the first stroke made into an I, to denote the N of the initial system. The S is so litttle used that IS might be sufficient. The C, M, R, have, fortunately, not been meddled with.

Mr. Chisholm, in his most useful work, *Commutation Tables*, &c., 1858, has adopted the initial system. His warnings are clear enough to those who examine before using, with one exception, which shows how apt slight differences are to be held no differences at all. He says (p. xlii.) that "the D and N columns for joint lives thus formed, being identical in construction with the preparatory tables of Mr. Jones on the Carlisle basis, the two sets of tables were carefully read over together . . . ." The words "identical in construction" are not correctly applied to two tables which always differ in tabular result throughout one of the two columns which compose them. This sentence, which comes very conspicuously at the end of a half page of print, once nearly led me into serious error. But Mr. Chisholm's work, like Mr. Sang's, is above slight criticism, both by the quality and the quantity of its matter.

Mr. Chisholm's work offers an incidental example of the superior legibility of figures with heads and tails. The type most used is large and even; but, from the necessity of the case, the commutation tables for single lives are in a much smaller type, which happens to have heads and tails. I think anyone who looks at the two will see that the smaller figures project themselves into the eye, while the larger ones must be picked up by the eye.

To return to the subject. It is much to be hoped that in all new undertakings, when any apparently desirable change presents itself, severe examination will be given to the question whether the alteration, be it what it may in itself, is an advantageous addition to what actually exists. But even this is of small importance compared with the warning of history, which tells us not to meddle with names and change their meanings. *Extension* is allowable, when advantageous; that is, inclusion of the old meaning, *and more*. But mere change, whatever may be right, is always wrong.

It may be worth remembrance that since D_0 is always 1000 . . . , the *initial* form immediately shows itself by the first two Ns, N_0 and N_1, differing in only one of their digits.

While on the subject of changes, I may notice a tendency which prevails to substitute one word for two, under the idea of abbreviation. If letters be saved, there is advantage gained in writing. If syllables be saved, there is often—not always—advantage gained in speaking. But if neither letters nor syllables be saved, there can be no advantage, unless some awkward sound be got rid of. What then shall we say to the word *predecease*, in the sense of *die before*, with one more letter, as many syllables, and an ugly double sibilant? The word *decease* itself is a disgrace to our language: it must have been made* by some one who imagined his word was connected with *cessare* instead of *cedere*. I wish some of our purists would write *decess*, as in *process*, *recess*, *access*, *success*, *excess*, from the same verb. In the seventeenth century, *decession* is noted as a word coming in. The etymological objection, however, might be overlooked if advantage were gained. In any case the word should be *predecede*, and *precede* would do; for an actuary's order of going almost always relates to death. But surely *die before* is better than either; and it allows its correlative *die after*. Are we also to have *post-deceasing?* To my ear *predecease*, as an active verb, conveys the idea of previous slaughter; and a man who precedeased his father is one, I suppose, who killed his father before he killed his mother.

When two words become a phrase, they are really one word, both in meaning and pronunciation. Etymologists are now agreed, I believe, that every syllable is a word, originally. The Chinese language, commonly called monosyllabic, is only so in this respect, that all its syllables have kept their separate meanings: the Chinese speaker runs his significant syllables together, and makes polysyllabic words, just as we do. If the phrase *annuity due* were used, it would be one word immediately.

Again, the introduction of a monosyllable, as more easy to

* The first notorious place in which I find it is in the authorised version of the Bible; but it also appears in the older Protestant versions. Omitting Isaiah xxvi. 14, which those who compare the Septuagint with the English will suspect of having a difficulty in the Hebrew, it is seen in Matth. xxii. 25, Luke ix. 31, 2 Pet. i. 15. In the first passage ὁ πρωτος γαμησας ἐτελευτησε is translated "the first, when he had married a wife, deceased." And here the Greek is really *came to an end, ceased off;* and here Tyndale (1534) and his followers have the neuter verb *deceased*. But in the other two passages, in which the substantive *decease* of our version is the translation of ἐξοδος, *departure*, the previous Protestant versions have *departing*, while the Rhemish (Roman, 1582) version has *decease*. It may be suspected that Tyndale meant *decessatio*, and the Rhemish translator *decessus*, and that our translators adopted both.

Shakspere has one use of the word: he makes Gower reproach Ancient Pistol with mocking at a memorial of "predeceased valour": and down goes the word into dictionaries as common English of Shakspere's time. Gower is making rather a stilted speech, and it may be surmised that the dramatist intended to represent him as inventing the word for his own use. If so, this is far from being the only case in which the English peculiar to a comic character has been made the English of the poet's contemporaries.

pronounce than a dissyllable, may be a mistake. When a monosyllable is long and emphatic, it very often *is* a dissyllable. When we say *mill*, unless we use great effort, the removal of the tongue from the palate before the sound has ceased makes *mill-e*, and gives the second syllable which the French recognise in *cel-le*, which English learners generally take for a monosyllable. Consequently, *miller*, pronounced without the *r*, as usual, is an easier word than *mill*: the second syllable is easier than the pull-up by which it is avoided. If the call-boys in the steam-packets were made to say "Ease," "Stop," "Back," instead of "Ease her," "Stop her," "Back her," they would soon learn the English truth of a Greek pun. They would find out that their new monosyllables are really dyssyllables, and would be glad to restore the old vowel. There is a Joe Miller about a man who heard a countrywoman call her son Hookey, and asked what his name was. "Sir," was the answer, "his name is *Hugh*, but we call him *Hookey* for short." The story-maker set this down as laughable for ignorance, but the good woman was right. And this is the true explanation of Willy and Charley, and a host of other abbreviations.

These things are not out of place when we are on the subject of word-making and word-changing; and for this reason, that some word-makers and word-changers seem not to know them.

There is one point about the printing of commutation tables which deserves notice and discussion. The N and S have so many figures before the decimal point that when, as often happens, only a few are wanted, the selection is perplexing—especially when the difference between two Ns in widely-separated parts of the table is wanted. We need a second decimal point, to separate higher from lower integers. Thus a separator for the thousands would be convenient: if this were used, Chisholm's N_0, at 4 per cent., would be printed 152,816·4338. In doing this we return to an old practice. Before the point or comma became the separator of integer and fraction, it was extensively used as an integer separator. The earliest tables of logarithms gave integer logarithms. In Brigg's *Arithmetica Logarithmica*, for instance, we have

$$\log 10 = 1,\ 00000,\ 00000,\ 0000$$
$$\log 12 = 1,\ 07918,\ 12460,\ 4762$$

It is not true that the first of the commas is our decimal point: Brigg's logarithm of 10 is a hundred million millions. When he wants a separator of fractions, he uses another method. Thus, in the *Lectori Salutem*, he wants the square and fourth roots of $85\frac{1}{3}$, and he gives them thus:—"Latus $85\frac{1}{3}$ est $9^{\underline{237604307}}$, et hujus

lateris latus est $3^{\underline{03984274}}$ cui numero æquatur $ll85\frac{1}{3}$." It would, perhaps, be desirable that the integer separator should be a semicolon formed with two commas; so that a casual fracture would certainly leave a comma, which would not be confounded with the decimal point.

The Letters from Francis Baily and William Morgan, referred to herein by Professor De Morgan, are as follows:—

EQUITABLE SOCIETY.

To the Editor of the Times.

SIR,—A pamphlet has lately been published by Mr. Morgan, containing (amongst other things) some remarks on what he is pleased to call "the late misrepresentations respecting the rules and practice of the Equitable Society;" and in the chapter which he has appropriated to this subject he has passed some very unmerited censures, and made many unfounded remarks, on the recent production of my friend Mr. Babbage, the present Lucasian Professor of Mathematics at Cambridge. Mr. Babbage is now making a tour on the continent, and the time of his return to this country is somewhat uncertain. But, as he honoured me with the perusal of his work prior to its being sent to the press, and was pleased to consult me in its progress, I consider myself in some measure called upon, under existing circumstances, to step forward on the present occasion to advocate his cause during his absence, not only by denying that spirit of misrepresentation of which he has been thus falsely accused, but also by challenging Mr. Morgan to a proof of the principal charges which he has thought proper to make, leaving to Mr. Babbage the task of answering more at length, on his return, the whole subject of Mr. Morgan's pamphlet, should he think such a step requisite or necessary.

Mr. Morgan commences his attack on Mr. Babbage, in page 52, by accusing him of stating that the Equitable Society has a large unfunded capital, which it conceals from the knowledge of the public. This charge is so utterly unfounded, and so totally at variance with any opinion that Mr. Babbage has expressed on the subject, that I am quite at a loss to conceive where Mr. Morgan could have met with any passage tending towards such a meaning, or that could even be forced into so opprobrious a construction.

Mr. Morgan next insinuates, in page 54, that Mr. Babbage treats the decennial investigations which are submitted to the members as a fallacy calculated only to delude the public. This passage also, like the former, exists only in Mr. Morgan's imagination, since there is no expression in Mr. Babbage's work tending in the most remote manner to such an opinion; and it can only be by a singular perversion of the ordinary meaning of words that such an interpretation could have been deduced from anything that he has said.

Mr. Morgan further states, in page 56, that Mr. Babbage accuses the Society of injustice, in the unequal method of distributing the surplus profits of the Society. Now, it is true that Mr. Babbage has expressed an opinion (not amounting, however, to an accusation) that the mode adopted by the Equitable Society is not the most equitable mode of distributing the surplus

profits of an Assurance Society, and in this opinion he is supported by many others as conversant with the subject as Mr. Morgan is; but Mr. Morgan ought in candour to have quoted the whole paragragh, which would have removed the sting that, under the colour of his misrepresentation, appears to exist. This formidable passage, which seems to have given so much offence, is in the following words:—"This inequitable mode of apportioning the profits is not, however, so disadvantageous in a system of mutual assurers, because the reserved surplus again accumulates for the benefit of the assured at the next period of division." In Mr. Morgan's opinion, it seems little short of high treason even to doubt the wisdom of any of the rules and regulations passed at Chatham Place.

Mr. Morgan, in page 59, represents Mr. Babbage as recommending that the division of profit should be made annually, and the whole of the surplus distributed amongst the members; but this is not the fact. Mr. Babbage certainly leans to the opinion that the divisions of the profits of an Assurance Office (not alluding, however, to the Equitable) ought to be more frequent; and I perfectly agree with him that, by the aid of an improved system of keeping accounts, it would by no means be difficult to assign annually to each assurer his proportion of the profits. But Mr. Babbage nowhere states that the whole of the profits ought to be so distributed; and these two opposite and distinct subjects ought not to have been thus unnecessarily united and placed in juxtaposition, in order to raise a prejudice against Mr. Babbage's opinion on these subjects.

Mr. Morgan further accuses Mr. Babbage, in page 61, of inaccuracy, in stating that no benefit attaches to an assurance (subsequent to its becoming one of the fortunate 5,000) until the next succeeding order; and, with an air of triumph, he announces that it immediately comes within the order for prospective additions, as soon as it falls within the golden number above mentioned. But Mr. Morgan must have well known that Mr. Babbage was speaking of the retrospective, and not of the prospective, additions—a distinction better known in its practical application than in the obscure wording of the rules of the Society. In this point of view Mr. Babbage was correct. The former term alludes to the large decennial divisions of profit, where the *bonus* is multiplied by the number of years that the assurer has been in the Society, whereas the other addition is very small in amount, and has not even been particularly alluded to by Mr. Babbage.

But I shall not proceed further in these remarks. I have already said enough to show the spirit in which Mr. Morgan's pamphlet is written, and how unfairly he has treated Mr. Babbage, both in his quotations and in his comments. The high character and splendid abilities of these two gentlemen ought to have removed all ground for even the suspicion of misrepresentation. Mr. Babbage's work was written with a view to point out the various shades of difference between the several Offices established in London for the assurance of lives, as well as to furnish some useful hints to those who might be disposed at any future time to establish new ones. In doing this he has certainly rendered an essential service to the public; and the Equitable Society, so far from being censured, as insisted upon by Mr. Morgan, is everywhere held up (as it deservedly is) as the best institution of the kind in existence.

Mr. Morgan seems very much annoyed at the interest which the public take in the proceedings at Chatham Place; and "cannot see what the public, distinct from its members, has to do with the concerns of the Society." But

Mr. Morgan ought to know, that when a Society issues its proposals to the public, thus inviting them to join their concern, the public have a right to canvass and discuss the proposals thus tendered to them, and that in such discussion the whole establishment comes under review. Besides, where so many millions of money are concerned, it is not the members alone that are interested, since the prospect of ultimately enjoying this property is diffused, not amongst the members themselves, but amongst a vast assemblage of relations and expectants, all of whom consider themselves as fairly entitled to give an opinion on what is going forward.

The Equitable Society, however, need not fear the test of any public examination or inquiry. For, though its origin, according to Mr. Morgan's own showing, was not very flattering, and although in its infancy it seems to have adopted practices which are deprecated in even the worst of the present Joint Stock Companies, yet, by a speedy reformation, the Society advanced in credit and renown; and, under the prudent and able administration of Mr. Morgan himself, it has arrived at a degree of opulence and importance unparalleled in the history of such undertakings.

I have the honour to be, Sir,
Your obedient servant,
FRANCIS BAILY.

37, *Tavistock Place, June* 24, 1828.

To the Editor of the Times.

SIR,—I feel no disposition to enter into any dispute or discussion respecting my late publication, and therefore have only to observe, in answer to Mr. Baily's letter in your paper, that he has put a construction on the remarks I made on Mr. Babbage's work which I never intended. In using the word misrepresentation, I had too much respect for that gentleman to suppose that it could have arisen from any other cause than misinformation, and have accordingly regretted that he should not have been better acquainted with the rules and practice of the Equitable Society before he made them the subject of his animadversion. With regard to the passages in which I am accused of having misrepresented him, I see no reason to alter my opinion, and consequently do not think it necessary to notice all the instances in which I am charged with acting unfairly towards Mr. Babbage. I shall just select the first and second articles in Mr. Baily's letter, which will be sufficient for my present purpose, and must refer to the works themselves for a justification of the rest.

In the first article it is said "that I commenced my attack on Mr. Babbage by accusing him of stating that the Equitable Society had a large unfunded capital, which they concealed from the knowledge of the public." Mr. Babbage's words are—"It is said that they possess a large undivided capital, of which the public, from want of sufficient information, have very vague, and, perhaps, extravagant ideas" (page 40). In giving this account, without expressing a doubt as to its authenticity, Mr. Babbage so far sanctions it, and I was naturally led to express my surprise from what source he could have derived his information, and how he could have given any credit to such a tale.

In the second article it is said "that I have accused Mr. Babbage with treating the decennial investigations which are submitted to the members as a fallacy calculated only to delude the public." Now this is not strictly

correct. I have only observed, that, from the manner in which these statements are represented to have been given, it might be inferred that the whole was a fallacy, calculated only to delude the public; and I wish to know what other inference could be deduced from the three following passages:—"Although two-thirds is the sum nominally divided amongst the assured, since that sum is only payable on the death of the respective nominees, it is not in reality equal to two-thirds, and is, in fact, different for different ages" (page 81). Again, "these facts show that the two-thirds apparently given to the assured, are, in reality, not equal to one-half the total profits" (page 82). And, in page 83, "that two-thirds of the profits are not divided among the assured, as they are led to believe in the prospectus." With such an accusation against them, is it possible to think respectfully of the abilities or integrity of those who conduct the affairs of the Society? Mr. Baily mentions his having read Mr. Babbage's work before its publication, and therefore makes himself more blameable on this occasion than Mr. Babbage, who must have naturally concluded himself to be correct when supported by a gentleman, not only so well acquainted with the subject, but also who had been so many years a member of the Society.

Had Mr. Babbage applied to myself before, as he did after, the publication of his work, I have no doubt that he would have corrected most of the mistakes which I pointed out to him, and I should have been spared the unpleasant task of noticing them in public, which has been really done, not with the view of "passing unmerited censures upon him," but of rectifying his mistakes, and thus preventing the Society from being misled by them.

I am, Sir, yours, &c.,

Equitable Assurance, June 30, 1828. WM. MORGAN.

On the proper Mode of estimating the Liabilities of Life Insurance Companies. By ROBERT TUCKER, *Actuary to the Pelican Life Insurance Company, and one of the Vice-Presidents of the Institute of Actuaries.*

[Read before the Institute, 24th November, 1862, and printed by order of the Council.]

THE object of this paper is to endeavour to show that the true method of estimating the liabilities of an Insurance Company should be based upon the principle of re-insurance by its own rates of premium, interest being reckoned at 3 per cent.

In submitting the following observations, I feel it necessary, in the first place, to apologise for recurring to a subject which has been so frequently and ably treated by the President of the Institute of Actuaries. But as my method of dealing with this important branch of our business differs in some respects from the plan laid down by that learned writer, I trust I may be allowed to record the present attempt to demonstrate what in my opinion are

the true principles which should guide us in determining the liabilities of Life Insurance Companies.

Let π'_x denote the Office annual premium at any age x,

a'_x the corresponding life annuity, and

A'_x the corresponding single premium;

then the liability of a Company in respect of each policy at the end of the first period of division, say n years, may be determined from any one of the following well-known formulæ, viz.:—

$$(\pi'_{x+n}-\pi'_x).(1+a'_{x+n});$$

or,
$$A'_{x+n}-\pi_x'.(1+a'_{x+n});$$

or,
$$1-\frac{1+a'_{x+n}}{1+a'_x};$$

or,
$$1-\frac{\pi'_x+d}{\pi'_{x+n}+d};$$

If π_x denote the pure annual premium, and k the percentage of loading, then

$$\pi'_x=(1+k)\pi_x.$$

Suppose an Office to charge premiums based upon the Carlisle 3 per cent. table, with a loading or addition to each premium of 25 per cent., then we have the following values:—

Age.	π'_x.	a'_x.	A'_x.
20	1·8667	19·9223	39·0613
30	2·4400	17·6828	45·5841
40	3·2491	15·2291	52·7202
50	4·5279	12·4398	60·8546

&c. &c.

The value of a policy of £100 at several decennial periods will be as follows:—

Age when Assured.	Period Elapsed.				
	10 Years.	20 Years.	30 Years.	40 Years.	50 Years.
20	10·7039	22·4315	35·7631	52·9084	67·3967
30	13·1334	28·0630	47·2635	63·4885	76·2871
40	17·1869	39·2903	57·9684	72·7020	82·3946
50	26·6907	49·2452	67·0366	78·7408	
60	30·7662	55·0351	71·0007		

The value of a policy at the same ages and periods, when the loading is thrown off and the pure premium only is taken into account on both sides, is

Age when Assured.	Period Elapsed.				
	10 Years.	20 Years.	30 Years.	40 Years.	50 Years.
20	9·4219	20·0609	32·5747	49·3662	64·2066
30	11·7455	25·5617	44·1001	60·4839	73·9026
40	15·6549	36·6598	55·2255	70·4304	80·7142
50	24·9041	46·9138	64·9412	77·1365	
60	29·3103	53·3151	69·5537		

On comparing these two sets of values, it is noticed that the reserve, when the *pure premium* only enters both sides of the account, is less than when the estimate is made by the *full premium*.

The reason of this difference will be apparent when we consider the effect of a pure premium valuation upon old and new members. A member now aged 60, but who entered at 20, pays, in addition to the net value of the risk, 7*s*. 6*d*. per cent. per annum, whilst a member entering at age 60 pays £1. 9*s*.

By the pure premium method of valuation the contribution of a member at age $(x+n)$ years is—the original premium,

$$\pi'_x \text{ or } (1+k)\pi_x;$$

and the reserve,

$$\pi_{x+n}-\pi_x;$$

together,

$$=\pi_{x+n}+k\pi_x.$$

By the other method, the contribution is—the original premium,

$$\pi'_x;$$

and the reserve,

$$\pi'_{x+n}-\pi'_x;$$

that is,

$$\pi'_{x+n}=\pi_{x+n}+k\pi_{x+n};$$

If the loading is a constant quantity, $k\pi_{x+n}=k\pi_x$, and the inequality above noticed would be corrected. But an uniform addition to the premium is objectionable in practice, because it makes the rates too high at the younger and too low at the older ages. Besides the reason assigned for this plan—viz., that each person should pay the same contribution, is true only as to amount, and not in respect of value. For A'_x-A_x will decrease as the age increases when the loading is constant, but this is not so when it

is by a percentage on the premium, as will be seen in the following examples deduced from the Carlisle 3 per cent. table—A'_x denoting the value of the annual premium + 25 per cent., A_x the pure premium :—

	Age 20.	30.	40.	50.	60.
A'_x	39·0613	45·5841	52·7202	60·8546	71·3030
A_x	33·8973	40·1254	47·1580	55·4303	66·5299
	5·164	5·4587	5·5622	5·4243	4·7731

The custom of paying an annual commission of 5 per cent. on a premium would seem to offer another practical objection to a constant loading. But this difficulty is sought to be overcome by making the loading to partake of both characters—a constant and a percentage. In whatever way the annual premiums are loaded, if they are not made the basis for calculating the other rates of the Office, it must lead to some anomalies in its premiums.

The following rates appear in the prospectus of an Office having a consulting actuary :—

Survivorship Rates.	*Joint Life Rates.*
20 *v.* 25 = £1 5 10 per cent. per annum.	
25 *v.* 20 = 1 10 9	20 and 25 = £3 13 1 per cent. per annum.
£2 16 7	
40 *v.* 45 = £2 0 1	
45 *v.* 50 = 2 14 8	40 and 45 = £5 13 0
£4 14 9	
29 *v.* 20 = £1 6 11	
2	20 and 20 = £3 9 10
£2 13 10	

And, generally, throughout the table similar irregularities are to be met with. So that, in all cases whenever it is desired to make a joint life insurance, a considerable saving may be effected by taking out two survivorship policies.

It is difficult to discover upon what principle these tables have been constructed—certainly without regard to the equation

$$\frac{A_{x,y}}{1+a_{x,y}} = \frac{A^1_{x,y} + A_{\overline{x,y}|}}{1+a_{x,y}}.$$

Indeed, it is obvious that, unless the relation between these and other algebraical values of different descriptions of risk is to be maintained, the elements employed must be those which are immediately deducible from the ordinary rates of premium.

For instance, if $\pi'_x = 1{\cdot}8667$ at age 20,

$$a'_x \text{ must} = 19{\cdot}9223,$$
$$\text{and } A'_x \quad = 39{\cdot}0613,$$

when interest is reckoned at 3 per cent.

Now, suppose an Office, as some have been known to do, to base their annual premiums upon the Carlisle 4 per cent. table, with a loading of 40 per cent., we should have, at age 75,

$$\pi_x = {\cdot}12182 \qquad a_x = 5{\cdot}239 \qquad A_x = {\cdot}76004$$
$$\pi'_x = {\cdot}17055 \qquad a'_x = 3{\cdot}785 \qquad A'_x = {\cdot}816$$

But if the custom had been to add 40 per cent. to all the Office premiums, the single premium at age 75 would be 1·06406; that is, an Office would require £106. 8*s*. 2*d*. to insure £100 at death, which is absurd.

A'_x being the premium to insure £1 at death, its value can never be made to appear greater than unity, and must, therefore, be determined from $\pi'_x.(1+a'_x)$, and not $\pi'_x(1+a_x)$.

In constructing a table of annual premiums it is not material whether the rate of interest employed is 3, 3½, or 4 per cent., provided the percentage added to each premium increases with the rate of interest. For instance, if we take the Carlisle table, it will be found that 3 per cent. premiums loaded with 25 per cent., or 3½ per cent. with 30 per cent., or 4 per cent. with 35 per cent. added to each premium, are nearly equal at all the important ages. It is only at the very early periods of life that the rates are lower, and at the more advanced ages higher, than we are accustomed to see them when 4 per cent. interest is used.

But it is otherwise when we come to estimate the liabilities of a Company, for then the higher the rate of interest used the smaller will be the reserve. It would therefore appear, that practically and for the sake of uniformity, as well as for safety, it is not desirable to use a higher rate of interest than 3 per cent. in the construction of assurance premiums, and in valuing the liabilities of any Company using them.

If we recognise the principle that from each person effecting an insurance for the whole term of life the Company should derive the same profit upon each £100 assured, the loading must be by a percentage on the annual premium; so that $A'_x - A_x$ may be constant, or nearly so, at all insurable ages. It should be borne

in mind that the entrant is for life; that the value of the risk is determined in a single payment, and that the annual contribution is merely its equivalent spread over the duration of the risk.

We have seen that a *pure premium* valuation not only gives a smaller reserve than a valuation by the *full premium*, but it is unfair to incoming members, because they are charged a higher rate of contribution than those already admitted. When the business of one Office is transferred to another, they are said to amalgamate;* and if, after a valuation made upon the same principles by both Offices, one is found to be richer than the other, the surplus in the one case is got rid of, or the deficiency in the other is made good, in order that they may amalgamate upon equal terms. The principle of equality is here clearly recognised, but it is not observed as between old and new members in an Office valuing its pure premiums only.

The following example is given at page 186 of vol. iii. of the *Magazine*:—It is supposed that "a Society charging the Northampton premiums, and making its calculations by that table, to have been in existence 40 years, and to consist of 4,000 members, each assured for £1,000, at the age of 30; but the whole body divided into four classes of 1,000 each, the duration of whose assurances is 10, 20, 30 and 40 years respectively. It is further supposed that the real rate of interest prevailing is 3 per cent., and the real mortality that called the 'experience.'"

To the liabilities thus deduced I will add those derived from the Carlisle 3 per cent. pure premiums and the Carlisle 3 per cent. premiums with a loading of 25 per cent.

	Northampton Table.	Experience.	Carlisle, Pure.	Carlisle, Loaded.
	£	£	£	£
Value of £1,000,000 assured 10 years†	141,978	145,947	136,974	155,734
" " " 20 "	276,590	305,089	275,136	305,030
" " " 30 "	425,051	480,059	460,520	497,035
" " " 40 "	594,938	648,611	624,358	659,285
	1,438,557	1,579,706	1,496,988	1,617,084

* See Mr. Jellicoe's paper "On the Principles which should govern Assurance Companies in Amalgamating," 28th Feb. 1858.

† The values given in Mr. Jellicoe's examples suppose the year's premium to have been just paid: I have, for the sake of comparison, made the same assumption in the other cases.

It will be observed that the valuation by the Carlisle loaded premiums gives a larger reserve than is shown by any of the other rates; that it is only about 2½ per cent. in excess of that recommended by Mr. Jellicoe, but is 8 per cent. higher than by the Carlisle pure premiums, and nearly 12½ per cent. by the Northampton Table.

We are warned that the "good old Northampton table" is rather a dangerous one to trust to. Undoubtedly it is, as Mr. Jellicoe proves by the example just quoted. But that "a similar result may be looked for whenever the rates of interest are the same and the true mortality less than that of the factitious table," is not quite so apparent. I have shown practically in the examples already given, and I shall presently demonstrate theoretically, that the reserve by a table of premiums loaded with a percentage is greater than the reserve by a table of pure premiums.

But suppose we value by what is called a *true table* and at a *true rate of interest*, and deduct from the value of the future premiums a percentage for profit and expenses: if the annual premiums, or some of them, have been commuted for a single payment, the plan fails, and the reserve becomes insufficient unless some contrivance is resorted to to give a greater margin. Besides the deduction, if arbitrary, is objectionable; and if it is the exact value of the loading, the reserve is the same as by the pure premium method. For

$$A_{x+n}-\pi'_x(1+a_{x+n})-k\pi_x(1+a_{x+n})=A_{x+n}-\pi_x(1+a_{x+n}).$$

The effect of any mode of valuation is to assume that certain premiums will be payable in future upon the policies existing and included in such valuation. And if an Office distributes its surplus according to this estimate, it places itself, in point of security, in the same position as if those premiums had been originally charged. Because, as the late Mr. David Jones remarks at page 1094 of his treatise *On the Value of Annuities and Reversionary Payments*, the sum reserved and the value of the future premiums are together equal to the single premium that would be charged to reassure the various lives at their present ages according to the particular table used for the valuation.

Suppose an Office to anticipate the value of the loading upon all its policies, then we should have, for the value of the future premiums,

$$\pi'_x.(1+a_{x+n});$$

and for the reserve or value of the policies,

$$(\pi_{x+n}-\pi'_x).(1+a_{x+n});$$

these together,

$$=\pi_{x+n}(1+a_{x+n}) \text{ or } A_{x+n},$$

the single premium required to reinsure all the lives at their present ages, without any margin for expenses, commission, &c.

If the valuation were made by the Carlisle 4 per cent. table, an

Office charging originally Carlisle 3 per cent. premiums, with 25 per cent. added thereto, would have in hand and in the value of the future premiums upon every £100 insured—

At age 20 . .	25·532	instead of	39·061
30 . .	31·338	„	45·584
40 . .	38·178	„	52·720
50 . .	46·658	„	60·855
60 . .	58·987	„	71·303

Let V'_{x+n} represent the value of a policy by Office premiums at age $(x+n)$ years,

V_{x+n} the value by pure premiums at same age,

then $$V'_{x+n}=(1+a'_{x+n}).(\pi'_{x+n}-\pi'_x).$$

Now,

$$1+a'_{x+n}=\frac{1}{\pi'_{x+n}+d}=\frac{1}{(1+k)\pi_{x+n}+d}$$
$$=\frac{1}{\frac{1}{1+a_{x+n}}+k\pi_{x+n}}=\frac{1+a_{x+n}}{1+k\pi_{x+n}(1+a_{x+n})},$$

and

$$(\pi'_{x+n}-\pi'_x)=(1+k)(\pi_{x+n}-\pi_x);$$

$$\therefore V'_{x+n}=\frac{1+k}{1+k\pi_{x+n}(1+a_{x+n})}\times(1+a_{x+n}).(\pi_{x+n}-\pi_x)$$
$$=\frac{1+k}{1+k\pi_{x+n}.(1+a_{x+n})}\times V_{x+n}.$$

If, instead of an Office debiting and crediting itself with the value of the pure premium or the full premium, it debits itself with value of the pure premium and takes credit for the value of the full premium, then its liability is represented by

$$(\pi_{x+n}-\pi'_x).(1+a_{x+n}), \text{ or}$$
$$\{\pi_{x+n}-(1+k)\pi_x\}(1+a_{x+n}), \text{ or}$$
$$(\pi_{x+n}-\pi_x)(1+a_{x+n})-k\pi_x(1+a_{x+n})=V''_{x+n} \text{ suppose,}$$

then $$V''_{x+n}=V_{x+n}-k\pi_x(1+a_{x+n});$$

and, since

$$V'_{x+n}=V_{x+n}\times\frac{1+k}{1+k\pi_{x_{x+n}}(1+a_{x+n})}$$

$$V''_{x+n}=V'_{x+n}\times\frac{1+k\pi_{x+n}(1+a_{x+n})}{1+k}-k\pi_x.(1+a_{x+n}).$$

These expressions serve to point out the difference in the value of a policy by the three methods here employed.

$$V_{x+n} \text{ is} < V'_{x+n} \text{ by the quantity } \frac{1+k}{1+k\pi_x(1+a_{x+n})},$$

$$\text{and } V''_{x+n} \text{ is} < V_{x+n} \text{ by the quantity } k\pi_x(1+a_{x+n});$$

showing, in this last case, that the remaining value of the original loading is anticipated. They also enable us to pass from one value to another; but nothing is saved in point of labour by doing so, because it is quite as easy to calculate the value of a policy by each method separately and compare the result.

To find the relation between A'_x and A_x, and between $1+a'_x$ and $1+a_x$:—

$$1+a_x=\frac{1}{\pi_x+d},$$

$$1+a'_x=\frac{1}{\pi'_x+d}=\frac{1}{(1+k)\pi_x+d}=\frac{1}{\dfrac{1}{1+a_x}+k\pi_x};$$

$$\begin{aligned}
A_x&=\pi_x.(1+a_x)\\
A'_x&=\pi'_x(1+a'_x)\\
&=(1+k)\pi_x.(1+a'_x)\\
&=\frac{(1+k)\pi_x}{\dfrac{1}{1+a_x}+k.\pi_x}\\
&=\frac{1+k}{\dfrac{1}{\pi_x(1+a_x)}+k}\\
&=\frac{1+k}{\dfrac{1}{A_x}+k}.
\end{aligned}$$

I think I have now shown that the proper mode of estimating the liabilities of Life Insurance Companies is by the principle of reinsurance. This principle is clearly recognised and acted upon by Fire and Marine Insurance Companies. In determining what profit these latter Companies have made, it is their practice to set aside a sufficient sum to reassure their outstanding risk. I mean such a sum as another Office charging similar premiums would be willing to undertake the liabilities for, and not an arbitrary sum which some one or more Offices desirous to extend their operations might be content to take.

This principle is also recognised in the standard works on life contingencies, and acted upon for many years by all actuaries.

When an Office values its liabilities upon the principle of reinsurance, no doubt, I submit, can arise as to its position at any given rest. For the value of the depreciation by reason of the increased age of the lives assured—that is, the value of the difference between the premiums that would be required to reinsure the amount at risk and the premium actually receivable—is determined; and this can only be ascertained by this particular mode of valuation. If the funds of the Company other than the paid-up capital or what belongs exclusively to the proprietors, exceeds the value of the outstanding risk, the *difference* is the *surplus* for division. If the assets fall short of this value, the *difference* is the *deficiency*, which ought to be made good out of the proprietors' fund. It is necessary, of course, in the first instance, to charge such premiums as may fairly be taken to represent the true value of the risk, and to add a percentage thereto sufficient for expenses and bonus purposes; otherwise inadequate rates of premium will lead to insufficient reserves by this as well as by the pure premium method. The complaint that is frequently made against the mode of valuation here contended for is that we are working in the dark, and we know not what margin is set aside at each periodical rest. This complaint has no foundation in fact—the method is the most simple that can be devised—the margin reserved being the loading at the present age of each assured life, instead of the original loading.

Suppose £1,000,000 to be now assured on lives all aged 60 years, the policies to have been in force 30 years, and the additions made by way of bonus to be £200,000. The value of this risk by the Carlisle loaded table is £615,241, and by the Carlisle pure table £574,061. In the former case the Office reserves the loading on £1,200,000, at age 60, which is £1. 9*s*. per cent., and amounts to £17,400 per annum. In the latter the reserve is the original loading on £1,000,000, at age 30, and only amounts, at 9*s*. 9*d*. per cent., to £4,875 per annum. I admit that it has its inconveniences, because a bonus can only be declared out of realised profits, and that it is not possible to draw upon futurity without showing what amount has been anticipated.

On the other hand, the practice of treating as an asset the value of what has been termed "the marginal guaranty," and which has yet to be realised, affords an opportunity of declaring profits out of capital which has not been neglected. Indeed instances are on record where the whole of this "marginal guaranty" has been anticipated.

This subject has of late years engaged the attention of many

persons interested in the welfare of Life Assurance Institutions. The aid of actuaries has been sought to test the solvency of a particular Company, or to ascertain whether the members of a Society receive their due share of profit. It is much to be regretted that these questions do not always receive a satisfactory solution, owing, it is believed, to the difference of opinion which is said to prevail amongst actuaries of more or less eminence in their profession. But is this question a mere matter of opinion? I think not. When an actuary is asked, How should an Office using a given table value its liabilities, so that no portion of its future profits may be anticipated, and that old and new members of the same age may stand upon an equality, there can be but one answer—Upon the principle of reinsurance here laid down.

On the Valuation of Policies of Assurance.

(Concluded from page 267.)

CASE 6.—*Surrender value, or the price the holder of a policy should receive from the Office on a mutual cancelling of the engagement.*

THE premium paid on a policy of assurance is divisible into two parts—the first, or net premium, representing the annual contribution absolutely necessary, on a calculation of averages, to provide a stipulated sum on death; and the second being an arranged contribution towards the expenses of the Office.

The contract entered into by the parties to the policy, is—

On the part of the assured, to pay a certain sum annually till his death.

On the part of the Office, to pay to the representatives of the assured, on his death, or at a reasonable time afterwards, the sum assured, on condition that all the agreed payments on the part of the assured have been duly made.

If the assured wish to break his part of the contract, *i.e.*, to withdraw those contributions on the expectation of the continuance of which the calculations of the Office are based—he would have no reasonable ground of complaint if forfeiture were the result; but it is customary with Offices to make an allowance in consideration of the surrender, on the ground that the payments already made have been more than sufficient to meet the risk already run.

Rule B would give the surrender value of the policy, on the following conditions, viz.:—

1st. That the premiums charged were absolutely net premiums, or increased by an equal charge per £1 upon the sum assured in order to meet expenses, and that the annuity value was also a net value.

2ndly. That the experience of the Office exactly coincided with the data used in the calculation of its tables.

3rdly. That no injury results to the Office or to other contributors by the withdrawal of the assured.

The first of these conditions is rarely, if ever, fulfilled; the second has never been; and the third is a question that can be decided only on a scrutiny of the Office accounts.

The real value of a policy, upon its surrender to the Office, depends on (1st) the mortality experienced by the Office, (2ndly) the rate of interest made on its investments, (3rdly) the proportion borne by its actual expenses to those which have been assumed in the preparation of its tables, and (4thly) on the constitution of the Society.

In a purely Mutual Society, there can be no doubt that each policy-holder is entitled to a share of any profits that may accrue from either a favourable rate of mortality, a more profitable rate of investment than that anticipated, or an expenditure less than that which might have been incurred in strict accordance with the premiums charged; and, on the other hand, there can be no doubt that he should bear his fair proportion of all losses resulting from the operation of contrary causes.

The actuary of the Office can alone procure, from the recorded transactions of the Society, the necessary data for the correct determination of the surrender value of a policy; and if he, on investigating the affairs of the Society, determine these, the problem may be solved thus:—

Ascertain the net value of an annuity due on the life at the present age, at the experienced rate of mortality and interest; multiply it by the net premium that would be chargeable at the present age, less the gross premium chargeable, on the same data, at the age when the policy was taken up; and add to, or deduct, from the result, the amount, in as many years as the policy has been in force, of an annuity of the difference between the gross premium that should have been charged and that which has been paid, according to the circumstances of the case. [*Rule* M.]

This Rule may be elucidated thus:—

The assured pays, at the time of settlement—

1st. The sum to which the adjusted gross premium for the younger age would have amounted at the time;

2ndly. The present value of all future payments of the same premium;

and, completing thus his part of the contract and entitling himself to the full benefit of the policy, receives—

1st. The present worth of the sum assured;

2ndly. A return of all the payments he has already made.

The surrender value of the policy before referred to, if the Office experience coincide with the English Life Table at 3 per cent. and the expenses are found to be fairly adjusted by an addition of £3 to the net premium at age 20, would be found thus:—

Net premium at age 60		£61·312
Gross premium at age 20		18·851
Difference . . .		42·461
Mutiply by worth of annuity due, at age 60 .		11·057
		469·49
Gross premium at 20 . .	£18·851	
Premium paid	17·290	
	1·561	
Multiply by amount of annuity in 40 years	75·4013	
Deduct the product, the premium paid having been insufficient		117·70
		£351·79

If, however, the Office make 4 per cent. interest, and an addition of £2. 10*s*. to the premium would have sufficed for expenses—

Net premium at age 60		£58·67
Gross premium at age 20		16·60
		42·07
Multiply by worth of annuity due at age 60 . .		10·295
		433·11
Gross premium at 20	£16·60	
Premium paid	17·29	
Excess payment . .	·69	
Multiply by amount of annuity in 40 years	95·026	
Add the product, the premium paid having been too large		65·568
		£498·678

If the Office be a Proprietary or a Mixed Office, the non-participating assured, by selecting a mode of assurance that deprives him of the right to a participation in profits, screens himself also against liability as regards losses; in such cases it appears only right that, on cancelling the contract, the consideration-money should be merely the difference between the net present value to the Office of the sum assured which the policy-holder surrenders and the net present value of the future premiums surrendered by the Office. The Rule will then be expressed thus:—

Add to the premium payable under the terms of the policy the discount for one year on the sum assured (reckoned at the rate of interest actually made by the Office), multiply by the net value of an annuity due on the surrenderer's life, and deduct the product from the gross sum assured. [*Rule* N.]

In the case before used, the valuation would be, if the Office is making 4 per cent. of its funds—

Premium	£17·29	
Discount on £1,000 at 4 per cent. . .	38·46	
	55·75	
Annuity due, at age 60, worth, by same table as in last example . . .	10·295	
Deducting the product	573·946	from £1,000

The surrender value, on these terms, is £426. 1*s.* 1*d.*

If the Office be making only 3 per cent., the valuation would be—

Premium	£17·29
Discount at 3 per cent.	29·126
	46·416
Annuity due, worth	11·057
Deducting the product	£513·22 from £1,000

The surrender value is found to be £486. 15*s*. 7*d*.

A Method of Multiplication which may be practised Mentally.

By LIEUT.-COL. WM. HENRY OAKES, *Bengal Presidency.*

1. WRITE down the multiplicand in the usual manner.

2. Write on a separate slip the multiplier with its figures reversed (as in contracted multiplication), being careful to space the figures so that they can be exactly placed over those of the multiplicand.

3. Thus arranged, place the multiplier so that the figure in the true unit's place may fall over the unit's place of the multiplicand; then multiply each figure of the multiplicand by that immediately over it, sum the products and write down the last figure of the sum, reserving the rest as a carriage.

4. Shift the multiplier one place to the left, multiply each figure as before and sum the products, adding thereto the carriage from the preceding operation; write down the last figure of this new sum, and reserve the rest as a carriage.

5. Proceed in this manner until the multiplier is exhausted; the sum last obtained must be written in full, and the work will then be complete.

Example.—$\overset{abcd}{9763} \times \overset{lmnr}{8452}$.

Process in detail, showing the several positions of the multiplier.

```
   2548  Multiplier reversed.
9763     Multiplicand.
 ----
      6
```

```
      2548
     9763
     ----
        15
        12
        --
        27

      2548
     9763
     ----
        12
        30
        14
Carriage . . .  2
        --
        58

     2548
     9763
     ----
        24
        24
        35
        18
Carriage . . .  5
       ---
       106

    2548
     9763
     ----
       48
       28
       45
Carriage . . . 10
      ---
      131

   2548
     9763
     ----
       56
       36
Carriage . . . 13
      ---
      105

  2548
    9763
    ----
    72
Carriage . .  10
    --
    82
```

Mental process.

```
                    6 = rd.
                   ---
                 1 5 = nd.
                 1 2 = rc.
                   ---
                 2 7   sum.
               1 2     = md.
               3 0     = nc.
               1 4     = rb.
               -----
               5 8     sum.
             2 4       = ld.
             2 4       = mc.
             3 5       = nb.
             1 8       = ra.
             -------
           1 0 6       sum.
           4 8         = lc.
           2 8         = mb.
           4 5         = na.
           -----
         1 3 1         sum.
         5 6           = lb.
         3 6           = ma.
         -----
       1 0 5           sum.
       7 2             = la.
       -----
       8 2 5 1 6 8 7 6
```

The figures of the *result only* are to be recorded, as shown by the dotted lines.

On the Methods pursued in Valuing the Risks of Life Assurance Companies, and on the Division of Surplus. By CHARLES JELLICOE, *Actuary of the Eagle Insurance Company.**

[Read before the Institute, 25th February, 1850, and printed by order of the Council.]

IN a paper read a short time ago before the Institute, I endeavoured to show the inexpediency of using, for the purposes of valuation, tables constructed on the principle which characterizes the Northampton table. I did not intend to pursue this particular branch of the subject further; but, finding that more difference of opinion exists in reference to it than I was aware of, and considering that the practice of making valuations by the Northampton table, and by tables framed upon similar principles, is still carried on to a great extent, I am led to offer a few more observations upon the matter previously to entering upon the consideration of some other questions, the correct solution of which materially depends upon a right understanding of this one.

In any table of premiums intended for actual use, it is evident that the rates charged must be more than sufficient to provide for the sum assured. This is obviously demanded by prudential considerations, and is, indeed, based upon an inevitable necessity; but in order to judge of the sufficiency of the premium to be charged, it is plain that we must know the amount of that which is required to provide for the risk merely, the determination of the one being essential to the proper adjustment of the other. Now, the premium for the risk, or the payment required to provide for the sum assured, can only be deduced, as is well known, by means of a mathematical process, from the true average rates (or from what are supposed to be the true average rates) of interest and mortality, whilst the addition to be made to the payment thus deduced, as a provision for other contingencies, is purely arbitrary. In this view of the matter, then, the test of the accuracy of the data will be their yielding a premium just sufficient, and no more than sufficient, to provide for the sum assured. It will be convenient to call such data, or such rates of interest and mortality, the true data or rates; and such premium, the true premium. Now, if the premiums to be charged be derived, by a direct process, from

* This and another paper on the same subject by the same writer are, we believe, the only ones published by authority of the Institute not in the *Journal*. For the sake of completeness, we avail ourselves of a convenient opportunity to reprint, with some slight additions, the longer one of the two.—ED. *A. M.*

rates of mortality and interest, it is evident that such rates cannot both be true ones, since they yield a premium more than sufficient for the sum assured; that is to say, they yield a false premium, and therefore must, one or the other, or both, be also false.

I will call tables of this description "factitious," *as being made up of quantities in their nature heterogeneous*, and as distinguished from tables having to do solely with the premium for the risk. Of these factitious tables, many, beside the Northampton, are extant, although not published; for it is well known to be a common practice to arrange, first, the premiums to be charged; then to obtain the values of the annuity from the premiums; and from the values of the annuity to deduce rates of interest and mortality; such values and rates being thereafter made use of in all official calculations. But although the distinguishing feature of tables thus constructed is strictly identical with that which characterises the Northampton table, its introduction in the case of the latter was owing to circumstances wholly different, and was, indeed, scarcely to be avoided by the original framers of the table, as I may presently have occasion to show; but I would first endeavour to point out more distinctly the relation which these factitious tables bear to the true ones; or, what is the same thing, the relation which exists between the elements of a table, when such elements include the provision for extra contingencies, and when they do not, and to call attention to some of the consequences resulting from their use. With this object, let us call p'_x the true premium—that is to say, the premium for the risk; and let d' denote the discount of £1 at the true rate of interest, then the expression for the corresponding value of the annuity will be

$$\frac{1}{p'_x+d'}-1=\mathrm{A}'_x.$$

In like manner the annuity answering to p_x, or the premium charged, may be expressed by

$$\frac{1}{p_x+d}-1=\mathrm{A}_x,$$

where p_x may be considered as equal to $p'_x+\phi_x$, and d taken at any rate of interest whatever. This, then, is the relation between the value of the annuity and the premium charged, which obtains in the Northampton table, and of course in all other tables similarly constructed.

Substituting, in the latter of the two expressions, the value of p'_x, as shown by the former, it will be found that

$$1+A_x=\frac{1+A'_x}{1+(\phi_x+d-d')(1+A'_x)},$$

an equation which exhibits the relation between the values of the true and the factitious annuity, or between the value of the annuity when the payment for extra contingencies is involved in it, and when such payment is not so involved.

Now, the mere inspection of this quantity ought to suffice, one would think, to show the entanglement and obscurity which must result from its introduction into our investigations: but it is worth while to examine the manner in which it operates a little more closely. Between the limits within which our practice is usually confined, the difference between d' and d can rarely exceed ·00933, that being their difference when taken at two consecutive and integral rates of interest; but whatever their difference may be, ϕ_x will generally be found in the cases which occur in practice to exceed it; hence (ϕ_x+d-d') is almost always positive, and, consequently, $1+A_x$ is almost always less than $1+A'_x$. The effect of this upon other quantities depending on the value of the annuity is obvious enough. Thus the value of a reversion by the factitious table will generally be greater than by the true one, since $1-d(1+A_x)$ evidently exceeds $1-d'(1+A'_x)$ (d being equal to or less than d'); and conversely, the present values being the same, the reversionary sums will of course be less. Moreover, the liability under an assurance will generally be less by the factitious table than by the true one, especially if ϕ_x be a constant or decreasing quantity, for in such cases

$$(p'_{x+n}+\phi_{x+n}-p'_x-\phi_x).(1+A_{x+n})$$

is clearly less than

$$(p'_{x+n}-p'_x).(1+A'_{x+n}).$$

But this will cease to be always true when ϕ_x becomes less than $d'-d$, as also when ϕ_x is a quantity increasing as the premium increases.

Let us now briefly compare the Northampton table at 3 per cent., and the Carlisle table at $3\frac{1}{2}$ per cent., taking the former as a specimen of a factitious table, and the latter as a sample of a true one. In this case, then, $d'-d=$ ·00469, and the value of ϕ_x at given ages is as follows:—

Age.		Age.	
25	00799	50	01041
30	00819	55	00908
35	00869	60	00709
40	00916	65	00890
45	01003	70	00758

being always greater than $d'-d$, whence (ϕ_x+d-d') is positive, and $1+A'_x$ must always exceed $1+A_x$. But it will be seen that ϕ_x sometimes increases with the age, and sometimes decreases; so that $(p'_{x+n}+\phi_{x+n})-(p'_x+\phi_x)$ will be sometimes greater and sometimes less than $(p'_{x+n}-p'_x)$. The effect of this is to make the liability, as determined by the factitious table, in some cases too little and in others too great. But, by a parity of reasoning, the additions made by way of bonus will always be too little, and, what is worse, will be so in a greater degree at some ages than at others, while at subsequent valuations these very additions will again be over-estimated, but in different degrees.

If in any particular case it be found that the average rate of interest annually realised has been 3 per cent., then the use of the Northampton table in making valuations must lead to still worse consequences. For d being then equal to d', the relation between the true and factitious annuities will be expressed by

$$1+A_x=\frac{1+A'_x}{1+\phi_x(1+A'_x)}.$$

Whence the latter will always be less than the former so long as ϕ_x has any positive value. Let the true table be that called the "Experience;" or, what amounts to the same thing, let the value of ϕ_x, at the undermentioned ages, be

Age.		Age.	
25	00738	50	00695
30	00762	55	00551
35	00781	60	00341
40	00793	65	00090
45	00761	69	00006

Then it will be seen that whilst $(p'_{x+n}-p'_x)$ is nearly the same as $(p'_{x+n}+\phi_{x+n})-(p'_x+\phi_x)$, $1+A_{x+n}$ must be less than $1+A'_{x+n}$; and that when these last approach to equality, $(p'_{x+n}+\phi_{x+n})-(p'_x+\phi_x)$ is much less than $(p'_{x+n}-p'_x)$. Thus it appears that, except in extreme cases, the values of assurances will at every age, and after any given time, be considerably greater by the true table than by the factitious one; and hence the total amount reserved for liabilities, as determined by the Northampton table,

will, under such circumstances be very much less than it ought to be.* These inconsistencies, it must be remembered, are not merely those between two tables selected at hazard, but result, in fact, from the introduction of a quantity into the investigation which cannot, with any degree of propriety, be mixed up with it. In other words, they arise from making valuations in accordance with the data derived by a direct process from the premiums charged; a practice which, however excusable in the infancy of life assurance, has nothing to justify a perseverance in it at the present day.

All this, however, serves only to confirm what has been said by the best authorities when speaking of these erroneously-constructed tables. Thus Mr. Gompertz, in a paper read before the Royal Society, in 1820, and quoted by Mr. Griffith Davies in his able work on life assurance, observes: — "That the proper method of regulating the premiums for life assurances, &c., is to employ what may appear to be the correct average rates of interest and mortality *to form the calculation upon*, and to the results, to make such additions as may leave an adequate portion for the security, profit, and expenses of the Insurance Company;" adding, "That it does not seem possible to adopt tables of mortality which are not correct in themselves, connected with a rate of interest which is not the average rate made in reality, so that the advantage may tend in one direction." Mr. Sang also, in one of his essays read before the Edinburgh Society of Arts, says, with his usual vigour of thought and expression:—"If, in this business, we use a life table

* This observation will apply with double force to instances in which no reserve is made beyond the amount so determined. For it must be carefully noted that the comparison here is with the liability indicated by a true table, and that it is invariably the practice to make a large addition to the amount of liability indicated in that way. Thus, after 20 years, the values of assurances by the Northampton 3 per cent. and Experience 4 per cent. tables are as follows:—

Age at Commencement.	Northampton 3 per cent.	Experience 4 per cent.
24	21·4	20·2
29	24·4	24·4
34	27·6	29·1
39	31·2	34·2
44	35·6	39·5
49	41·2	44·7
54	47·7	49·7
59	54·1	54·3
	283·2	296·1

The latter being, in the aggregate, a good deal in excess of the former, although the rate of interest is higher. But the advocates of this method would, nevertheless, consider a large additional reserve necessary, whereas those of the former would, as a general rule, contemplate no other than that here exhibited!

known to be erroneous, we can neither tell the amount of the error nor the side on which it is likely to be, except by contrasting the results with those drawn from a life table believed to be nearer the truth; and we are thus placed in the awkward position of virtually making two valuations and using the bad one, augmenting the absurdity, perhaps, by correcting the erroneous results, so as to bring them to the true ones."

Lastly, Mr. Babbage, speaking to the same point, says:—"This method of determining the premiums to be taken is, in the present state of our knowledge, both inexpedient and unscientific, although when first employed it deserved a very different character. I imagine it will not require much argument to show that the more proper course would be to determine, as nearly as we can, the real value of the risk, and consequently the amount of premium just sufficient to meet it, and to add to this such percentage as will defray the expenses of management, &c."*

When the table of premiums so well known as the Northampton table was first constructed, it is probable that its framers did not anticipate that the payments to be made in accordance with it would be more than sufficient to provide for the sum assured. The addition of 15 per cent. originally made to the premiums, would rather lead to the inference that they were considered insufficient without it to meet other contingencies. At the outset, then, we find established the relation between the sum guaranteed under contract and the premium derived from what were then supposed to be the true average rates of mortality and interest: the 15 per cent. being, no doubt, disregarded in any official calculations, or at least dealt with separately, since we have never heard of any table of annuities which would, by a direct process, give Northampton premiums plus 15 per cent. When the payments

* Since writing this paper, my attention has been called by Mr. Peter Gray to the following observations made by Mr. Babbage in his evidence on Friendly Societies (*Report*, 1827, pp. 29, 30):—"If I am asked which tables I should wish to use in making any calculations relative to the poorer orders of society, I should state that my object would be to get such tables as exactly and perfectly represented those classes throughout all their ages; but it may be said that this is unsafe, and that others should be taken where the deaths are more numerous than those that really happen. My view in all cases is, let us get, as nearly as we can, the law of mortality of the class for which we want to calculate, and add to the prices computed from it some proportional part sufficient to insure the safety of the establishment which uses them. I strongly object to using tables giving a greater mortality than is expected to take place, a course which has sometimes been defended on the ground of safety to the establishment. Safety is much more certainly secured by judging, as nearly as possible, the true risk, and adding an additional sum for security. If tables not representing the mortality of the class for whom they are designed are employed, every step in the reasonings that are deduced from them is liable to increased error; and if the calculations are at all complicated, the errors so introduced may not improbably act on the opposite side to that which they were introduced to favour."

thus adjusted were found, as they soon were, to be more than sufficient, it is evident that the consistent course was to revise the rates of mortality and interest. The circumstance of a surplus being found, to arise uniformly throughout a given period, proved incontestably that the premiums charged for the risk were excessive, and consequently that some error existed in the assumed rate of mortality or in that of interest. Not to be guided by such evidence was to persevere in error, and to neglect the most direct indications of being in it. It would behove those concerned, then, to determine in what the error consisted, or wherein a false assumption had been made, and to correct it accordingly, so that thenceforward no surplus should arise in respect of the premium for the risk. It by no means followed, however, that the excessive portion of the premium should cease to be charged. It could of course be retained in its new character as a mere guaranty against adverse fluctuations, to be relinquished, if so arranged, at stated intervals, with such deductions as might be necessary, or with such additions as circumstances might warrant. But it will be perceived that it no longer formed any part of the true premium, and was therefore quite out of the pale of investigations dealing solely with the risk and with the payment required to meet it. Hence the premium actually charged might thenceforth be regarded as consisting of two distinct parts, the one being simply a provision for the sum guaranteed under the policy, derived by a direct process from the best authenticated rates of interest and mortality; the other, a merely arbitrary and independent addition, providing against contingencies of a totally different character, and therefore the subject of a treatment altogether distinct. In this way p_x would come to be looked upon as $p'_x + \phi_x$; and the elements which, at the outset, corresponded with p_x, would finally adjust themselves, so as to yield p'_x, ϕ_x being thus excluded, as it were, from the arena within which the theorems of the doctrine of probabilities were to have their legitimate play.

Thus, then, we again arrive at the conclusion that whatever the rates of premium charged may be, *the difference between them and the true premium must be discarded, or dealt with apart in all official calculations, if we would have anything like consistency in our deductions.* What the amount of the true premium may be under any given circumstances, can only be got at by close and careful investigation; but at the present day we can have no great difficulty in arriving at an approximation, which will at least show that some distinction is to be drawn between the two, and which will

extricate us from the inconsistencies invariably arising whenever we persist in confounding the one with the other.

From what has been said we shall be enabled, I think, without leaving room for much difference of opinion, to arrive at the true principles which should govern us in the distribution or division of surplus; for, since the payments of the true premium will evidently, from their nature and character, be always absorbed in the amount which constitutes the liability, whatever surplus is found at any time to exist must clearly arise from payments of ϕ_x, and the interest upon them; and therefore nothing more remains to be done than to redistribute these last to the several contributors. In short, we may imagine that when the payments of p_x are made, p'_x is carried to one fund and ϕ_x to another; and that interest being properly apportioned between the two, the remainder of the latter fund, after payment of expenses and other charges, is returned, at given intervals, in due proportion to those who have created it; the fund arising from the payments of p'_x, and the interest upon them, being of course sufficient to meet the claims as they fall in: so that if we call S the real surplus found to exist after any valuation, and $\Sigma.\phi_x a^n$ the artificial one determined from the payments of ϕ_x improved at the true rate of interest, then the share of each contributor may be represented by $\frac{S}{\Sigma.\phi_x a^n} \times \phi_x a^n$; this latter quantity denoting the amount of the surplus payments actually contributed by each.

This method will have the effect of distributing the surplus with probably as much fairness as we can expect practically to arrive at. But that it is but a near approximation will be observed, when it is considered that, for the expenses to be duly apportioned, the collateral payments of ϕ_x must each extend through the same period, a condition which will rarely obtain. The methods usually adopted, however, at the present day are objectionable, as involving other inaccuracies beside the one mentioned. I will select two which are most in vogue, viz., the division in proportion to the amount, at compound interest, of the premiums paid, and that in proportion to the same amount after deducting the value of the assurance. Now, it will be remembered that the total liability corresponds with the amount at the true rate of interest of the payments of the true premium, less the claims; whilst the surplus owes its existence to the accumulations of ϕ_x, less the expenses, which, for our present purpose, we may express as follows, viz.:—

$$\Sigma.p'_x a^n - \Sigma.c = \Sigma.l = \text{the liability},$$
$$\Sigma.\phi_x a^n - \Sigma.e = \Sigma.s = \text{the surplus}.$$

The division, then, in accordance with the one principle, will be in the proportion which $\Sigma.p_x a^n$ bears to $p_x a^n$; and that in accordance with the other in the ratio expressed by $\Sigma.p_x a^n - \Sigma.l : p_x a^n - l$; that is to say, they will be in the ratio respectively of

$$\Sigma.p'_x a^n + \Sigma.\phi_x a^n : p'_x a^n + \phi_x a^n, \text{ and}$$
$$\Sigma.\phi_x a^n + \Sigma.c \quad : \phi_x a^n + e, \text{ or } p_x a^n - l.$$

But it has been shown that the surplus arises from $\Sigma.\phi_x a^n$ alone, and not from either of the sources above denoted, and hence it will be seen that the ratios adopted in these two methods are, under ordinary circumstances, neither of them properly applicable.

I do not propose, on this occasion, to consider the several modes in which the share of surplus may be returned or placed to the account of the assured; but before I conclude I would say a few words as to the nature of the addition made to the true premium—that is, as to the relation which ϕ_x should bear to p'_x. In the Northampton table we have seen that the former is an irregular quantity, if anything, tending to decrease as the age increases. If the true mortality be represented by the Carlisle table, and the true rate of interest by 4 per cent., then ϕ_x in the Northampton table is nearly constant. But the usual practice is to make the addition a percentage on the premium, the effect of which is to make the entrants, at all ages, contribute to extra contingencies in proportion *to the intensity of the risk*, without any reference to the time relatively occupied in completing the transaction, or to the natural duration of the assurance.* But in the light in which we have been regarding these extra payments, there is no one object to which they are devoted, or purpose for which they are required, which is not of periodical recurrence; and therefore it would seem that the equity of the case would be better met by making the assurers contribute to extra contingencies in proportion to the time and the sum, and not in the proportion above stated; that is to say, ϕ_x should be a constant quantity,† varying only with the

* This is obviously unjust, as well as unmeaning! Whatever is paid over and above the premium for the risk is in the nature of a contribution to rent, taxes, salaries, &c. &c.; and what more simple or equitable plan can be devised than the requiring the like annual contribution to these charges on account of each one hundred pound assurance? A member's policy does not entail greater expense on the Society because he happens to be older than another!

† See Professor De Morgan's *Essay on Probabilities*, page 271.

sum assured, and of such an amount as the objects of the assurers may induce them to make it.*

It will, perhaps, be necessary, however, to admit of a certain departure from this principle where commission is usually allowed to agents. This charge, it is well known, instead of being regulated, as it ought evidently to be, by the amount assured only, is always in proportion to the premium, and therefore it will be proper to take this circumstance so far into consideration as to make a separate addition to the premium on account of it. The premium charged will thus consist of the quantities $p'_x + \phi + \frac{p'_x + \phi}{19}$ or $\frac{20}{19}\left(p'_x + \phi\right)$, the rate of commission allowed being 5 per cent.; and it will be found that, in a table so formed, the abnormal and distorted features so frequently presenting themselves in the rates for assurance put forward from time to time are no longer to be met with.

On Weights and Measures. By Professor Leoni Levi.†

PUBLIC attention has for some time past been earnestly directed to the introduction of the decimal system in our weights, measures, and coins. The nation is generally convinced, that the adoption of such a system would prove of immense benefit—that it would afford great facilities for calculations of all kinds, that it would shorten the work of education, that it would economise labour, and that it would diminish the chances of error. The Society of Arts, and other scientific societies, have investigated the subject in all its phases and bearings, and we have been expecting the speedy adoption of some practical plan which would be certain to confer so great a boon. Unfortunately, the Russian war, the Indian mutiny, and other political events, have rendered it necessary to put aside the consideration of many social reforms, and this, among the rest, shared the same fate. We have bestowed, also, far too much attention to the pound and mil scheme, as if upon it rested the entire

* If S denote the sum resulting from the addition of the true premiums *per cent.*, taken at the ages 21, 22, 23, 24, &c., to 65, and if it be proposed to increase them one third, then $\phi = \frac{1}{45} \times \frac{S}{3}$, or $\frac{S}{135}$; and the total annual contribution to extra contingencies, irrespective of commission, will always be equal to $\frac{\phi}{100}$. (total sum assured).

† Reprinted, by permission, from No. 9 of *The Exchange*.

question of decimalisation, and thus years have passed without a single step of a definite character being taken.

From this state of torpor, however, we are suddenly awakened, by the publication of a large Blue-book, being the Report of Mr. Ewart's Committee of the House of Commons "On the practicability of adopting a simple and uniform system of weights and measures, with a view not only to the benefit of our internal trade, but to facilitate our trade and intercourse with foreign countries." There is one advantage in the present Report, that it leads us to the bottom of the question, and enables us to form a correct view of the entire system. Hitherto we have only heard of decimal coinage, and we have omitted to realise the close relation of the coinage to weights and measures. It was well to direct our attention to these. Indeed, we quite agree with Lord Overstone, that we should take a comprehensive view of the whole before we begin to make any change:—

"A decimal coinage," said Lord Overstone, in his Draft Report for the Decimal Coinage Commissioners, "is one element of a complete decimal system, in the same sense in which the florin is one element in a decimal coinage; and to decide in favour of a change in the coinage without determining what would be the best course to take with reference to other parts of the metric system, would be to fall into the same mistake in a far more serious and important shape than was committed in issuing florins in their present form before any final adjudication on the general question of decimal coins. Any precipitate decision on an isolated part of what is really one great question is the more earnestly to be deprecated, because a change once introduced into the coinage will be irrevocable, and ought not to be regarded until every consideration bearing upon the question has been fully entered into. When we consider the intimate relation between weights and measures and coins—that is, between the instruments by means of which commodities are divided and distributed, and the instruments by which the commodities so divided are paid for—it seems scarcely credible that a proposition can be seriously made to introduce, at the cost of much temporary inconvenience, a new system of coinage, without reference to the nature or principle of that simplification of our weights and measures which the anomalies of the present system appear to render indispensable."

Nor are those anomalies light and unimportant. The bare description of them will suffice to establish an absolute necessity for a prompt legislative remedy. We can afford to wait for the introduction of a decimal coinage, but we cannot delay to introduce uniformity in our weights and measures. As many as ten different systems are actually in use in this country. There is, first, the grain, computed decimally, which is used for scientific

purposes; second, the troy weight, under 5 Geo. IV., c. 74, and 18 and 19 Vict., c. 72; third, the troy ounce, with decimal multiples and divisions, called bullion weight, under 16 and 17 Vict., c. 29; fourth, bankers' weight, to weigh 10, 20, 30, 50, 100, and 200 sovereigns; fifth, apothecary weight; sixth, diamond weight and pearl weight, including carats; seventh, avoirdupois weight, under 5 Geo. IV., c. 75, and 18 and 19 Vict., c. 72; eighth, weights for hay and straw; ninth, wool weight, using as factors 2, 3, 7, 13, and their multiples; and tenth, coal weight, decimal, under 1 and 2 Wm. IV., c. 76, and 8 and 9 Vict., c. 101. We have also in occasional use the weights of the metric system. For measures of length we have the ordinary inch, foot, and yard. In cloth measure, we have yards, nails, and ells. There are four different sorts of ells. For nautical purposes we have fathoms, knots, leagues, and geographical miles, differing from the common mile. The fathom of a man of war is 6 feet; of a merchant vessel, 5½ feet; of a fishing smack, 5 feet. We have also the Scotch and Irish mile, and the Scotch and Irish acre. There are several sorts of acres in the United Kingdom, and there are a great variety of roods. We have in almost every trade measures of length specially used in those trades. For the measurement of horses, we have the hand; shoemakers use sizes; and we are compelled to adopt gauges where the French use the millimetre. The gauges are entirely arbitrary. The custom of the trade is the only thing which would decide the question in case of dispute. For measures of capacity, we have twenty different bushels. We can scarcely tell what the hogshead means: for ale, it is 54 gallons; for wine, 63. Pipes of wine vary in many ways; each sort of wine seems to claim the privilege of a different sort of pipe. For measures of weight, we have about ten different stones; a stone of wool at Darlington is 18 lbs., a stone of flax at Downpatrick is 24 lbs., a stone of flax at Belfast is 16¾ lbs., but it is also at Belfast 24½ lbs.; having in one place two values. The cwt. may mean 100 lbs., 112 lbs., or 120 lbs. If you buy an ounce or pound of anything, you must inquire if it belong to Dutch, troy, or avoirdupois weight. Such are only a few of the many anomalies exhibited by our weights and measures. Can anyone say that such a state as this is creditable to our civilisation, or compatible with our extensive commercial interests? It cannot be said that we are deficient in laws on the subject—indeed, as far as legislation is concerned, there has been a superabundance of it. And yet we are as distant as ever from this great desideratum. What, then, is the reason?

What is the cause that we are behind almost every other nation in this most essential object? Partly because the law has left many loopholes whereby local usages have been allowed to remain; but much more because, in the want of a clear and intelligible system capable of meeting the manifold requirements of society, every trade and every class had to seek its own way of weighing and measuring commodities.

In dealing with a question of such a momentous and practical character, the Committee of the House of Commons had three alternatives—first, to retain the present system, though defective, and simply to endeavour to amend the law—as it has been done again and again, without result—a mode which no one recommended; second, to create a separate decimal system of our own—that is to say, to decimatise our present units, the pound, the yard, and the gallon—as recommended by Professors Airy, De Morgan, and others; third, to adopt, in common with other countries, the metric system, in accordance with the recommendations of a host of witnesses. The first of these modes appeared to the Committee quite useless. The second would necessitate a complete change, and cause much confusion and trouble, without corresponding results. The latter—the introduction of the metric system—seemed by far the simplest and best. By adopting this course, the Committee were enabled to set aside the chaos of our irregular method, and all the discrepancies by which it is distinguished, and to enter at once into a plain and symmetric principle. And when they considered that their instructions were not only to seek a system perfect and complete itself, but one which should facilitate trade and intercourse with foreign countries, they could not well hesitate in their choice.

The metric system has the great advantage of being in existence and in great favour in many countries. France, Holland, Belgium, Italy, Spain, Portugal, Switzerland, and Greece, and several countries of South America, all have adopted the same. The public mind has long ago pointed to this system as the most convenient for all purposes, and the only basis of international uniformity—a matter of great importance in these days of international movements. From all sides we hear expressions of a want of such uniformity. Our trade with the continent has immensely increased since the conclusion of our treaties of commerce. For some time past our commerce with the countries using the metric system has increased much more rapidly than our commerce with countries using the English or other systems of weights and measures. Our

exports to countries using the metric system, in 1847, amounted to £23,696,000; in 1861 it had reached £55,242,000—showing an increase of 133 per cent.; and our exports to countries using the English system, in 1847, amounted to £16,261,568; and in 1861 to £24,211,449—though it would have been a larger amount had it not been for the American war, which has reduced our exports to that country more than £10,000,000. The Associated Chambers of Commerce of the United Kingdom, at their annual meeting in 1861, speaking as delegates on behalf of their various districts, and representing some of the most important towns in the country, as well as various branches of industry, unanimously passed the following resolution:—"It is highly desirable to adopt the metric system, which has been introduced into many European countries with great advantage to the saving of time in trading and other accounts." The Jurors of the International Exhibition found difficulty in arriving at a common standard, in consequence of the various weights and measures used by the exhibitors of different countries. The International Statistical Congress has again and again expressed an opinion in favour of international units of weights and measures; and our men of science complain of the confusion arising in scientific observations from the uncertainty of the weights and measures used by observers in different countries. "The majority of the facts in natural history," said Professor Owen, in his speech as a member of the deputation to the Board of Trade, "include the elements of weights and measures. An English anatomist and physiologist gives the weight of the brain, lungs, &c., in relation to the weight of the body of some rare animal which he has had the opportunity to examine: the foreign physiologist desires to reduce the English weights to those of his country. If the kind of weights used by the Englishman be not specified—viz., avoirdupois or troy—the description is useless to the foreigner in regard to the important constants of the proportion of parts or organs to the whole body. Similar difficulties are experienced as regards linear measures. And although, when the system of weights or measures is noted by the observer, its reduction, or the finding the equivalent in another system, may be a small demand upon his time, yet the repetition of that act takes a serious amount from the working hours of the individual; and when multiplied by the number of observers, obstructed by conflicting systems of weights and measures, the impediment to the progress of the sciences of observation becomes so great as to render the subject quite worthy of the consideration of legislative

authority." These and other facts of a like nature were present to the mind of the Committee of the House of Commons, and were enforced by the evidence of some of the most distinguished men from various countries. Seldom, indeed, have we seen such a galaxy of eminent witnesses as appeared before Mr. Ewart's Committee—men such as Mr. Graham, the Master of the Mint; Mr. Fairbairn, the late President of the British Association; Professor Airy, the Astronomer Royal; Professor Miller, of Cambridge; Professor De Morgan, Dr. W. Farr, Mr. James Yates, Fellow of the Royal Society; M. Michel Chevalier, Senator, Member of the Institute; M. Visschers, Conseiller des Mines, of Belgium; Dr. Karmarsh, Principal of the Polytechnic Institution, of Hanover; Dr. Steinbeis, President of the Board of Trade, of Wurtemberg; and many others. And seldom, too, have we found such a concurrence of opinion as was exhibited by them. We do not wonder that they produced like unanimity in the resolutions of the Committee.

Let us now examine the nature of the metric system, and ascertain what relation it bears to our own. It had long been the desire of France to have a uniform system of weights and measures. But under the old regime there was too much indolence, and too great a disregard of material interests, to allow them to think of such things. As soon, however, as the nation awoke to a sense of its own importance, the *Assemblée Constituante* took the matter up in earnest, and on May 8, 1790, it passed a resolution desiring the King to obtain the co-operation of the English Legislature with the National Assembly for the determination of a natural unit for the comparison of weights and measures. The proposal was, that an equal number of commissioners, chosen from the Academy of Sciences and from the Royal Society, should meet, in order to find, at the parallel of latitude half way between the equator and the pole, or any suitable parallel, the length of the second's pendulum. No response having been given by Britain to this invitation, the Academy proceeded in their labours by themselves, and their first decision was that all the multiples and subdivisions of the system should be according to the decimal scale, and that the units of surface, capacity and weight, should all depend on the unit of length. The Academy then appointed as commissioners, Lagrange, La Place, Borda, Monge, and Condorcet. These commissioners, having carefully discussed the relative merits of the invariable length which is known to be required for the exactness of a second's pendulum at any given latitude, and of a unit taken from the

dimensions of our planet, decided upon preferring the latter, as not involving the heterogeneous element of time, and being also necessarily of a more cosmopolitan character. The ten-millionth part of the arc of the meridian comprised between the equator and the north pole was therefore selected as the unit of linear measure. At first the commissioners considered it quite sufficient to take the standard of length from some previously executed measurement of the earth; but eventually they resolved to undertake all the preliminary work afresh, and the astronomers Delambre and Mechain were appointed to undertake the geodesical operations. We shall not attempt to follow these astronomers in their arduous labours, or to detail the strange adventures which they experienced during the worst times of the French revolution. It is sufficient to say that ten years elapsed before the measurements terminated; but as soon as they were concluded, the French Government again issued invitations to neutral and allied countries to send deputies to assist in the final settlement of the metric system, so that it might be adapted to the usage of all nations. It was then that the exact measure of the metre was fixed, and the other standards of weight and capacity also definitively settled. For the standard of surface for land measure, the commissioners took the square of ten metres on each side, or one hundred square metres. The standard of capacity for liquids was determined by finding a cylindrical volume equal to a cube, whose edges are formed by tenths of the linear standard. This is the litre; and the weight of a litre of distilled water at its maximum density was adopted for standard of weight, and called a kilogram.

Having once adopted the units, the commissioners proceeded to determine the multiples and subdivisions, according to the decimal scale, and they also resolved upon a nomenclature totally new and distinct from any in use—taking Latin and Greek words for ten, hundred, thousand, and ten thousand, to express respectively the decimal gradation, upward and downward, of the respective units. It was, perhaps, unfortunate that the commissioners adopted such a nomenclature. We are quite sure that it retarded the popular adoption of the system. But the commissioners objected to take words derived from the vernacular tongue—not on account of any preference for the Latin or Greek, but because they expected they would excite less prejudice, and would be more readily adopted by other nations. There is, however, great value in the fact that each name expresses exactly and clearly, not only the quantity therein represented, but the relation in which it

stands to all the other quantities of the same. Indeed, a glance at the system shows at once how superior it is to our own uncouth and irregular gradation, with binary and duodecimal factors, and a considerable admixture of other modes of progression. The units of measures and weights are as follows:—The *Metre*, the unit of the measure of length, is the ten-millionth part of the fourth of the terrestial meridian: the *Are* is the unit of superficial measure, and is a square of ten metres on each side; the *Litre*, the unit of capacity, is a cubic decimetre; the *Stere*,* the unit of the measure of solidity, is a cubic metre; and the *Gram*, the unit of weight, is the weight of a cubic centimetre of distilled water at the temperature of its greatest density. For multipliers, there are added to the respective units the following Greek prefixes:—*Deca*, viz. 10 times; *Hecto*, viz. 100 times; *Kilo*, viz. 1,000 times; *Myria*, viz. 10,000 times. For divisors, there are added to the respective units the following Latin prefixes:—*Deci*, viz. 10th part; *Centi*, viz. 100th part; *Milli*, viz. 1,000th part. But perhaps a tabular statement will bring the whole out even more clearly:—

NOMENCLATURE OF WEIGHTS AND MEASURES AND THEIR EQUIVALENTS.

Measure of Length.

Names.	Value.	English equivalents.
Millimetre	The thousandth part of a metre	0·03937
Centimetre	„ hundredth „ „	0·39371
Decimetre	„ tenth „ „	3·93708
METRE	——	39·37079
Decametre	Ten metres	32·80916 feet
Hectometre	Hundred do.	328·09167 „
Kilometre	Thousand do.	1093·63890 yards
Myriametre	Ten thousand do.	10936·38900 „
		or 6 miles 1 furlong 28 poles

Land Measure.

Centiare	The hundredth part of the are, or square metre	1·1960 square yards
ARE	——	119·6046 „
Decare	Ten ares	1196·0460 „
Hectare	Hundred do.	11960·4604 „
		or 2 acres 1 rood 35 perches

Measure of Capacity.

Millilitre	The thousandth part of a litre	0·06103 cubic inches
Centilitre	„ hundredth „ „	0·61028 „
Decilitre	„ tenth „ „	6·10280 „

* The Stere, used in France for measuring stacks of firewood, is not wanted in England.

Measure of Capacity (continued).

LITRE	——	61·02803 cubic inches, or 2·1135 wine pints
Decalitre	Ten litres	610·28028 cubic inches, or 2·642 wine gallons
Hectolitre	Hundred do.	3·5317 cubic feet, or 26·419 wine gallons
Kilolitre	Thousand do.	35·3171 cubic feet, or 1 tun 12 wine gallons
Myrialitre	Ten thousand do.	353·17146 cubic feet

Measure of Solidity.

Decistere	The tenth of a stere	3·5317 cubic feet
STERE	——	35·3174 „
Decastere	Ten steres	353·1741 „

Weights.

Miligram	The thousandth	0·0154 grains
Centigram	„ hundredth	0.1543 „
Decigram	„ tenth	1·5434 „
GRAM	——	15·4340 „
Decagram	Ten grams	154·3402 „
Hectogram	Hundred do.	3·527 oz. avdpois.
Kilogram	Thousand do.	2 lbs. 3 oz. 4½ drs. avdps.
Myriagram	Ten thousand	22·0485 lbs. „
Quintal	. . . . .	1 cwt. 3 qrs. 25 lbs.
Ton	Thousand kilograms.	

This is the system recommended by the Committee, and we congratulate the nation on the fair prospect of a satisfactory and permanent settlement of this great question. Any further patching up of the present method would, we are sure, have proved fruitless.

The recommendations of the Committee are as follows:—

"1. That the use of the metric system be rendered legal, though no compulsory measures should be resorted to until they are sanctioned by the general conviction of the public.

"2. That a Department of Weights and Measures be established in connection with the Board of Trade. It would thus become subordinate to the Government, and responsible to Parliament. To it should be intrusted the conservation and verification of the standard, the superintendence of inspectors, and the general duties incident to such a department. It should also take such measures as may from time to time promote the use and extend the knowledge of the metric system in the departments of Government and among the people.

"3. The Government should sanction the use of the metric system, together with our present one, in the levying of the customs duties; thus familiarising it among our merchants and manufacturers, and giving facilities to foreign traders in their dealings with this country. Its use, com-

bined with that of our own system, in Government contracts has also been suggested.

"4. The metric system should form one of the subjects of examination in the competitive examinations of the civil service.

"5. The gram should be used as a weight for foreign letters and books at the post office.

"6. The Committee of Council on Education should require the metric system to be taught (as might easily be done, by means of tables and diagrams) in all schools receiving grants of public money.

"7. In the public statistics of the country, quantities should be expressed in terms of the metric system in juxtaposition with those of our own, as suggested by the International Statistical Congress.

"8. In private Bills before Parliament, the use of the metric system should be allowed.

"9. The only weights and measures in use should be the metric and imperial, until the metric has generally been adopted.

"10. The proviso in the 5th and 6th William IV., c. 63, s. 6, should be repealed.

"11. The department which it is proposed to appoint should make an annual report to Parliament."

According to these recommendations, the first thing to be done is the legalisation of the metric system. It is needless to say that, under the present law, a sale of wheat by the kilogram, or of cloth by the metre, is illegal, and could not be enforced. It is now proposed to make all contracts embodying the weights and measures of the metric system legal. Yet it is only the permissive use which is to be authorised, and no one is to be compelled to use the metre instead of the yard, or the kilogram instead of the pound. The creation of the Department of Weights and Measures is absolutely necessary. At present, the Comptroller of the Exchequer has some superintendence over the weights and measures, but it is of a very slender kind. The Inspectors of Weights and Measures, appointed by the magistrates of different towns, have nothing to do with the central office. They only report to the magistrates, and with a divided responsibility there is no guarantee whatever for the due enforcement of the law. If the metric system is to be adopted, there is great need, moreover, of diffusing information on the subject, in the shape of comparative tables and scales. The new standards must be introduced and supervised. Measures must be taken for the verification of the standards, and for the allocation of local standards. All this requires care, and can only be done by officers appointed for the purpose. The next recommendation of the Committee is to the effect that customs duties may be levied according to the metric system. Continental traders must find it very difficult to calculate the amount of English duties

on their goods. Let us remove this difficulty by issuing our tariff, not only in the present form, but reduced according to the metric system. This will facilitate trade, and greatly aid in familiarising merchants with the new weights and measures. A recommendation of a practical nature is also made to allow the use of the gram, in the post-office, for foreign letters. Sir Rowland Hill gave many illustrations of the complicated and inconvenient operation of the different weights in different countries in the postage of letters. In Germany, for instance, the unit of weight is the loth, equivalent to $257\frac{3}{10}$ths grains; in this country it is the half-ounce, equivalent to $218\frac{75}{100}$ths grains—showing a difference of 17 per cent. And the consequence is, that though the same rates are charged in both countries, the German correspondent has an advantage, and it becomes cheaper to send a letter unpaid than prepaid. With France, again, there is another difficulty, arising from the difference in the scale of internal postage. France proposed to adopt a 10-gram scale, in lieu of their $7\frac{1}{2}$-gram, as the unit of weight for letters to England, with a view to uniformity; but we have no weight equivalent to 10 grams, or a third of an ounce—and the 15 grams, equivalent to our half-ounce, is too heavy. The adoption of the gram will smooth this difficulty. The Committee suggested other special means (educational and otherwise) for diffusing information of the subject, and recommended also the abolition of a clause in the present law, which shelters many of the present irregularities in our measures. It is pleasing to see, in the whole of this Report, a perfect combination of what is purely scientific with all that is eminently practical.

The Committee have fully appreciated the difficulties connected with any changes in weights and measures. If they have come to the determination to recommend the adoption of the metric system, it is because, all things considered, it is the best that could be offered. But in doing so, they were alive to the necessity of great caution in adopting such a new system, and, taking the example suggested by the experience of other countries, they have deemed it necessary to stimulate the voluntary adoption of the system, before any compulsory measure is resorted to for the enforcement of it. But why, after all, should we be so apprehensive of changes? Changes have been made before this, and they have been attended with little or no inconvenience. What have other nations done when they introduced their new systems? What have the United States done when they abandoned their pound for the dollar? What has Ireland done when the Irish currency was abolished? They

faced the difficulties, and conquered them. We are not underrating the inconvenience of the change. The weights, measures, and coins of the realm are so identified with national ideas, and embodied in our mental vocabulary, that no change can be made in them without producing a sudden disturbance. But with the means now in existence for diffusing information, any ignorance of the subject will soon be dispelled. Newspapers, magazines, and reviews will, by means of letters, articles, and dissertations, throw a flood of light on the question in all its bearings. The teachers in our British and National schools will enter in right earnest into the tuition of the decimal system. The merchant will not be slow in reducing the prices to the new denominations, and hard necessity will soon teach the humblest individual new ideas of quantities and of value. There is no reason, therefore, for fear and hesitation. If we can answer affirmatively, that the change is wanted, and that the reform is beneficial, we may courageously give ourselves to carry it out with as little delay as possible.

NOTES AND QUERIES.

Suggestion by Mr. Newmarch as to the Federation of certain cognate Societies.—I have long entertained the opinion that the time has come when it will be found advantageous, and perhaps necessary, that the six or seven societies now existing in London for the cultivation of different branches of social science, should form themselves into a Federation, not so complete as to be subversive of individual independence, but sufficiently compact to secure the great objects of (1) concentrated libraries and places of meeting; (2) economy in management and expenses; (3) moral and intellectual power arising from the combination of several parts into one consistent whole.

Several members of the council will remember that at various times during the last two years, I have suggested the desirableness of an arrangement of the kind now indicated, and that some progress has been made in the consideration of details.

I employ, without any hesitation, the phrase *Social Science*, not perhaps as the most exact term that could be found, but as the title of a new branch of knowledge which has already acquired, in the public apprehension, a definite place and a recognised function.

There are in London at the present time the following seven Societies, all engaged, in one way or the other, in the cultivation of social science, viz.:—

1. Statistical Society.
2. Institute of Actuaries.
3. Juridical Society.
4. Society for Amendment of Law.
5. Reformatory Union.
6. Association of Sanitary Officers.
7. National Association for promoting Social Science.

It seems to me that the manifest policy of these seven separate Societies —to say nothing of manifest duty—is to form themselves into a powerful "Institute of Social Science," on the model of the British Association and the Social Science Association—that is to say, full sectional action and independence under the supervision of a central authority.

It will be observed that, in the list of Societies just given, there is no provision for the investigation and discussion of questions of *Economic Science* as a separate and special pursuit; and yet sound economic views are indispensable to the successful treatment of most of the subjects which engage the attention of the learned bodies now enumerated. There is, moreover, the striking anomaly, that in the native land of political economy, and in the country which has done, and is doing, the most to discover its laws and illustrate their application, there is in the multitude of scientific associations not one which specially cultivates a branch of knowledge so essentially English and practical. In France there have been for a long period the Academy of Moral and Political Sciences, besides other special means of promoting economical studies. An Institute of Social Science would be well able, by means of concentrated strength and resources, to establish a separate section of political economy, and so supply a defect and a want which has been long confessed.

It may be sufficient to say here, that conformity to at least four principles may be assumed to be indispensable in any efforts which may be made to establish a federal union of Societies, viz., (1) that each existing Society shall remain in possession of its own property, shall continue to be governed by its own internal rules, and shall continue to choose its own managers and officers; (2) that similar independence shall be preserved as regards the control of the publication of its own papers and proceedings; (3) that each meeting of each of the federated Societies shall be open to the members of each of the other federated Societies, so as to concentrate upon each department the force of the entire body; and (4) that the authority to be exercised by the officers and council of the Federation itself should be limited to the purposes and objects rather of advising than of actively interfering with, the associated Societies.

It has been stated that the memorial to the late Prince Consort, to be erected at Kensington, will include a hall or college available for the use of learned Societies. If this statement should be verified, it is allowable to say that no plan would more happily fulfil some of the favourite schemes of the lamented Prince himself, than a union in his memory of those learned bodies which cultivate that social science which is so greatly beholden to him as a founder, guide, and expositor.

CORRESPONDENCE.

ON MR. SAMUEL YOUNGER'S PLAN FOR THE ASSURANCE OF INVALID LIVES.

To the Editor of the Assurance Magazine.

SIR,—I have been for some time expecting to see a notice in the *Assurance Magazine* of the remarkable plan for the assurance of invalid

lives, suggested by Mr. Morrice Black, of the London and Yorkshire Office, and I have read with interest the paper of my friend, Mr. Younger, in the Number just issued. It appears to me, however, that there is a fundamental error of principle which renders both of the proposed plans unsound, and which the palliative considerations set forth by Mr. Younger do not affect.

The payment for a contingency ought never to depend on the issue of the event. An ordinary assurance on a healthy life, it has often been pointed out, is a transaction in the nature of a wager—morally harmless, it is true, but still a wager upon the happening of an event unknown to either party. If the life survive the year of payment of the premium, the Company stands to win; if it die, the Company will lose: and as the latter event is antecedently less probable, the Company gives odds to the assured. But as soon as the year has expired the transaction is closed: the assured cannot say, "The life has survived, return me my money;" nor the Company, if the death has happened, "We will pay you back what we have received from you, and no more." It is true the Company has entered simultaneously upon many other similar transactions, and expects to be neither a loser nor a gainer on the whole extent of them; but that is its own risk, not the risk of the assured. The proportion of the premium to the sum assured—that is, the amount of the odds given by the Company—is settled according to what are erroneously termed the "laws" of mortality, which are really only the results of past experience as extensive as we can collect. Those "laws" promise no certainty of life equivalent to the odds given, but merely enable the Company to say at what rate they can afford to run the risk.

An assurance upon an invalid life differs in no respect from an ordinary assurance, except that the rate of odds the Company can afford to give is matter of more anxious calculation. Whatever that rate may be, when fixed upon an antecedent investigation into the case, it must not be disturbed when the contingency is at an end. If, upon an individual case, or any number of cases, the Company find the transaction result in a loss, they must abide it; if they have made profit, the profit is legitimate, having regard to the antecedently equivalent risk of loss. There cannot be an "error" or "mistake," for subsequent remedy, in the estimate put originally upon the risk.

It is a second fallacy, I conceive, to "assume that the fact of a person, whose life was considered an extra-hazardous one, living to the end of the 'expectation' period, is complete evidence that the medical opinion was wrong, and that no additional premium was necessary." Reflection will show that the "expectation of life" means nothing whatever. It is merely an arithmetical result. It does not import that any one person, or any number of persons, will live that term of years; but only that, if you add the years lifetime of a number of persons together, and divide by the number of lives observed, you obtain a given result. If some die n years earlier, others will die n years later. Those who live beyond have not really "exceeded their expectation;" if it were so, all would live till the last age in the tables, for the "expected" age at death increases with every year of life completed. The "expectation" is not an element in the premium to be charged, nor is it a measure of time available for any purpose whatever.

By way of illustration, take the case of a person aged 30, as set forth in

Mr. Younger's example, to whose age the medical adviser recommends an addition of 10 years to be made for the purpose of calculating the premium to be charged—thus removing him from a class whose supposed "expectation" is 34 years, to one whose supposed "expectation" is 28 years. Now, even if it were admitted that the payment should abide the issue of the event, that issue would not arise at the end of 28 years, or at the end of 34 years, as suggested by Mr. Younger, but at the end of such a period as would make the average duration of the assurance for all who have been exposed to the risk, including those who have died previously to the supposed expectation, not less than 28 years.

I see that Mr. Younger admits he has "departed from theory;" but I think that the experience of all Offices which have entertained invalid assurance would show that such departures on the wrong side are very dangerous. It is a most hazardous class of business, and the Company requires every practical and theoretical barrier against loss that can be devised. Every plan which charges a debt upon the policy, to be wiped off at a future day if the death has not intervened, whether the debt be estimated upon the careful method of Mr. Younger, or by the more empirical modes adopted by the Offices which advertise the system, deprives the Company of the chance of profit which alone can compensate them for the concurrent chance of loss; and enables the assured in effect to say to the Company, "Heads, I win; tails, you lose." Indeed, any debt whatever upon a policy is unsound, and subversive of the principles of assurance.

It may be worth while to add, as collateral to the subject, that what has been said does not affect the question of returning a bonus to the assured out of the resulting profit of a series of transactions, if such profit has arisen from prudent investment, gain of interest beyond that involved in the tables, economy in expenditure, or other like causes. But bonus ought never to be divided out of profit supposed to arise from a more favourable past experience than that for which the rates of premium have provided; and those Companies who boast that their claims have been less than the expected amount, unless they retain in their hands the money so saved, are congratulating themselves upon what is really a presage of future disaster, as far as the policies under observation are concerned. For the deaths that have been hitherto avoided are yet to happen, inasmuch as every death must occur at some time; and the fact that, in the early years of a Company, or of a series of policies, the mortality has been less than the tables would have allowed for, leads to the inference that in future years (as regards those policies) the deaths will exceed the expected amount to the same extent. And this, which theory leads us to anticipate, experience shows to be the actual result; for the advantage of medical selection ceases in a very few years, and the self-selection of the assured against the Company overbalances it—the good lives dropping their policies upon slight temptation, but the impaired lives keeping theirs in force at all risks, knowing them to be a property which, once lost, cannot be replaced. Reserve must, therefore, be made, not only for the expected mortality according to the "law," but also for the probability that the real future mortality will exceed it.

These may be "old-school" views; but I am inclined to think that the avidity of policy-holders for bonus, and the extraordinary measures of advertising now adopted, are evil results of the competition of the last few

years, and that it should be the part of your *Magazine* to maintain soundly conservative doctrines in life assurance. I crave your pardon for troubling you with these remarks, not having the honour of belonging to the Institute which your *Journal* represents with so much ability; but I believe you do not consider that qualification essential to correspondence with you.

I am, Sir,

Your obedient servant,

27, *Regent Street,*
October, 1862.

EDWARD W. BRABROOK, F.S.A.

P.S.—I add one line, to congratulate Mr. Younger and the Institute on his election as a Fellow; though we happen to differ on this occasion, a long acquaintance with him has led me to hold him and his merits in very high regard.

E. W. B.

ON MR. SAMUEL YOUNGER'S PLAN FOR THE ASSURANCE OF INVALID LIVES.

To the Editor of the Assurance Magazine.

SIR,—Will you allow me space for a few remarks on Mr. Younger's paper, in your last Number, on the assurance of diseased lives, as I think that gentleman rather needlessly complicates a comparatively simple matter.

A person desirous of making a specific annual payment or investment for the assurance of his life, has, obviously, various alternatives to choose from. He may assure the same sum over the whole period of life—a larger sum for but a part of his life—or a sum increasing by a certain ratio, as in most ordinary "Bonus" assurances. He may stipulate that the risk of the Office shall only commence a fixed number of years hence, as in a printed table now before me; or, as in the case detailed by Mr. Younger, it may be an understanding that the risk of the Office shall be for one (smaller) sum during an earlier period of the assurance, and for a larger sum thereafter—the *fixed* terms being, the amount of premium to be paid, the larger sum for which the Office is to be responsible, and the period when the larger risk is to commence; the *variable* term being the amount of risk during the earlier period. Of course, such a system can be carried out only within certain limits, and the amount of the smaller risk is obtained by the simple equation—

$$S = \frac{AM_x - \pi N_{x-1}}{M_x - M_{x+n}},$$

where S is the smaller amount required, π the premium agreed to be paid, reduced by its proportion of Office loading or commission, A the larger assurance agreed on, and n the number of years for which the risk of it is deferred.

By this we may work out the first example given by Mr. Younger, where $x=35$ (the *assumed* age of the diseased life), $n=34$, $A=100$, and $\pi=1{\cdot}7554$, the net annual premium by the Carlisle 4 per cent. table for a life assurance of £100, at age of 30. This premium is perfectly arbitrary,

and any other might have been taken for illustration—the result gives £82·4536 for the smaller sum to be assured, or £17·5464 less than the £100 to be finally covered. Mr. Younger proposes to deduct only £6. 12*s*.

The method has lately been made applicable to the assurance of diseased lives; the benefit being calculated at the increased age of the assured assumed by the Office, and the premium payable being that applicable to his real age. Certainly, more has been made of it than is warrantable, as it has been asserted to be a means of correction of any erroneous estimate of the deterioration of the life, although that estimate is strictly adhered to in the equation.

Mr. Younger now, however, proposes to deal in another way with the latter problem. Reasoning, from an assumed tendency of the medical officer to overrate a special risk, and from other causes, he thinks to correct the error by making his computation at the real age of the assured, deducting from the benefit, during the earlier period of the assurance, such an amount as is represented by the extra premium arbitrarily imposed by the exigencies of the case. The propriety of this view is, to say the least, very doubtful; but without stopping to discuss it, you will see that, assuming the extra premium payable for the whole of life, the deduction is to be calculated from the expression—

$$\frac{\pi N_{x-1}}{M_x - M_{x+n}},$$

π being the extra premium imposed; and dealing with Mr. Younger's first example, where $x = 30$, $n = 34$, and $\pi = \cdot 35833$, the result gives the equivalent short assurance (by Carlisle 4 per cent.) to be £28·9808.

I am not very sure, however, of Mr. Younger's meaning, when he "suggests that the extra premium proposed to be charged should be left unpaid until the expiration of the period termed the expectation of life;" but assuming that his proposal is, to require payment from the assured, not for the whole of life, as recommended by his medical adviser, but for that term only during which the deduction from the sum assured is to be in force, and which he fixes at the tabular "Expectation," then, abiding by his first example, and the Carlisle 4 per cent. data, the present value of the 34 years' assurance, which he is to deduct from the benefit under the policy in question, is—

$$\frac{6 \cdot 6 (M_{30} - M_{64})}{D_{30}} = 1 \cdot 45684,$$

while the present value of but the first 34 payments of the premium, which he is to forego in respect of this risk, is—

$$\frac{\cdot 358333 (N_{29} - N_{63})}{D_{30}} = 5 \cdot 89234,$$

or upwards of four times the first amount.

Such a discrepancy invites an inquiry into Mr. Younger's investigation, but I confess my inability to follow his reasoning. Assuming the truth of his own expression (1), as a correct estimate of the present value of the annuity forborne, he should, surely, have equated it, not with an assurance payable at death, but with one payable should the life fall within *e* years,

the early period he deals with; but I must leave it to himself, or any other of your contributors who will kindly give us some farther information on the point.

I am, Sir,

Your most obedient servant,

H. AMBROSE SMITH.

Aberdeen, 24th November, 1862.

ON MR. YOUNGER'S SCHEME FOR THE ASSURANCE OF DETERIORATED LIVES.

To the Editor of the Assurance Magazine.

SIR,—My attention has been directed to Mr. Younger's paper in your last Number (p. 268), on a scheme for the assurance of diseased and deteriorated lives, and I shall be glad to be allowed to make a few remarks on it.

The usual mode of dealing with deteriorated lives is to treat them as if so many years older than their actual age, the effect being the imposition of an additional amount of premium, which may or may not be commuted into a reversionary deduction. Mr. Younger is dissatisfied with this mode of treatment, and he propounds a scheme of his own, with a view to obviate his objections to that at present in use.

Mr. Younger's main position is, that the decision of "the medical officer [which regulates the number of years to be added to the age of the life, and, consequently, the amount of the extra premium], if incorrect, is far more likely to be in favour of the Company than otherwise"; and upon this, as a foundation, he erects his superstructure. Retaining the extra premium, imposed in accordance with the medical report, he proposes:—

1. That this premium shall be payable, not during the whole of the after-lifetime, but only during a portion of it.

2. That it shall not be payable even during the term in question, unless the assured die during the said term. And,

3. Commuting the premium thus curtailed and rendered contingent into a *whole-life* reversionary deduction, he proposes, finally, to restrict the duration of this deduction to the term already referred to.

Add to all this that Mr. Younger, in his valuations and commutation, uses the *real*, and not the *increased* age, and it will be strongly surmised that the concessions enumerated are vastly more than sufficient to meet a presumed *likelihood* that the medical report, *if incorrect*, errs in favour of the Office.

Let us examine a particular case. I take at random the first given by Mr. Younger—a life of 30, which is estimated by the medical officer as deteriorated to the extent of five years, and on which, accordingly, an additional premium of 7*s.* 2*d.*=·358333 per cent. is imposed. The present value of this premium (Carlisle 4 per cent.), as at age 35, is 6·1065. Mr. Younger, for this premium, substitutes a reversionary deduction of £6. 12*s.*=6·600, during the first 34 years—the present value of which is 1·7086. If, with Mr. Younger, we use the real age, the values come out 6·3970 and 1·4568 respectively, exhibiting a still greater disparity. The

medical report is thus, in fact, all but nullified; and it is hence easy to predict the consequences to an Office which should adopt Mr. Younger's scheme. Mr. Younger can hardly have instituted such a comparison as the above previous to propounding his scheme. If he had, I cannot believe that he would have considered the striking off of 77 per cent. from the penalty for deterioration, *all round*, the right thing to do in compensation of a *possible* error in a few cases, in favour of the Office.

The problem implied in Mr. Younger's scheme possesses interest as involving an allocation—I cannot call it payment—of premium which has not been heretofore considered. For that reason, therefore (as a useful exercise), and for others besides, I here give a solution of it.

PROBLEM.

A temporary premium, π, to last t years, is exigible on (x), but is to be payable only if the life fail during the said term of t years. The *periodical* payment of such a premium being impracticable, it is to be commuted into a temporary reversionary deduction, V, for the same term of t years, from a whole life assurance on (x), which assurance may be one either already subsisting, or to be now constituted. Required V.

Solution.—A premium exigible as described, as a little consideration will serve to show, is the same thing, so far as pecuniary result is concerned, as the same premium, π, payable during t years, and subject to return, with compound interest, at the end of the term, if (x) shall be then alive.

The present value of the temporary premium is

$$\frac{\pi N_{x-1|t}}{D_x},$$

and the return being $\pi(1+r)(A)_t$, where $(A)_t$ is the tabular *amount* of an annuity certain of £1 for t years (the factor $1+r$ adapting it to the case of payment in advance), its present value is

$$\frac{\pi(1+r)(A)_t D_{x+t}}{D_x}, \text{ or } \frac{\pi[(A)_{t+1}-1]D_{x+t}}{D_x}.$$

Hence the value, subject to the return, is

$$\frac{\pi\{N_{x-1|t}-[(A)_{t+1}-1]D_{x+t}\}}{D_x}.$$

Equating this to

$$\frac{VM_{x|t}}{D_x},$$

the present value of the deduction, we get

$$V=\frac{\pi\{N_{x-1|t}-[(A)_{t+1}-1]D_{x+t}\}}{M_{x|t}}.$$

Applying this to the foregoing example, where $x=30$, $t=34$, and $\pi=\cdot358333$, we find for V, 9·8784. Mr. Younger has 6·600, the difference arising from his having, as already stated, determined the deduction as if it were to last for life, and *then* arbitrarily restricted its duration to 34 years.

Writing the above expression for the value of the premium, thus,

$$\frac{\pi\{N_{x-1}-[N_{x+t-1}+\overline{(A)_{t+1}-1}\,.\,D_{x+t}]\}}{D_x},$$

we see at once the nature and the effect of Mr. Younger's dealing with the medical report. The first portion, $\pi N_{x-1} \div D_x$, is the value of the premium arising from the medical officer's estimation of the risk, equal (if, with Mr. Younger, we erroneously used the *real* age) to 6·3970; and the remaining portion, $\pi\{N_{x+t-1}+[(A)_{t+1}-1]D_{x+t}\} \div D_x$, is the value of the portion of this premium that Mr. Younger abandons, equal to 4·3269. And this is irrespective of his subsequent curtailment of the duration of V.

Mr. Younger's expression for the value of the premium is reducible to that given above. It is needless to occupy space by showing it.*

It may interest some of the readers of the Magazine if I add a solution of the problem according to the old, and now almost disused, method.

That a premium will become due in respect of the nth year will depend on the concurrence of these two events; first, that (x) enters on that year, the probability of which is, $p_{x.n-1}$; and, secondly, that $(x+n-1)$ will not attain the age $x+t$, the probability of which is, $1-p_{x+n-1.t-n+1}$. The probability of the compound event therefore is,

$$p_{x.n-1}(1-p_{x+n-1.t-n+1})=p_{x.n-1}-p_{x.t}.$$

Multiplying by πv^{n-1} (the payments being due at the *commencement* of the year), and summing from 1 to t, we obtain for the value of the premium,

$$\pi\left(1+a_{x.t-1\rceil}-p_{x.t}\frac{1-v^t}{1-v}\right),$$

which may be put in either of the forms,

$$\pi\{1+a_{x.t-1\rceil}-p_{x.t}(1+r)(a)_t\},$$

$$\text{or } \pi\{1+a_{x.t-1\rceil}-p_{x.t}[(a)_{t-1}+1]\},$$

where $(a)_t$ and $(a)_{t-1}$ denote the tabular *present values* of annuities certain of £1 for t and $t-1$ years respectively.

To return for a moment to the scheme: Mr. Younger assumes, for the purposes of his investigation, that the incorrectness of the medical report is established, if the life reported on live over the term of years that he calls its "expectation." Mr. Younger knows theoretically that this is not true, for he admits as much. He nevertheless uses the hypothesis as if it were true, and we have seen the result. But I go a great deal further than a simple denial of the hypothesis, and I say that it is maintainable that no amount of *survivance* on the part of a particular life, although extending to the utmost limit of human existence, will suffice to prove that there was not, *at the date of the medical report*, sufficient ground for the relegation of that life to a class of a greater age than the age assigned by the date of its birth. If, in consequence of increased care (which is frequently engendered by delicacy of constitution), or from any other cause, the life in question should, contrary to anticipation, attain a good old age, the case must be

* Mr. Younger, like some other writers, denies himself the use of the very convenient symbol v, equivalent to $1+(1+r)$, or $(1+r)^{-1}$. In consequence, his expressions are more cumbrous than they need have been.

considered as just one of those, the decision of which in favour of the Office, enables it to meet the claims arising on account of those that are decided against it. Having stood the risk of an adverse decision, the Office must not be called upon to surrender the consideration in respect of which it undertook that risk.

Mr. Younger intimates that one object he proposed to himself in the devising of his scheme was to exhibit a less value of the measure of deterioration than that exhibited by the existing method. The simplest way of doing this would have been to strike off a percentage from the result of the usual method, which way, moreover, would have had the further advantage of letting us know exactly what we were about. This, Mr. Younger's way of proceeding does not do. It is too complex for that. In fact, it is apt to remind one of the proceedings of the scientific tailors of Laputa, who, disdaining the use of a tape for the measuring of their customers, employed a sextant instead. The customers were, to be sure, very badly fitted. But what of that? The process was conducted on strictly scientific principles.

I am, Sir,

Yours most obediently,

Camden Town, 3rd December, 1862. P. GRAY.

PROFESSOR DE MORGAN'S QUERY ABOUT INTEREST ACCOUNTS.

To the Editor of the Assurance Magazine.

SIR,—Under the head of "Notes and Queries," in your *Magazine* for October, I find a notice by Professor De Morgan of a mismanaged interest account.

The Professor does not state the method his friend followed; so with your permission I shall endeavour to point out what has to be considered in making up an interest account of the nature described, on equitable principles—the course the debtor most likely followed—and the errors he fell into.

When money is lent at a certain rate of interest, no dates for the payment of such interest being mentioned, it is understood to be paid once a year; and if interest falls in arrear, and no penalty has been mentioned in the agreement, the least that can in equity be expected of the debtor is that he pay interest at the same rate on the arrears.

In framing an ordinary account current it is usual to calculate the interest on each Dr. and Cr. balance for the time it exists (within a year), keeping a note of the Dr. and Cr. interest, and to add or deduct, as the case may be, the difference at the end of each year. If interest be charged and allowed at the same rate, this method is the same as charging interest on each advance, and allowing interest on each payment, from the date it is made to the end of the year. A new accounting then commences, and the process is carried on from year to year during the continuance of the account. If there is but one payment made in each year, and that on the day the interest falls due, this process becomes similar to that described by Professor De Morgan—interest for the period is added to the principal, and the payment just made deducted; and this is the proper plan, whether the payment exceeds the interest or not. It would thus appear that the

balance of interest arising from the transactions of the year or other period agreed on should be added or deducted only at the end of such period. If the balance of interest is against the debtor, it then becomes part of the sum on which the next year's interest is calculated, unless paid by him, when he would lose the use of his money while the creditor gained it. In either case he pays *compound interest.*

Now the majority of accounts are made up once a year with a view to a settlement, or for reasons quite independent of the interest: so that many who are accustomed to make them up quite overlook the effect this has on the interest, if they ever think about it at all. When such a person is instructed to make up an account such as that described by the Professor, he would naturally proceed in the way he was accustomed to, quite overlooking the necessary balance at the end of each period at which interest *fell due*, balancing only at the end of the account, and thus charging merely simple interest; and the difference between this method and compounding interest yearly for a term of years could not fail to produce a startling result.

If the payments had been made regularly, the Professor's method of accounting was strictly correct; but if, as he states, they were irregular, it follows from what has been said that he should have balanced at the end of each year, not at the date of each irregular payment.

I think it was the first of these errors, that of charging simple interest only, that the debtor fell into when accounting with his friend and creditor; and afterwards, from not properly understanding Professor De Morgan's method, he applied it to an account on which there were several operations in a year, compounding interest at the date of each operation; or else, falling into the second error, he did not balance at the proper time, which may account for the solicitor requesting him to adhere to his original plan.

There appears to be a misprint in the debtor's reply to the Professor's question, as it is evident that whatever method gave him as a debtor *less* to pay would confer a similar benefit on any other debtor.

There are, I understand, certain accounts, as that between an agent who acts as a *quasi* banker and his client, which must be rendered before interest can be legally compounded. It is held that, until the account is rendered, the client is ignorant of its state, and is deprived of the power of paying the balance should it be against him; while, on the other hand, if the account is rendered and he does not pay the balance, which includes interest, it is held that he consents to the compounding. These considerations can have no weight in such a case as that described by the Professor, but it is just possible that the solicitor who objected to the compounding of interest was influenced by the practice arising from them, such practice being to allow simple interest only to the date at which the account is rendered.

I fear I have already trespassed on your space, but enclose a few practical illustrations of the subject, which you can insert if you think proper.

I am, Sir,
Your most obedient servant,

Edinburgh, Nov. 15th, 1862. A. H. T.

ILLUSTRATIONS.

1. *Simple Interest.*—A borrows £100 for 5 years, to be repaid at the end of that period, with interest at the rate of 5 per cent. per annum.

At the end of the 5 years he pays the principal, with £25 of interest; in all, £125.

2. *Compound Interest.*—B borrows £100 for 5 years, agreeing to pay interest *yearly*, at £5 per cent. per annum.

The interest which B pays is nominally the same as that paid by A (No. 1), viz., £25, but it is paid yearly, by which arrangement B loses the interest on the

1st year's interest for 4 years = . . .	£1	0	0
2nd " for 3 " = . . .	0	15	0
3rd " for 2 " = . . .	0	10	0
4th " for 1 " = . . .	0	5	0
In all,	£2	10	0

And his payments are equal to £127. 10*s.*, viz.,—

Principal	£100	0	0
Interest, £25 and £2. 10*s.* . . .	27	10	0
	£127	10	0

3. If, instead of repaying the principal in one sum at the end of the 5 years, the debtor pays £50 at the end of the second year, £25 at the end of the third year, and the remaining £25 at the end of the fifth year.

1st. If *simple* interest were payable the account would be stated thus:—

1855.	Jan. 1.	To sum lent		£100	0	0
1857.	"	To interest on £100—2 years,	£10 0 0			
"	"	By cash then received		50	0	0
		Balance . . .		£50	0	0
1858.	"	To interest on £50—1 year,	£2 10 0			
"	"	By cash then received		25	0	0
		Balance . . .		£25	0	0
1860.	"	To interest on £25—2 years,	£2 10 0			
		Add interest . . .		15	0	0
		Balance, principal and interest .		£40	0	0
"	"	By cash then received		40	0	0

The debtor thus pays—

Principal				£100	0	0
And interest, 5 years, on £100 . .		£25	0	0		
Less on £50 for 3 years .	£7 10 0					
" on £25 for 2 years .	2 10 0					
		10	0	0		
				15	0	0
				£115	0	0

2nd. If the interest were *compounded annually* the account would be as follows:—

1855. Jan. 1.	To sum lent	£100	0	0
1856. „	To interest for 1 year	5	0	0
		£105	0	0
1857. „	To interest, 1 year, on £105	5	5	0
		£110	5	0
„ „	By cash then received	50	0	0
	Balance	£60	5	0
1858. „	To interest, 1 year, on £60. 5*s.* . . .	3	0	3
		£63	5	3
„ „	By cash then received	25	0	0
	Balance	£38	5	3
1859. „	To interest, 1 year, on £38. 5*s.* 3*d.* . . .	1	18	3
		£40	3	6
1860. „	To interest, 1 year, on £40. 3*s.* 6*d.* . . .	2	0	2
		£42	3	8
„ „	By cash then received	42	3	8
The debtor thus pays £50, £25, and £42. 3*s.* 8*d.*= .		£117	3	8
Of which principal		100	0	0
The balance being interest		£17	3	8
Consisting of simple interest .	£15 0 0			
Interest on interest	2 3 8			
		17	3	8

In the above example interest is accumulated each year, but not paid. If it were paid each year the result would be the same—the debtor would lose £2. 3*s.* 8*d.* of interest on the interest so paid.

INSTITUTE OF ACTUARIES.

PROCEEDINGS OF THE INSTITUTE.

First Ordinary Meeting, Session 1862-63.—*Monday, 24th November,* 1862.

The President in the Chair.

The minutes of the annual general meeting were read and confirmed.

The Secretary announced various donations to the library.

Messrs. J. W. Holland and D. H. McGregor, duly nominated at the last ordinary meeting, were unanimously elected Associates of the Institute.

The following gentlemen were nominated for ballot at the next ordinary meeting:—

Official Associates.

J. H. Evens.	W. T. Hancock.

Associates.

Francis Addiscott.	Lieut.-Col. Oakes.
Benjamin Cunningham.	Chas. H. Ogbourne.
Wm. Hughes.	Henry Parminter.
T. H. Johnson.	Arthur Smither.
A. E. Middleton.	E. Waterhouse, B.A.

Mr. Tucker read a paper "On the proper mode of estimating the liabilities of Life Insurance Companies."

Thanks were voted to Mr. Tucker, and the meeting adjourned to Monday, 29th December, 1862.

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END OF VOL. X.

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THE

ASSURANCE MAGAZINE,

AND

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

GENERAL INDEX

TO THE

FIRST TEN VOLUMES.

LONDON:
CHARLES & EDWIN LAYTON, 150, FLEET STREET.
1864.

LONDON:
CHARLES AND EDWIN LAYTON,
150, FLEET STREET.

PREFACE.

THE great bulk to which the Journal of the Institute has grown, and the number and variety of the matters discussed in it, have impressed upon the minds of the Council of the Institute the desirableness of providing for the use of the Members and of readers generally, such an Index of the Contents as will enable them to refer, without needless trouble or loss of time, to whatever passage in it they may desire to consult.

After due consideration, they determined on having an Index compiled for the first ten volumes of the Journal, and after inquiries as to whom they could best entrust the conduct of the work, they were fortunate enough to secure for it the services of Mr. John Nicholson, of Lincoln's Inn Library, who had had a good deal of experience in such compilations, and by the skill and ability displayed in them had shown himself well fitted for the task.

By this gentleman the Index has been constructed on the following plan:—the name of the author being known, the title of his paper and an abstract of its contents have been placed under his name; the name not being known, the title of the paper and the abstract are found under the heading of the principal subject treated of. The more important matters set forth in the abstracts also appear in their order in the general alphabetical arrangement, and under this arrangement have been placed in addition, references more or less frequent to every statement or remark of the least interest or importance.

It has been of course necessary to make use of some varieties of type in the printing of this work, but it has been thought desirable to introduce only such as would conduce to a due degree of symmetry, and to a clearer and more ready apprehension of its plan. Thus:—

The alphabetically arranged headings are in Small Capitals.

The entire Titles of Papers, and the general body of the Index, are in Brevier Type.

The Abstracts of Papers are in Nonpareil.

The Counter References are made in Small Capitals, except those occurring in very comprehensive headings, such for instance as "Correspondence." In such, these references will be found to be in Brevier Type, it being intended thereby to indicate that the references are made to the alphabetical arrangement under the heading, and not to that of the whole volume.

A very limited use has been made of Italics; they have been employed in conjunction with the Counter References, and also where it has been desired to make an expression or sentence more conspicuous. Where information is supplied for the first time, parentheses have been introduced, to indicate that such is the case.

It will probably be observed that the references to particular subjects are sometimes very frequent, and it may be thought by some unnecessarily so; Mr. Nicholson's desire has been, however, to provide as ready means as possible of extracting such information on any given point as the Journal affords, and we doubt not that his labours will be duly appreciated by all who have occasion to consult the pages of the work he has so well and so thoroughly analysed.

Ed. A. M.

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